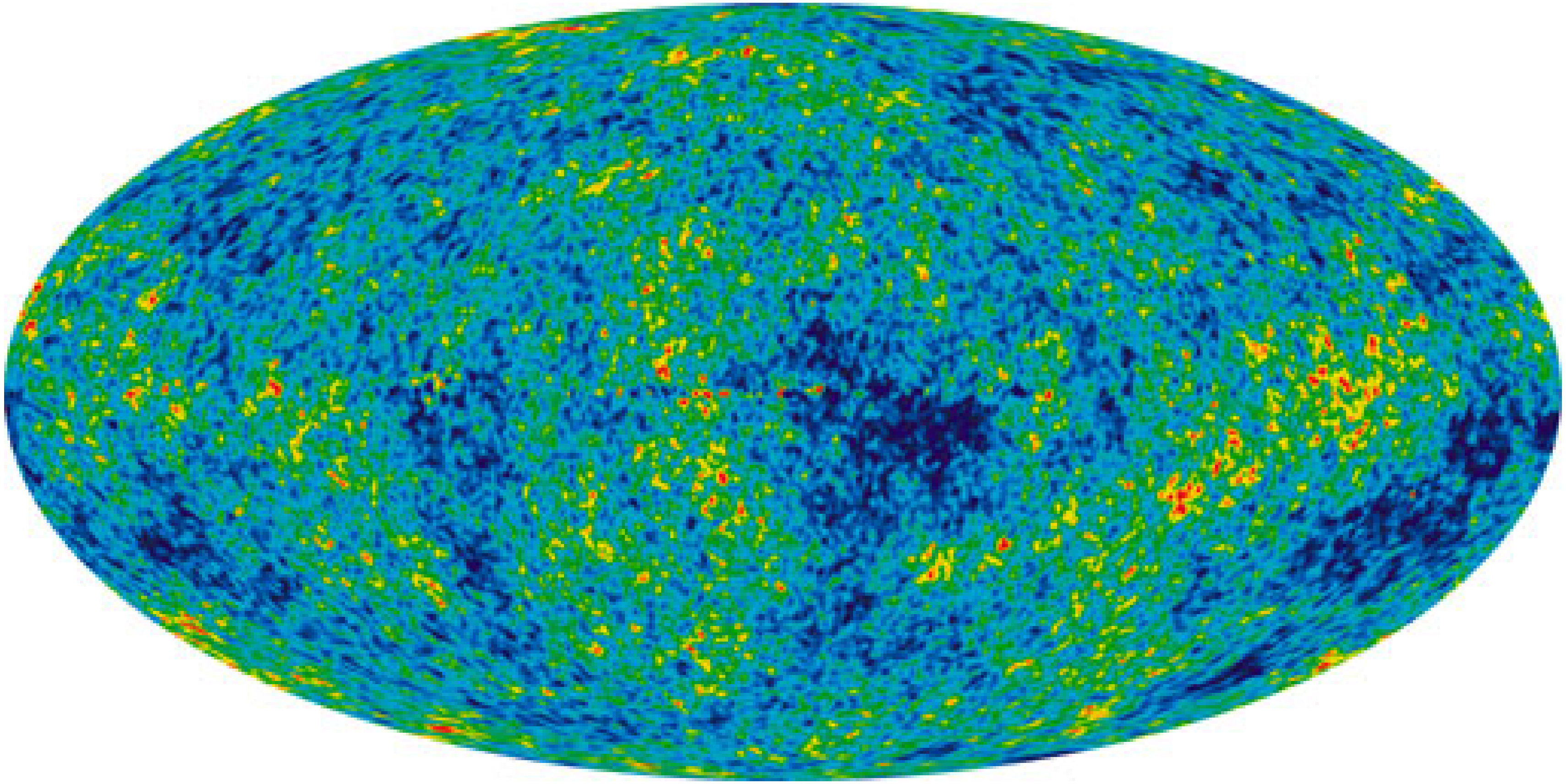
Thermal Fluctuations



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Introduction

The standard theory of inflation predicts that the large scale distribution of galaxies can be traced back to quantum vacuum fluctuations of a weakly coupled field during the inflationary era.

If the universe did not supercool, then...

- can reduce the need for very small couplings
- more opportunities for particle phenomenology (F-terms)
- make structure from thermal fluctuations
- can produce observable amounts of non-gaussianity
- exit nicely

The standard picture

Inflaton ϕ and expansion rate H satisfy:,

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0,$$
$$3H^2 = 8\pi G\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

There are two slow-roll parameters:

$$\epsilon = \frac{m_p^2}{16\pi}\left(\frac{V_{,\phi}}{V}\right)^2, \qquad \eta = \frac{m_p^2}{8\pi}\left(\frac{V_{,\phi\phi}}{V}\right),$$

where $m_p = G^{-1/2}$. We want $\epsilon \ll 1$ and $\eta \ll 1$.

The scale of the potential V determines the amplitude of fluctuations during inflation. Their power spectrum,

$$\mathcal{P}_s(k,t) = \frac{H^4}{\dot{\phi}^2} \sim \frac{1}{\epsilon}\frac{V}{m_p^4} \quad \text{at} \quad t_k.$$

Cooling and heating

- The radiation density ρ_γ satisfies,

$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0$$

The radiation density during inflation redshifts away.

- Suppose the radiation density ρ_γ satisfies,

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2.$$

The radiation density during inflation stabilises.

Radiation production determined by $\Gamma \equiv \Gamma(\phi, T)$.

Warm Inflation

The production of radiation is associated with a friction term in the inflaton equation,

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V_{,\phi} = 0$$

Effectiveness of warm inflation measured by

$$r = \frac{\Gamma}{3H} \gg 1$$

- Cold or standard inflation: $T \ll H$
- Weak regime of warm inflation: $r \ll 1$ and $T \gg H$
- Strong regime of warm inflation: $r \gg 1$ and $T \gg H$

IGM 1985, Berera 1995

Slow-roll parameters

The slow-roll parameters are

$$\varepsilon = \frac{m_p^2}{16\pi}\left(\frac{V_{,\phi}}{V}\right)^2, \eta = \frac{m_p^2}{8\pi}\left(\frac{V_{,\phi\phi}}{V}\right), \beta = \frac{m_p^2}{16\pi}\left(\frac{V_{,\phi}\Gamma_{,\phi}}{V\Gamma}\right), \delta = \frac{TV_{,\phi T}}{V_{,\phi}},$$

where m_p is the Planck mass. Inflation for

$$\eta, \epsilon, \beta \ll r = \Gamma/3H, \qquad \delta \ll 1$$

- The need for small couplings in reduced.
- There is no η problem in F-term SUGR models

Hall, IGM, Berera 2004

Heated arguments[1]

- thermal corrections to the potential
- thermalisation timescales
- thermal field theory is unpopular

Transport theory gives $\Gamma \sim g^n m^2 \tau$, $n = 1, 2$, g, m and τ are coupling, mass and thermal relaxation time.

- Large g: potential corrections problem
- Large τ: thermalisation problem

Two-stage decay[2]:

- decouple the radiation from the inflaton
- inflaton couplings of order 0.1
- SUSY to reduce vacuum corrections

1 Yokoyama and Linde 1999, 2 Berera and Ramos 2003

Two-stage decay

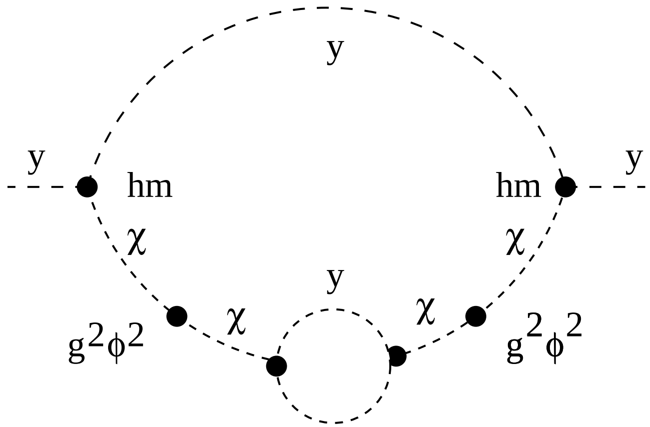
Two-stage reheating

$$\underset{\text{inflaton}}{\phi} \rightarrow \underset{\text{heavy boson}}{\chi} \rightarrow \underset{\text{light fermion}}{\tilde{y}}$$

with Lagrangian

$$\mathcal{L}_I = -\frac{1}{2}g^2\phi^2\chi^2 - h\chi\bar{\psi}_y\psi_y$$

Take the light fields to be close to equilibrium, temperature T.



Friction coefficient for $T \ll m_\chi$ is $\Gamma \sim g^2h^4(T^3/m_\chi^2)$.

Density fluctuations

Density fluctuations are made by thermal fluctuations.

Let $\phi \equiv \phi(x,t)$ and average out the other fields. This gives a stochastic system,

$$\ddot{\phi}(x,t) + (3H + \Gamma)\dot{\phi}(x,t) + \frac{\partial V}{\partial \phi} - \frac{1}{a^2}\nabla^2\phi(x,t) = \xi(x,t).$$

Now we have a spatial gradient term, a scale factor a, and a stochastic source ξ. The source has a gaussian distribution with local correlation function

$$\langle \xi(x,t)\xi(x',t')\rangle = a^{-3}(3H + \Gamma)T\delta(x - x')\delta(t - t')$$

Gleiser and Ramos 1994

Density fluctuations: power spectrum

We perform the usual type of perturbation analysis with

$$\phi(x,t) = \phi(t) + \delta\phi(x,t)$$

Dark energy terms $\partial V/\partial\phi$ are small, and we get a linear equation for $\delta\phi$

$$\delta\ddot{\phi}(x,t) + (3H+\Gamma)\delta\dot{\phi}(x,t) - \frac{1}{a^2}\nabla^2\delta\phi(x,t) = \xi(x,t)$$

- Scalar fluctuations 'freeze out' before horizon crossing
- The amplitude $\delta\phi^2 = (H\Gamma)^{1/2}T$
- The density fluctuations $\zeta = H\delta\phi/\dot{\phi}$
- The spectral index $n_s = 1 -$ small parameters
- Reduced gravity wave amplitude

CMB amplitude $\mathcal{P}_s = 2\times 10^{-9}$ fixes the energy scale V.

Density fluctuations: non-gaussianity

Introduce a radiation fluid velocity **v** and an advection term

$$\ddot{\phi}(x,t) + (3H + \Gamma)\dot{\phi}(x,t) + \Gamma \mathbf{v} \cdot \nabla \phi(x,t) - \frac{1}{a^2}\nabla^2 \phi(x,t) = \xi(x,t)$$

The non-gaussianity arises at next-to-leading order in perturbations. Let $\delta\phi = \delta_1\phi + \delta_2\phi + \ldots$, then

$$\delta_2\ddot{\phi}(x,t) + \Gamma\delta_2\dot{\phi}(x,t) - \frac{1}{a^2}\nabla^2 \delta_2\phi(x,t) = -\Gamma \mathbf{v} \cdot \nabla \delta_1 \phi(x,t)$$

This produces a large amount of non-gaussianity with a characteristic form. IGM and C. Xiong astro-ph/0701302

Shape of the bispectrum

Parameterise the bispectrum B_ζ by a polynomial expansion,

$$B_\zeta(k_1,k_2,k_3) = \sum_{\rm cyc} f_{ln} p_n(k_1,k_2) P_\zeta(k_1) P_\zeta(k_2) P_l(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2).$$

- Standard inflation $f_{00} \approx 0.05$
- Standard inflation plus curvaton $f_{00} \approx -0.75$ or larger
- Sachs-Wolfe $f_{11} \approx 1$
- Warm inflation $f_{11} \approx -15 \ln(1 + r/14)$

Limits from the WMAP data $-375 < f_{11} < 37$

(and $f_{11} < 0$ at 94% c.l.)

IGM and C. M. Graham astro-ph/0707.1647

Quantum cosmology predicts warm inflation

SWH has suggested† weighting the probability of universe nucleation with the volume,

$$e^{3N-2I}.$$

In the slow roll approximation,

$$N = -\kappa^2 \int_{\phi_b}^{\phi} \frac{V}{V_{,\phi}} d\phi$$

$$I = -\frac{32\pi^2}{3\kappa^4 V_b}$$

In warm inflation, with the friction coefficient Γ,:

$$N = -\kappa^2 \int_{\phi_b}^{\phi} \left(1 + \frac{\Gamma}{3H}\right) \frac{V}{V_{,\phi}} d\phi$$

Given identical potentials, models with $\Gamma \gg 3H$ are prefered!

† Hartle, Hawking and Herzog, this meeting

Outlook

- When $\Gamma_{,T} \neq 0$ the effects of entropy perturbations during inflation become important, and numerical work is required. Oscillations can occur in the power spectrum.
- If strong warm inflation took place, then Planck should see evidence of non-gaussianity.
- The bi-spectrum would enable us to say whether the source of non-gaussianity was thermal fluctuations, rather than, say, multi-scalar field models of inflation.
- Some model building has been done-with moderate success. The weak regime appears to be the more likely. (But maybe quantum cosmology says otherwise).

Thanks to my students Chun Xiong and Chris Graham.