Why did the Universe Inflate?
Proceedings of the Nuffield Workshop, Cambridge, 1982
Old Inflation Scenario

- Bubble Nucleation
- Bubble Collision
New Inflation Scenario

\[ V \]

\[ \phi \]

Slow roll down

Cosmological Constant
Microwave Background Radiation
Why did the Universe Inflate?

Why did the scalar field start with a high value of the potential, and run down to a minimum?

Why didn’t it just start at the minimum?
No boundary amplitude

$$\psi [h_{ij}, \Sigma] = \int Dg \, e^{-S[g]}$$

Sum over all metrics that have \( \Sigma \) a boundary and where the induced metric on \( \Sigma \) is \( h_{ij} \).
Wheeler DeWitt Equation

\[
\left[ -G_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} - \frac{3}{2} R(h) h^{1/2} + 2\Lambda h^{1/2} \right] \Psi [h_{ij}] = 0
\]

Where \( G_{ijkl} \) is the metric on superspace,

\[
G_{ijkl} = \frac{1}{2} h^{-1/2} (h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})
\]

and \( 3R \) is the scalar curvature of the intrinsic geometry of the three-surface.
Wheeler DeWitt Equation

\[
\frac{1}{2} \left[ \frac{\partial^2}{\partial a^2} - a^2 - \frac{1}{a^2} \frac{\partial}{\partial \varphi^2} + a^4 V \right] \Psi(a, \varphi) = 0
\]
WKB Approximation

\[ \Psi = B \, e^{iC} \]

\[ \nabla C \cdot \nabla C + J = 0 \]

\[ \nabla B \cdot \nabla C = 0 \]
No Boundary Amplitude

Amplitude

$N$
Volume Weighting

\[ \text{P( Observing a Hubble Volume )} = |\Psi|^2 \times \text{Number of Hubble Volumes} \]

\[ \propto |\Psi|^2 e^{3N} \]
Volume Weighted Probability

Quadratic Potential

Probability

Initial $\phi$

Planck Density
Volume Weighted Probability

Potential with a Maximum at $\varphi = \varphi_m$
Why did the Universe Inflate?

• Volume weighting gives a high probability of a large amount of Inflation