# From k-essence to generalised Galileons and beyond…

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*From k-essence to generalised Galileons,*
arXiv:1103.3260v1 [hep-th]

# Outline

- Motivation and examples
- Main result
- Proof
    - Lorentz invariance
    - Second order EOM
    - Curved space-time
- Summary and future directions

# Motivation

- Goal: finding consistent modifications of gravity on large scales (e.g. self-accelerating cosmologies) that satisfy solar system tests (e.g. Vainshtein mechansim)
- Scalar-Tensor modifications of GR with derivative interactions of the scalar field might do the trick!
- Higher derivative equations of motion lead to *ghosts*
- We need theories with second order equations of motion
  - k-essence
  - Galileon
  - … ?

# Example: the Galileon (I)

- General expression (in flat space-time)

$$L_{Gal}^{n} \equiv \frac{1}{(D-n)!} \pi^{\mu} \pi_{\mu} \varepsilon^{\mu_1 \cdots \mu_n \rho_1 \cdots \rho_{D-n}} \varepsilon^{\nu_1 \cdots \nu_n}{}_{\rho_1 \cdots \rho_{D-n}} \pi_{\mu_1 \nu_1} \cdots \pi_{\mu_n \nu_n}$$

$$= -\pi^{\mu} \pi_{\mu} \sum_{\sigma \in S_n} \epsilon(\sigma) \pi_{\mu_1}{}^{\mu_{\sigma(1)}} \cdots \pi_{\mu_n}{}^{\mu_{\sigma(n)}} \; (\equiv 0 \text{ for } n > D)$$

- Equation of motion of order (exactly) 2

$$E_{Gal}^{n} \equiv \underbrace{\sum_{\sigma \in S_{n+1}} \epsilon(\sigma) \pi_{\mu_1}{}^{\mu_{\sigma(1)}} \cdots \pi_{\mu_{n+1}}{}^{\mu_{\sigma(n+1)}}}_{\equiv 0 \text{ for } n > D-1} = 0$$

where

$$\pi_{\alpha} \equiv \partial_{\alpha} \pi$$
$$\pi_{\alpha\beta} \equiv \partial_{\alpha} \partial_{\beta} \pi$$

A. Nicolis, R. Rattazzi, E. Trincherini
*The Galileon as a local modification of gravity*
arXiv:0811.2197v2 [hep-th]

# Example: the Galileon (II)

- Other form of the Galileon terms

$$\tilde{L}^n_{Gal} \equiv \frac{1}{(D-n)!}\pi^{\lambda}\pi_{\mu_1\lambda}\pi_{\nu_1}\varepsilon^{\mu_1\cdots\mu_n\rho_1\cdots\rho_{D-n}}\varepsilon^{\nu_1\cdots\nu_n}{}_{\rho_1\cdots\rho_{D-n}}\pi_{\mu_2\nu_2}\cdots\pi_{\mu_n\nu_n}$$

$$= -\sum_{\sigma\in S_n}\epsilon(\sigma)\,\pi^{\lambda}\pi_{\mu_1\lambda}\pi^{\mu_{\sigma(1)}}\pi_{\mu_2}{}^{\mu_{\sigma(2)}}\cdots\pi_{\mu_n}{}^{\mu_{\sigma(n)}}\;\left(\equiv 0 \text{ for } n > D\right)$$

- Equivalent up to a total divergence

- Not the only two ways of writing the Galileon Lagrangian

# Assumptions and result

- Flat space-time
- Most general scalar theory such that
  i. its Lagrangian contains derivatives of order 2 or less of the scalar field π
  ii. its Lagrangian is polynomial in the second derivatives of π
  iii. the corresponding field equations are of order 2 or lower in derivatives
- Main result $\boxed{L = \sum_{n=0}^{D-1} f_n(\pi, X) L^n_{Gal}}$ where $X \equiv \partial_\mu \pi \partial^\mu \pi$

# Lorentz invariance (I)

- How do we build a Lagrangian obeying conditions i. and ii. ?
- Lorentz invariant terms involving first and second derivatives of π

$$\pi_\mu \pi^\mu \equiv X$$

$$\pi_{\mu_1}\pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2} \equiv \langle 1 \rangle$$

$$\pi_{\mu_1}\pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2}{}_{\mu_3}\pi^{\mu_3} \equiv \langle 2 \rangle$$

$$\vdots$$

$$\pi_{\mu_1} \underbrace{\pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2}{}_{\mu_3}\cdots\pi^{\mu_{i-1}}{}_{\mu_i}\pi^{\mu_i}{}_{\mu_{i+1}}}_{i \quad \text{second derivatives}} \pi^{\mu_{i+1}} \equiv \langle i \rangle$$

$$\pi^{\mu_1}{}_{\mu_1} = \Box\pi \equiv [1]$$

$$\pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2}{}_{\mu_1} \equiv [2]$$

$$\vdots$$

$$\underbrace{\pi^{\mu_1}{}_{\mu_2}\pi^{\mu_2}{}_{\mu_3}\cdots\pi^{\mu_{i-1}}{}_{\mu_i}\pi^{\mu_i}{}_{\mu_1}}_{i \quad \text{second derivatives}} \equiv [i]$$

# Lorentz invariance (II)

- General Lagrangian $$L = \sum_n \sum_{\{p_i, q_j\}} L^n_{\{p_i, q_j\}}$$

where $$\boxed{L^n_{\{p_i, q_j\}} = f^n_{\{p_i, q_j\}}(\pi, X) \prod_{i=1}^{n} [i]^{p_i} \prod_{j=1}^{n} \langle j \rangle^{q_j}}$$

and $\sum_i ip_i + \sum_j jq_j = n \equiv$ number of twice differentiated $\pi$

- What about condition iii. ?

# Second order EOM (I)

- Generic term

$$L_{gen} \equiv f^n_{\{p_i, q_j\}}(\pi, X)\,(\Box\pi)^{p_1}\,[2]^{p_2}\cdots[i-1]^{p_{i-1}}\,[i]^{p_i}\,[i+1]^{p_{i+1}}\cdots$$

$$\times\,\langle 1\rangle^{q_1}\cdots\langle j-1\rangle^{q_{j-1}}\,\langle j\rangle^{q_j}\,\langle j+1\rangle^{q_{j+1}}\cdots$$

- Variation with respect to π

$$\delta L_{gen} \supset f^n_{\{p_i, q_j\}}(\pi, X)\,(\Box\pi)^{p_1}\,[2]^{p_2}\cdots[i-1]^{p_{i-1}}\,\delta[i][i]^{p_i-1}\,[i+1]^{p_{i+1}}\cdots$$

$$= f^n_{\{p_i, q_j\}}(\pi, X)\,(\Box\pi)^{p_1}\,[2]^{p_2}\cdots[i-1]^{p_{i-1}}\,\Box[i-1]\delta\pi[i]^{p_i-1}\,[i+1]^{p_{i+1}}\cdots$$

- Fourth derivatives cancelled by varying

$$L'_{gen} \propto f^n_{\{p_i, q_j\}}(\pi, X)\,(\Box\pi)^{p_1+1}\,[2]^{p_2}\cdots[i-1]^{p_{i-1}+1}\,[i]^{p_i-1}\,[i+1]^{p_{i+1}}\cdots$$

# Second order EOM (II)

- But also

$$\delta L_{gen} \supset f^n_{\{p_i,q_j\}}(\pi,X)\cdots\langle 1\rangle^{q_1}\cdots\langle j-1\rangle^{q_{j-1}}\,\delta\langle j\rangle\langle j\rangle^{q_j-1}\langle j+1\rangle^{q_{j+1}}\cdots$$

$$= f^n_{\{p_i,q_j\}}(\pi,X)\cdots\langle 1\rangle^{q_1}\cdots\langle j-1\rangle^{q_{j-1}}\,\Box\langle j-1\rangle\delta\pi\langle j\rangle^{q_j-1}\langle j+1\rangle^{q_{j+1}}\cdots$$

- Fourth derivatives cancelled by varying

$$L'_{gen} \propto f^n_{\{p_i,q_j\}}(\pi,X)\,(\Box\pi)^{p_1+1}\cdots\langle 1\rangle^{q_1}\cdots\langle j-1\rangle^{q_{j-1}+1}\langle j\rangle^{q_j-1}\langle j+1\rangle^{q_{j+1}}\cdots$$

- One has to add all these terms to the initial $L_{gen}$ in order for the EOM to stay second order
- Last term: $L_0 \propto f^n_{\{p_i,q_j\}}(\pi,X)\,(\Box\pi)^p\left(\pi_\mu\pi^{\mu\nu}\pi_\nu\right)^q$

$$\boxed{q=0 \quad \text{or} \quad q=1}$$

# Second order EOM (III)

- Two families of theories where the functions $f^{n}_{\{p_i, q_j\}}$ are *necessarily* related

$$L = \sum_{n} f^{n}(\pi, X) \sum_{\{p_i\}} c^{n}_{\{p_i\}} \prod_{i=1}^{n} [i]^{p_i}$$

$$\tilde{L} = \sum_{n} f^{n}(\pi, X) \sum_{\{p_i\}} \tilde{c}^{n}_{\{p_i\}} \prod_{i=1}^{n} [i]^{p_i} \left\langle n - \sum_{i=1}^{n} i p_i \right\rangle$$

where the $c^{n}_{\{p_i\}}$ $\tilde{c}^{n}_{\{p_i\}}$ are independent of the $f^{n}(\pi, X)$

- The Galileon must therefore *necessarily* be of this form…

$$L^{n}_{Gal} = X \sum_{\{p_i\}} c^{n}_{\{p_i\}} \prod_{i=1}^{n} [i]^{p_i} \quad \text{and} \quad \tilde{L}^{n}_{Gal} = \sum_{\{p_i\}} \tilde{c}^{n}_{\{p_i\}} \prod_{i=1}^{n} [i]^{p_i} \left\langle n - \sum_{i=1}^{n} i p_i \right\rangle$$

# Main result

- Only scalar theories obeying conditions i. – iii.

$$L = \sum_{n=0}^{D} f^n(\pi, X) L^n_{Gal}$$

$$\tilde{L} = \sum_{n=0}^{D} g^n(\pi, X) \tilde{L}^n_{Gal}$$

- Generalisation of:
  - k-essence
  - Galileon theories
  - kinetically braided scalars
  - ...

# Main result

- Only scalar theories obeying conditions i. – iii.

$$L = \sum_{n=0}^{D-1} f^n(\pi, X) L^n_{Gal}$$

- Generalisation of:
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# Curved space-time (I)

- Minimal covariantization leads to third derivatives of the metric in the $\pi$ EOM
e.g. $\propto R_{\mu\nu\rho\sigma;\lambda}\pi^{\lambda}$

- Idea: adding counterterms which exactly cancel the "dangerous terms" when varied e.g.

$$L_{cov} \propto \pi^{\lambda}\pi_{\lambda}R_{\mu\nu\rho\sigma}$$

- No more than second derivatives of the field and metric

# Curved space-time (II)

- Covariantization of the generic term

$$L^n \equiv f(\pi, X) L^n_{Gal}$$

- Generic form of the counterterms

$$L^{n,p} \equiv \frac{1}{(D-n)!} \varepsilon^{\mu_1 \cdots \mu_n \rho_1 \cdots \rho_{D-n}} \varepsilon^{\nu_1 \cdots \nu_n}{}_{\rho_1 \cdots \rho_{D-n}} \left( \int \cdots \int f(\pi, X_p) X_p \, dX_1 \cdots dX_p \right) \prod_{i=1}^{p} R_{\mu_{2i-1}\mu_{2i}\nu_{2i-1}\nu_{2i}} \prod_{j=0}^{n-2p-1} \pi_{\mu_{n-j}\nu_{n-j}}$$

- Covariantized form

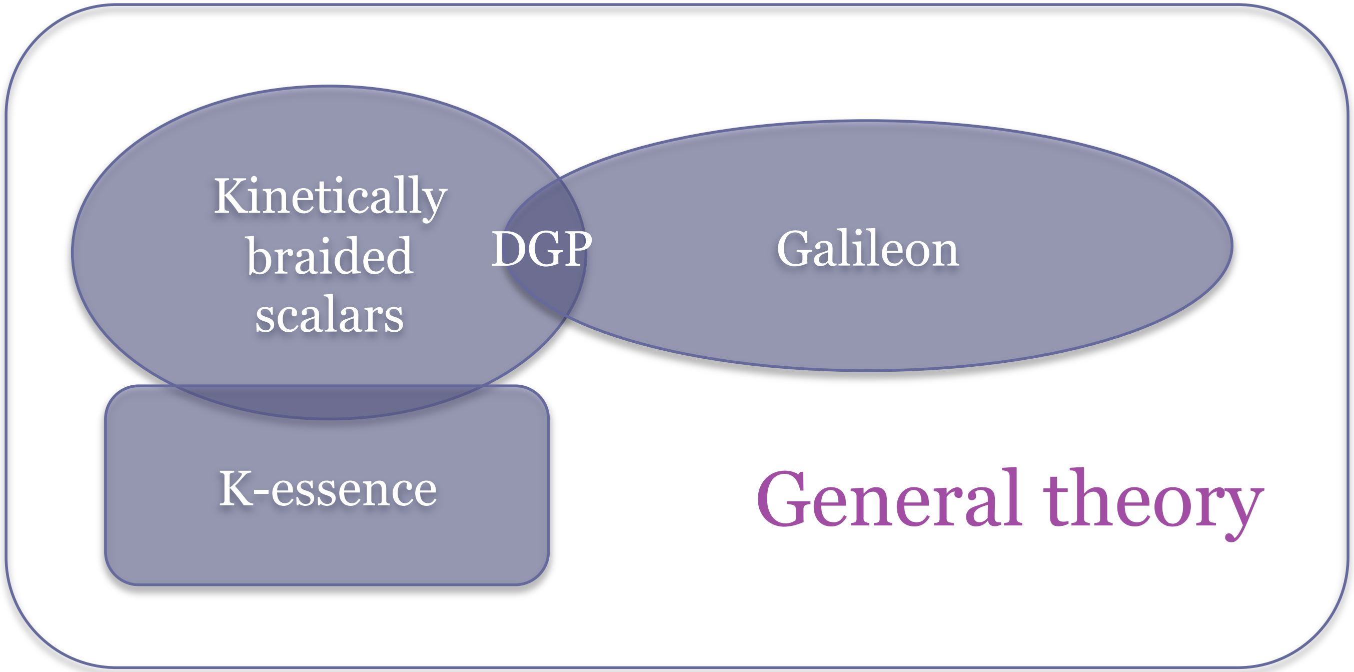
$$\boxed{L^n_{cov} \equiv \sum_{p=0}^{\lfloor n/2 \rfloor} C^{n,p} L^{n,p}}$$

# A general theory?

- Uniqueness proof in flat space **of arbitrary dimension** (under assumptions i, ii, iii) + generalisation to curved space
- Horndeski (1973) found the most general scalar tensor theory in arbitrary background **of dimension four** under the assumption that **both** the metric and field EOMs are second order
- Kobayashi et al. (2011): the covariantization of the generalised Galileon is equivalent to Horndeski's theory in 4D

➢ The two classes of theories are equivalent in 4D but they rely on different assumptions, and ours is valid in arbitrary dimension

# Summary and future directions




Valid in curved space-time

Opens up new horizons for phenomenology…