Probing Inflation with CMB Polarization

Daniel Baumann

School of Natural Sciences
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Cambridge, August 2009
“What would we learn from a B-mode detection at the $r' = 0.01$ level?”

Daniel Baumann

School of Natural Sciences
Institute for Advanced Study

Cambridge, August 2009
Today’s Talk

I will present results from discussions of the Inflation Working Group of the CMBPol Mission Concept Study

arXiv: 0811.3919

see also Scott Dodelson’s talk
Today’s Talk

I will focus on the implications of gravitational waves for the physics of inflation in the context of:

Wilsonian Effective Field Theory

String Theory

and illustrate this with concrete calculations for the example of D-brane inflation

see also talks by Silverstein, Burgess and Kaloper

this won’t be a representative review, but will be biased toward my own work with McAllister, Dymarsky, Klebanov and Kachru.
Based on

CMBPol Mission Concept Study: Probing Inflation with CMB Polarization


White Paper of the Inflation Working Group

arXiv: 0811.3919

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See also

DB and Liam McAllister

**Advances in Inflation in String Theory**

DB

**TASI Lectures on Inflation**

DB and Liam McAllister

**A Microscopic Limit on Gravitational Waves from D-brane Inflation**

Papers with McAllister, Dymarsky, Klebanov and Kachru


and work in progress.
Outline

1. Tensor Modes and the Lyth Bound

2. Inflation in EFT

3. Inflation in String Theory
Quantum Fluctuations

Any light field experiences quantum fluctuations during inflation with amplitude set by the expansion rate $H$

\[
\delta \phi \sim H \\
\delta g_{ij} \sim H
\]

\text{inflaton} \quad \text{graviton}

\text{scalar} \quad \text{tensor}
Quantum Fluctuations

tensors \( (\delta g_{ij}) \)

\[
P_t = \frac{8}{M_{pl}^2} \left( \frac{H}{2\pi} \right)^2
\]

scalars \( (\zeta) \)

\[
P_s = \left( \frac{H}{2\pi} \right)^2 \left( \frac{H}{\dot{\phi}} \right)^2
\]

\( \delta \phi \rightarrow \zeta \)
Quantum Fluctuations

tensors \((\delta g_{ij})\)

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P_s = \left( \frac{H}{2\pi} \right)^2 \left( \frac{\dot{H}}{\dot{\phi}} \right)^2
\]

\(\delta \phi \rightarrow \zeta\)

tensor-to-scalar ratio

\[
r \equiv \frac{P_t}{P_s} = 8 \left( \frac{d\phi}{d\mathcal{N}_e} \frac{1}{M_{pl}} \right)^2
\]

where

\[
d\mathcal{N}_e \equiv H dt
\]

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The Lyth Bound

tensor-to-scalar ratio

\[ r \equiv \frac{P_t}{P_s} = 8 \left( \frac{d\phi}{dN_e} \frac{1}{M_{\text{pl}}} \right)^2 \]

field evolution over 60 e-folds

\[ \frac{\Delta \phi}{M_{\text{pl}}} \approx \left( \frac{r}{0.01} \right)^{1/2} \]
The Lyth Bound

\[ \frac{\Delta \phi}{M_{\text{pl}}} \approx \left( \frac{r}{0.01} \right)^{1/2} \]

If we observe tensors it proves that the inflaton field moved over a super-Planckian distance!

\[ r > 0.01 \]

\[ \Delta \phi \gg M_{\text{pl}} \]
The Lyth Bound

\[ \frac{\Delta \phi}{M_{\text{pl}}} \approx \left( \frac{r}{0.01} \right)^{1/2} \]

In any inflationary model with \( r > 0.01 \):

Must ensure flatness of the potential

\[ \epsilon, \eta \ll 1 \]

over a super-Planckian range!

\[ \Delta \phi \gg M_{\text{pl}} \]
The Lyth Bound

i.e. we require a smooth potential over a range $\Delta \phi \gg M_{pl}$

few $\times M_{pl}$
The Lyth Bound

i.e. we require a smooth potential over a range $\Delta \phi \gg M_{pl}$

But, in an effective field theory with cutoff $\Lambda < M_{pl}$

we generically don’t expect a smooth potential over a super-Planckian range
Requirements for Observable B-modes

Defined by the target sensitivity of a future CMB Polarization satellite

\[ r > 0.01 \]

**Kinematic Requirement**

\[ \Delta \phi \gg M_{pl} \]

**Dynamical Requirement**

flat potential over a super-Planckian range
Inflation in EFT
Effective Field Theory

Inflation is often described as a low-energy effective theory valid below a scale $\Lambda > H$

- field content
- potential, kinetic terms
- interactions
- symmetries
- couplings to gravity
- etc.

The inflationary action is then *assumed* rather than *derived*.
Effective Field Theory

If we knew the *complete theory*

*i.e. the spectrum of particles and their interactions up to the Planck scale*

then we could in principle derive the effective action for the inflationary degrees of freedom:

UV-completion
e.g. string theory

low-energy EFT
If we knew the complete theory

i.e. the spectrum of particles and their interactions up to the Planck scale

then we could in principle derive the effective action for the inflationary degrees of freedom:

- integrate out heavy fields $M > \Lambda$

\[ V(\phi, \psi) \rightarrow V_{\text{eff}}(\phi) \]
Effective Field Theory

If we knew the complete theory

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then we could in principle derive the effective action for the inflationary degrees of freedom:

• integrate out heavy fields $M > \Lambda$

$$V(\phi, \psi) \rightarrow V_{\text{eff}}(\phi)$$

• effective potential receives (computable) corrections

$$\Delta V = \mathcal{O}_\delta \frac{M^{\delta-4}}{M}$$
Effective Field Theory

Typically, we **don’t** know the complete theory:

- *parameterize our ignorance* about the UV, i.e. add **all** corrections consistent with symmetries.

\[
\Delta V = \sum_{\delta} \frac{\mathcal{O}_\delta}{\Lambda^{\delta - 4}}
\]

Wilson
Effective Field Theory

Typically, we don’t know the complete theory:

- parameterize our ignorance about the UV, i.e. add all corrections consistent with symmetries.

\[ \Delta V = \sum_{\delta} \frac{O_\delta}{\Lambda^{\delta - 4}} \]

\[ \Lambda < M_{pl} \]
Effective Field Theory

Typically, we don’t know the complete theory:

• parameterize our ignorance about the UV, i.e. add all corrections consistent with symmetries.

\[ \Delta V = \sum_{\delta} \frac{\mathcal{O}_\delta}{\Lambda^{\delta-4}} \]

\[ \Lambda < M_{\text{pl}} \]

Inflation is sensitive even to couplings to Planck scale degrees of freedom.
Large-Field Inflation

UV sensitivity of inflation is especially strong in any model with observable gravitational waves
Large-Field Inflation

1. No Shift Symmetry in the UV

Effective Field Theory with Cutoff \( \Lambda < M_{\text{pl}} \)

\[
V_{\text{eff}}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \sum_{p=1}^{\infty} \lambda_p \phi^4 \left( \frac{\phi}{\Lambda} \right)^{2p}
\]
Large-Field Inflation

1. **No Shift Symmetry** in the UV

Effective Field Theory with Cutoff \( \Lambda < M_{\text{pl}} \)

\[
V_{\text{eff}}(\phi) = \frac{1}{2}m^2 \phi^2 + \frac{1}{4}\lambda \phi^4 + \sum_{p=1}^{\infty} \lambda_p \phi^4 \left( \frac{\phi}{\Lambda} \right)^{2p}
\]

we generically don’t expect a smooth potential over a super-Planckian range
Large-Field Inflation

2. **Shift Symmetry in the UV**

If the action is invariant under

\[ \phi \rightarrow \phi + \text{const.} \]

then the dangerous corrections are forbidden by symmetry

\[ V_{\text{eff}}(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \sum_{p=1}^{\infty} \lambda_p \phi^4 \left( \frac{\phi}{\Lambda} \right)^{2p} \]
Large-Field Inflation

2. **Shift Symmetry** in the UV

\[ \phi \rightarrow \phi + \text{const.} \]

With such a shift symmetry chaotic inflation is “technically natural”

\[ V_{\text{eff}}(\phi) = \frac{1}{2} m^2 \phi^2 \]

\[ \text{few} \times M_{\text{pl}} \]
Large-Field Inflation

2. *Shift Symmetry* in the UV

Seeing B-modes would show that the inflaton field respected a *shift symmetry* up to the Planck scale!
Large-Field Inflation

2. **Shift Symmetry** in the UV

Seeing B-modes would show that the inflaton field respected a *shift symmetry* up to the Planck scale!

We know fields with that property:

[axions](#)

---

1 The first controlled large-field models using axions have recently been constructed in string theory.
Inflation in String Theory
In string theory we have the complete theory so we can address these questions very explicitly.

i.e. identify a larger field content, integrate out the massive fields and compute the effective action for the remaining light fields
Example: D-brane Inflation

Dvali and Tye

D3

extra dimension

\[ r \]

D3
Example: D-brane Inflation

Dvali and Tye

extra dimension

\( r \)

- **inflaton field** = separation of spacetime-filling D3-branes (‘pointlike in the extra dimensions’)

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Example: D-brane Inflation

• Inflaton field = separation of spacetime-filling D3-branes (‘pointlike in the extra dimensions’)

• Coulomb potential flattens with increasing brane separation, BUT:

Dvali and Tye
Example: D-brane Inflation

Dvali and Tye

- **inflaton field** = separation of spacetime-filling D3-branes (‘pointlike in the extra dimensions’)

- Coulomb potential flattens with increasing brane separation, **BUT:**

- on a torus of size $L$:

$$\eta \sim \left( \frac{L}{r} \right)^6$$

i.e. **run out of space in the extra dimensions**, before you can separate enough to inflate!
Warped D-brane Inflation

Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi (KKLMMT)
Warped D-brane Inflation

Fluxes backreact on the Calabi-Yau metric to produce a warped throat region

\[ ds^2 = e^{2A(r)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} \left[ dr^2 + r^2 ds^2_{X_5} \right] \]

4d spacetime

6d throat
Warped D-brane Inflation
Warped D-brane Inflation

• *warping* flattens the potential
Warped D-brane Inflation

- warping flattens the potential
- ‘naively’, i.e. ignoring moduli stabilization and compactification effects:

\[ \eta \ll 1 \quad \text{for} \quad r \lesssim L \]
Warped D-brane Inflation

- **warping** flattens the potential

- ‘naively’, i.e. ignoring moduli stabilization and compactification effects:

  \[ \eta \ll 1 \quad \text{for} \quad r \lesssim L \]

- **BUT**: *moduli stabilization* gives important corrections

  \[ \eta = \frac{2}{3} + \Delta \eta \]
**Warped D-brane Inflation**

- **Warping** flattens the potential.
- ‘Naively’, i.e. ignoring moduli stabilization and compactification effects:
  \[ \eta \ll 1 \quad \text{for} \quad r \lesssim L \]
- **BUT:** moduli stabilization gives important corrections
  \[ \eta = \frac{2}{3} + \Delta \eta \]

**Generic eta problem!**

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Warped D-brane Inflation

- **warping** flattens the potential
- ‘naively’, i.e. ignoring moduli stabilization and compactification effects:
  \[
  \eta \ll 1 \quad \text{for} \quad r \lesssim L
  \]
- BUT: *moduli stabilization* gives important corrections
  \[
  \eta = \frac{2}{3} + \Delta \eta
  \]
  generic eta problem!
The Potential

8 + 2 Years Later
Why is Life so Hard?

Task: Compute inflaton action in string compactifications with stabilized moduli.

Challenge: Common approximation schemes often fail to incorporate relevant effects of massive moduli.★

★best understood for brane inflation, but true for most models of string inflation.
3 Ways to Compute the Potential

10d SUGRA

AdS/CFT

compactification

4d CFT

4d SUGRA

DB, Dymarsky, Kachru, Klebanov and McAllister
D-brane Potentials from AdS/CFT, work in progress
Type IIB string theory (and its low-E supergravity limit) contains a good dozen fields that in principle all couple to D-branes.

However, \( \text{D3-branes only couple to a very specific combination of background fields:} \)

\[
V(\phi) = T_3 \left( e^{4A(\phi)} - \alpha(\phi) \right)
\]

warp factor potential of 5-form flux
10d SUGRA

\[ V(\phi) = T_3 \left( e^{4A(\phi)} - \alpha(\phi) \right) \]

whose equation of motion is

\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

Laplacian on \( M_6 \)

3-form flux

imaginary-anti-self-dual component

4d Ricci scalar

branes, O-planes
10d SUGRA

\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

plus

UV Boundary Conditions

“coupling the throat to the bulk”
10d SUGRA

\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

*Homogeneous* Solution

\[ \nabla^2 V(\phi) = 0 \]
\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

**Homogeneous Solution**

\[ \nabla^2 V(\phi) = 0 \]

\[ V(\phi) = \sum \phi^\Delta f(\Psi) \]

where

\[ \Delta = \frac{3}{2}, 2, \ldots \]

sourced by UV deformations of the throat.
\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

**Inhomogeneous Solution**

\[ \nabla^2 V(\phi) = |G_-|^2 \]
\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

**Inhomogeneous Solution**

\[ \nabla^2 V(\phi) = |G_-|^2 \quad \text{d}G_- = 0 \]
10d SUGRA

\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

**Inhomogeneous Solution**

\[ \nabla^2 V(\phi) = |G_-|^2 \quad \text{d}G_- = 0 \]

\[ V(\phi) = \sum \phi^\Delta f(\Psi) \]

where

\[ \Delta = 1, 2, \frac{5}{2}, \ldots \]

DB, Dymarsky, Kachru, Klebanov and McAllister
D-brane Potentials from AdS/CFT, work in progress
\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

**Inhomogeneous Solution**

\[ \nabla^2 V(\phi) = R \propto V \]
Inhomogeneous Solution

\[ \nabla^2 V(\phi) = R \propto V \]

\[ V(\phi) = H^2 \phi^2 \]

This is the KKLMMT eta problem term!
10d SUGRA

\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

Spectrum of Corrections

\[ \phi^1, \phi^{3/2}, \phi^2, \phi^{5/2} \]

a subset of these is turned on in explicit models.
10d SUGRA

\[ \nabla^2 V(\phi) = |G_-|^2 + R + \text{local sources} \]

**Spectrum of Corrections**

- \( \phi^{1/2} \)
- \( \phi^{3/2} \)
- \( \phi^2 \)
- \( \phi^{5/2} \)

A subset of these is turned on in explicit models.

Inflection point models
Gauge Theory

string theory on

$AdS_5 \times T_{1,1}$

dual

$SU(N) \times SU(N)$

susy gauge theory

Maldacena

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Gauge Theory

string theory on

$AdS_5 \times T_{1,1}$

dual

$SU(N) \times SU(N)$

susy gauge theory

large radius
perturbations of
the geometry

UV perturbations of the
gauge theory Lagrangian

$\Delta V \sim \frac{\mathcal{O}_\Delta}{M^{\Delta-4}}$
Gauge Theory

$\Delta V \sim \frac{O_\Delta}{M^{\Delta-4}}$

We are able to match the spectral dimensions (and symmetries) of gauge theory operators $O_\Delta$ to the dimensions of 10d supergravity modes.

DB, Dymarsky, Kachru, Klebanov and McAllister
D-brane Potentials from AdS/CFT, work in progress
4d SUGRA

DB, Dymarsky, Klebanov, Maldacena, McAllister and Murugan

\[ W_{np} = A(\phi)e^{-\alpha \rho} \]

bulk CY

warped throat

gaugino condensation on D7-branes

\[ D = 4 \quad \mathcal{N} = 1 \text{ SUGRA} \]
4d SUGRA

DB, Dymarsky, Klebanov, Maldacena, McAllister and Murugan

gaugino condensation on D7-branes

warped throat

dimensions: 1024.0x788.0

\[ W_{np} = A(\phi) e^{-\alpha \rho} \]

\[ \Delta V(\phi) = \phi^1 + \ldots \]

spectrum of corrections (including angular symmetries) matches (a subset of) the flux-induced corrections in 10d SUGRA!
4d SUGRA

DB, Dymarsky, Klebanov, Maldacena, McAllister and Murugan

\[ \Delta V(\phi) = \phi^1 + \ldots \]

spectrum of corrections (including angular symmetries) matches (a subset of) the flux-induced corrections in 10d SUGRA!

Coincidence or deeper connection between non-perturbative effects on D7-branes and IASD 3-flux

DB, Dymarsky, Kachru, Klebanov and McAllister
D-brane Potentials from AdS/CFT, work in progress

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A Geometric Field Range Bound
A Scaling Argument

\[ \Delta \phi \propto L \]

size of the compact space
A Scaling Argument

\[ \Delta \phi \propto L \]

Can we make the field range large by increasing the size of the compact space?
Can we make the field range large by increasing the size of the compact space?

No! We need to remember that this changes the ruler:

$$M_{pl}^2 \propto \text{Vol}(M_6) \propto L^6$$
A Scaling Argument

\[ \Delta \phi \propto L \]

\[ M_{\text{pl}}^2 \propto L^6 \]

\[ \frac{\Delta \phi^2}{M_{\text{pl}}^2} \propto \frac{1}{L^4} \]

inversely proportional to the size of the compact space!
A Scaling Argument

\[ \Delta \phi \propto L \]
\[ M_{\text{pl}}^2 \propto L^6 \]

\[ \frac{\Delta \phi^2}{M_{\text{pl}}^2} \propto \frac{1}{L^4} \]

\textit{inversely proportional to the size of the compact space!}

NB: \textit{Monodromy} decouples \( \Delta \phi \) from \( M_{\text{pl}} \).

This changes the scaling argument in an essential way.

see talk by Silverstein

Thursday, 3 September 2009
Conifold Geometry

\[ ds^2 = e^{2A(r)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} \left[ dr^2 + r^2 ds^2_{X_5} \right] \]
Conifold Geometry

\[ ds^2 = e^{2A(r)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} \left[ dr^2 + r^2 ds^2_{X_5} \right] \]

\[ e^{4A(r)} = \left( \frac{r}{L} \right)^4 \]
Conifold Geometry

![Diagram of warped throat and bulk with variables r, X5, ψ, rmax, bulk, warped throat]

\[ ds^2 = e^{2A(r)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} \left[ dr^2 + r^2 ds^2_{X_5} \right] \]

\[ e^{4A(r)} = \left( \frac{r}{L} \right)^4 \]

\[ T_3 L^4 = \frac{\pi N}{2 \text{Vol}(X_5)} \]

flux integer \( \gg 1 \)

dimensionless volume \( O(\pi^3) \)
Compactification Volume

\[ (V_6)_{\text{throat}} = \frac{1}{2} \text{Vol}(X_5) L^4 r_{\text{max}}^2 \quad < \quad V_6 \]

Upper bound on the volume sufficient for a conservative lower bound on the Planck mass.
The Geometric Bound

\[ M_{\text{pl}}^2 > T_3 r_{\text{max}}^2 \frac{N}{4} \]

\[ \Delta \phi^2 = T_3 r^2 < T_3 r_{\text{max}}^2 \]

\[ \frac{\Delta \phi^2}{M_{\text{pl}}^2} < \frac{2}{\sqrt{N}} \]

DB and L. McAllister (2006)
g-waves in Brane Inflation

$$\frac{\Delta \phi^2}{M_{\text{pl}}^2} < \frac{2}{\sqrt{N}}$$
g-waves in Brane Inflation

\[ \frac{\Delta \phi^2}{M_{\text{pl}}^2} < \frac{2}{\sqrt{N}} \]

flux integer

\[ N > 1 \]

computational control requires:

\[ N \gg 1 \]
g-waves in Brane Inflation

\[ \frac{\Delta \phi^2}{M_{pl}^2} < \frac{2}{\sqrt{N}} \]

flux integer

\[ N > 1 \]

*computational control requires:*

\[ N \gg 1 \]

\[ \Delta \phi \ll M_{pl} \]
g-waves in Brane Inflation

\[ \frac{\Delta \phi^2}{M_{pl}^2} < \frac{2}{\sqrt{N}} \]

flux integer
\[ N > 1 \]

computational control requires:
\[ N \gg 1 \]

\[ \Delta \phi \ll M_{pl} \]

\[ r \ll 0.01 \]
Conclusions
Requirements for Observable B-modes

Defined by the target sensitivity of a future CMB Polarization satellite

\[ r > 0.01 \]

Kinematic Requirement

\[ \Delta \phi \gg M_{\text{pl}} \]

Dynamical Requirement

flat potential over a super-Planckian range

see Panel Discussion:

“How generic are observable gravitational waves from inflation?”
Gravitational Waves in String Inflation

A Geometric Bound in Brane Inflation

\[ \Delta \phi \ll M_{\text{pl}} \]

DB and L. McAllister (2006)
Gravitational Waves in String Inflation

A Geometric Bound in Brane Inflation

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DB and L. McAllister (2006)
Gravitational Waves in String Inflation

A Geometric Bound in Brane Inflation

\[ \Delta \phi \ll M_{pl} \]

DB and L. McAllister (2006)

Large-Field Models with Shift Symmetry

\[ \Delta \phi > M_{pl} \]

- possible in axion models
- potential protected by a shift symmetry

Silverstein and Westphal (2008), McAllister, Silverstein, and Westphal (2009)
Flauger et al. (2009)
Gravitational Waves in String Inflation

A Geometric Bound in Brane Inflation

\[ \Delta \phi \ll M_{pl} \]

DB and L. McAllister (2006)

Large-Field Models with Shift Symmetry

\[ \Delta \phi > M_{pl} \]

Silverstein and Westphal (2008), McAllister, Silverstein, and Westphal (2009)
Flauger et al. (2009)

...a very active research field.

The first controlled string inflation models with large tensors are appearing!

see talks by Silverstein and Burgess

Thursday, 3 September 2009
A B-mode detection would teach us a great deal about the **physics of inflation**:
A B-mode detection would teach us a great deal about the **physics of inflation**:

1. Inflation occurred!
A B-mode detection would teach us a great deal about the **physics of inflation**:

1. Inflation occurred!

2. It happened near the **GUT-scale**!
A B-mode detection would teach us a great deal about the **physics of inflation**:

1. Inflation occurred!

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3. The inflaton field moved over a super-Planckian distance!
A B-mode detection would teach us a great deal about the **physics of inflation**:

1. Inflation occurred!

2. It happened near the **GUT-scale**!

3. The inflaton field moved over a **super-Planckian distance**!

3’. Its potential was controlled by a **shift symmetry**. i.e. the inflaton was something like an axion
Thank you for your attention!

and thanks to my collaborators:

Liam McAllister, Anatoly Dymarsky, Igor Klebanov, Shamit Kachru, Paul Steinhardt, Juan Maldacena, Hiranya Peiris and Arvind Murugan.

and thanks to the members of the CMBPol Inflation Working Group
Large-Field vs. Small-Field
Small-Field

$$\Delta \phi \ll M_{\text{pl}}$$

Primordial B-modes undetectable

- **The Patch Problem**

For inflation to start the inflaton field has to be *homogeneous* over a distance that is *a few times the horizon size* at that time!
Small-Field

\[ \Delta \phi \ll M_{\text{pl}} \]

Primordial B-modes undetectable

• The Patch Problem

For inflation to start the inflaton field has to be homogeneous over a distance that is \textit{a few times the horizon size} at that time!

Since

\[ H^{-1} \propto r^{-1/2} \]

this seems to be a bigger problem for small-field (low-r) models.
Small-Field

\[ \Delta \phi \ll M_{\text{pl}} \]

Primordial B-modes undetectable

- The Patch Problem

- The Overshoot Problem

For inflation to start the field has to reach the flat part of the potential with small speed.

This problem is worse for small-field (low-r) models.
Small-Field

\[ \Delta \phi \ll M_{pl} \]

Primordial B-modes undetectable

- The Patch Problem
- The Overshoot Problem
- “Fine-Tuned” Potentials
  Boyle, Steinhardt and Turok (2005)

For single-field models
the transition

\[ r = 16 \epsilon \ll 1 \Rightarrow \epsilon = 1 \]

within 60 e-folds requires “fine-tuned” potentials!
Small-Field

\[ \Delta \phi \ll M_{\text{pl}} \]

Primordial B-modes undetectable

- The Patch Problem
- The Overshoot Problem
- “Fine-Tuned” Potentials

These \textit{qualitative} fine-tuning problems of small-field inflation are \textbf{hard to quantify}!

Typically, the arguments run into the ill-understood \textbf{measure problem}!

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Small-Field

$\Delta \phi \ll M_{\text{pl}}$

Primordial B-modes undetectable

- The Patch Problem
- The Overshoot Problem
- “Fine-Tuned” Potentials

Large-Field

$\Delta \phi \gg M_{\text{pl}}$

Primordial B-modes detectable!

- Potentials are “Simple” Functions
  
  e.g.
  
  $V(\phi) = \frac{1}{2} m^2 \phi^2$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{potential_diagram.png}
\end{figure}

\textbf{Thursday, 3 September 2009}
Small-Field

\[ \Delta \phi \ll M_{\text{pl}} \]
Primordial B-modes undetectable

• The Patch Problem
• The Overshoot Problem
• “Fine-Tuned” Potentials

but are corrections under control?

Large-Field

\[ \Delta \phi \gg M_{\text{pl}} \]
Primordial B-modes detectable!

• Potentials are “Simple” Functions

e.g.

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 \]
<table>
<thead>
<tr>
<th>Small-Field</th>
<th>Large-Field</th>
</tr>
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<tbody>
<tr>
<td>$\Delta \phi \ll M_{pl}$</td>
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- The Patch Problem
- The Overshoot Problem
- “Fine-Tuned” Potentials

- Potentials are “Simple” Functions
- **Attractor Solutions**
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A convincing argument that the tensor amplitude has to be large does NOT exist!
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A convincing argument that the tensor amplitude has to be large does NOT exist!

HOWEVER, there is also no convincing argument that it has to be small!