Primordial Gravitational Waves: Inflation & Bouncing Cosmologies

Primordial Gravitational Waves Workshop
Cambridge University
August, 2009
What kind of pattern is this? (Notice the pentagons)

For out-of-towners only: Where can you find this pattern?

Email answers to: steinh@princeton.edu
“Best Test” of Inflation
(as opposed to tests of consistency)

According to the inflationary paradigm, any given mode $k^*$ …

\[
\frac{a(t)H(t)}{k^*}
\]

reached 1 at some point during inflation

…grew by $N_{\text{before}}(k^*)$ e-folds before inflation ended

…shrank by $N_{\text{after}}$ e-folds after inflation ended

\[
\frac{a_0 H_0}{k^*} = e^{N_{\text{before}}(k^*)} \cdot e^{-N_{\text{after}}} < 1
\]

“closure condition”
Testing Inflation: bootstrap test

\[ \frac{a_0 H_0}{k^*} \sim e^{N_{\text{before}}(k^*)} \cdot e^{-N_{\text{after}}} < 1 \]

\[ H(\phi) = H_* + H'_* \phi + \frac{1}{2} H''_* \phi^2 + \frac{1}{6} H'''_* \phi^3 + \ldots \]

\[ H_* = \frac{\pi (\Delta_R^2)^{1/2}}{2} (2r)^{1/2}, \]
\[ H'_* = \frac{\pi (\Delta_R^2)^{1/2}}{8} (-r), \]
\[ H''_* = \frac{\pi (\Delta_R^2)^{1/2}}{32} (2r)^{1/2} [r + 4(n_s - 1)], \]
\[ H'''_* = \frac{\pi (\Delta_R^2)^{1/2}}{128} [64 \alpha_s - 3r^2 - 20r(n_s - 1)] \]

\[ r = \ln \frac{a_0 H_0}{k^*} + \ln \frac{8 \pi^2 \Omega_{rad} \Delta_R^2}{H_0^2} + \frac{1}{2} \]

\[ = 0.13 \]

for WMAP

\[ k^* = 0.002/\text{Mpc} \]
Testing Inflation: bootstrap test

If test is passed -- VERIFICATION TESTS

\[ r = \frac{8}{\ln \frac{a_0 H_0}{k^*} + \ln \frac{8\pi^2 \Omega_{\text{rad}} \Delta_R^2}{H_0^2} + \frac{1}{2}} \]

If test is failed -- Go to BOOTSTRAP TEST 2

For WMAP

\[ k^* = 0.002/\text{Mpc} \]

- Test result: 0.13
- For WMAP
- Closure condition

If test is passed -- VERIFICATION TESTS

\[ n_s = 1 - r/4 \]

\[ \alpha_s = \frac{3r^2 + 20r(n_s - 1)}{64} \]

If test is failed -- Go to BOOTSTRAP TEST 2

- Test result: 0.96
- Test result: -0.00053
Testing Inflation: bootstrap test

If test is passed -- VERIFICATION TEST

\[ H(\varphi) = H_0 + H_0' \varphi + \frac{1}{2} H_0'' \varphi^2 + \frac{1}{6} H_0''' \varphi^3 + \ldots \]

\[ \alpha_s = \left[ 3r^2 + 20r (n_s - 1) \right] / 64 \]
Testing Inflation: bootstrap test

\[ H(\varphi) = H_* + H'_* \varphi + \frac{1}{2} H''_* \varphi^2 + \frac{1}{6} H'''_* \varphi^3 + \ldots \]

confined to 2d surface in \((r, n_s, \alpha_s)\) plane
Testing Inflation:

What does it mean if all 3 bootstrap tests fail?

For practical reasons, not more observables on CMB scales, so…

1) Inflation is right, but requires more parameters than observables: *INHERENTLY UNPROVABLE*!

   the best you can do is show consistency

*Corollary: detecting G-waves essential to “proving” inflation*

2) Inflation is wrong – and there is a more predictive theory that explains the observations more efficiently
Gravitational Waves & Bouncing Cosmologies
Ekpyrotic Phase: Ultra-slow contraction

\[ V \]

\[ \phi \]

or bouncing branes

\[ w = \frac{1}{2} \frac{\dot{\phi}^2 - V(\phi)}{\dot{\phi}^2 + V(\phi)} \]

\[ >> 1 \]

\[ a \sim t^{2/3(1+w)} \]
Ekpyrotic Phase: Ultra-slow contraction

(w > 1)

causal horizon problem
flatness problem

\[ H^2 = \frac{8\pi G}{3} \left( \frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \ldots \right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \Lambda \]
Ekpyrotic Phase: Ultra-slow contraction

(w > 1)

causal horizon problem
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\[ H^2 = \frac{8\pi G}{3} \left( \frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \ldots \right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \Lambda \]

\[ + \frac{8\pi G}{3} \frac{\rho_\phi^0}{a^3(1+w)} \]

Ultra-slow contraction \(\rightarrow\)
Minkowski “blue” spectrum of g-waves
Scalar-induced Fluctuations (2\textsuperscript{nd} order)

\[ \Omega_{GW} \]

\begin{align*}
\ln(k) & \quad \Omega_{GW} \\
10^{-20} & \quad 10^{-13} \\
10^{-15} & \\
10^{-14} & \\
\end{align*}

$k_0$, $k_{eq}$, $k_c$

scalar-induced spectrum @ equality

spectrum today

from Baumann, PJS, Takahashi, Ichiki
see also Mollerach, Harari, Mattarese
Ananda, Clarkson, Wands
Scalar-induced Fluctuations (2\textsuperscript{nd} order)

\[ \Omega_{GW} \]

- $k_0$
- $k_{eq}$
- $k_c$

- $r=0.1$ maximal first-order tensors

- spectrum today
from Baumann, PJS, Takahashi, Ichiki
see also Mollerach, Harari, Mattarese
Ananda, Clarkson, Wands