The Classical Universes of the No-Boundary Quantum State

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The Quasiclassical Realm - A feature of our Universe

The wide range of time, place and scale on which the laws of classical physics hold to an excellent approximation.

- Time --- from the Planck era forward.
- Place --- everywhere in the visible universe.
- Scale --- macroscopic to cosmological.

What is the origin of this quasiclassical realm in a quantum universe characterized fundamentally by indeterminacy and distributed probabilities?
Classical spacetime is assumed in all formulations of inflationary cosmology.

Classical spacetime is the key to the origin of the rest of the quasiclassical realm.
Origin of the Quasiclassical Realm

- Classical spacetime emerges from the quantum gravitational fog at the beginning.
- Local Lorentz symmetries imply conservation laws.
- Sets of histories defined by averages of densities of conserved quantities over suitably small volumes decohere.
- Approximate conservation implies these quasiclassical variables are predictable despite the noise from mechanisms of decoherence.
- Local equilibrium implies closed sets of equations of motion governing classical correlations in time.
Only Certain States Lead to Classical Predictions

- Classical orbits are not predictions of every state in the quantum mechanics of a particle.
- Classical spacetime is not a prediction of every state in quantum gravity.
Classical Spacetime is the Key to the Origin of the Quasiclassical Realm. The quantum state of the universe is the key to the origin of classical spacetime.

We analyze the classical spacetime predicted by Hawking’s no-boundary quantum state for a class of minisuperspace models.

$$\Psi = \int_C \delta g \delta \phi \exp(-I[g, \phi])$$
Minisuperspace Models

Geometry: Homogeneous, isotropic, closed.

\[ ds^2 = (3/\Lambda) \left[ N^2(\lambda) d\lambda^2 + a^2(\lambda) d\Omega_3^2 \right] \]

Matter: cosmological constant \( \Lambda \) plus homogeneous scalar field moving in a quadratic potential.

\[ V(\Phi) = \frac{1}{2} m^2 \Phi^2 \]

Theory: Low-energy effective gravity.

\[ I_C[g] = -\frac{m_p^2}{16\pi} \int_M d^4x (g)^{1/2} (R - 2\Lambda) + \text{(surface terms)} \]
Classical Pred. in NRQM ---Key Points

Semiclassical form:

\[ \Psi(q_0) = A(q_0) e^{iS(q_0)/\hbar} \]

- When \( S(q_0) \) varies rapidly and \( A(q_0) \) varies slowly, high probabilities are predicted for classical correlations in time of suitably coarse grained histories.

- For each \( q_0 \) there is a classical history with probability:

\[ p_0 = \nabla S(q_0) \quad p(\text{class.hist.}) = |A(q_0)|^2 \]
NRQM -- Two kinds of histories

\[ \Psi(q_0) = A(q_0)e^{iS(q_0)/\hbar} \]

- \( S(q_0) \) might arise from a semiclassical approximation to a path integral for \( \Psi(q_0) \) but it doesn’t have to.

- If it does arise in this way, the histories for which probabilities are predicted are generally distinct from the histories in the path integral supplying the semiclassical approximation.
No-Boundary Wave Function (NBWF)

\[ ds^2 = \left( \frac{3}{\Lambda} \right) \left[ N^2(\lambda) d\lambda^2 + a^2(\lambda) d\Omega_3^2 \right] \]

\[ \Psi(b, \chi) \equiv \int_C \delta N \delta a \delta \phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar). \]

The integral is over all \((a(\lambda), \phi(\lambda))\) which are regular on a disk and match the \((b, \chi)\) on its boundary. The complex contour is chosen so that the integral converges and the result is real.
Semiclassical Approx. for the NBWF

$$\Psi(b, \chi) \equiv \int_C \delta N \delta a \delta \phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar)$$

- In certain regions of superspace the steepest descents approximation may be ok.
- To leading order in $\hbar$ the NBWF will then have the semiclassical form:

$$\Psi(b, \chi) \approx \exp\{[-I_R(b, \chi) + iS(b, \chi)]/\hbar\}.$$  

- The next order will contribute a prefactor which we neglect. Our probabilities are therefore only relative.
Instantons and Fuzzy Instantons

In simple cases the extremal geometries may be real and involve Euclidean instantons, but in general they will be a complex --- fuzzy instantons.
Classical Prediction in MSS and The Classicality Constraint

\[ \Psi(b, \chi) \approx \exp\{[-I_R(b, \chi) + iS(b, \chi)]/\hbar\} \]

- Following the NRQM analogy this semiclassical form will predict classical Lorentian histories that are the integral curves of \( S \), ie the solutions to:

\[ p_A = \nabla_A S \quad p(\text{class. hist.}) \propto \exp(-2I_R/\hbar) \]

- However, we can expect this only when \( S \) is rapidly varying and \( I_R \) is slowly varying, i.e.

\[ |\nabla_u I_R| \ll |\nabla_u S|. \]

This is the classicality condition.

Hawking (1984), Grischuk & Rozhansky (1990), Halliwell (1990)
Class. Prediction --- Key Points

• The NBWF predicts an ensemble of entire, 4d, classical histories.

• These real, Lorentzian, histories are not the same as the complex extrema that supply the semiclassical approximation to the integral defining the NBWF.
No-Boundary Measure on Classical Phase Space

The NBWF predicts an ensemble of classical histories that can be labeled by points in classical phase space. The NBWF gives a measure on classical phase space.

The NBWF predicts a one-parameter subset of the two-parameter family of classical histories, and the classicality constraint gives that subset a boundary.
Singularity Resolution

- The NBWF predicts probabilities for entire classical histories not their initial data.

- The NBWF therefore predicts probabilities for late time observables like CMB fluctuations whether or not the origin of the classical history is singular.

- The existence of singularities in the extrapolation of some classical approximation in quantum mechanics is not an obstacle to prediction by merely a limitation on the validity of the approximation.
\[ ds^2 = \left( \frac{3}{\Lambda} \right) \left[ N^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2 \right] \]

- \( N \) is arbitrary but can be complex. If we write \( d\tau = N \, d\lambda \) then different choices for \( N \) correspond to different contours in the complex \( \tau \) plane.

- Cauchy equivalent contours give the same action.

- We pick a convenient contour to find the extrema.
Equations and BC

\[ \ddot{a}^2 - 1 + a^2 + a^2 \left( -\dot{\phi}^2 + \mu^2 \phi^2 \right) = 0 \]

Extremum Equations:

\[ \ddot{\phi} + 3(\dot{a}/a)\dot{\phi} - \mu^2 \phi = 0, \]

\[ \ddot{a} + 2a\dot{\phi}^2 + a(1 + \mu^2 \phi^2) = 0. \]

Regularity at South Pole:

\[ a(0) = 0, \quad \dot{a}(0) = 1, \quad \dot{\phi}(0) = 0 \]

Parameter matching:

\[ \phi(0) \equiv \phi_0 e^{i\gamma} \]

\[ (\phi_0, \gamma, X, Y) \leftrightarrow (b, \chi, 0, 0) \]
Equations and BC

\[ h = c = G = 1, \quad \mu \equiv (3/\Lambda)^{1/2} m, \quad \phi \equiv (4\pi/3)^{1/2} \Phi, \quad H^2 \equiv \Lambda/3 \]

You won't follow this.
I just wanted to show how much work we did.

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The only important point is that there is one classical history for each value of the field at the south pole \( \phi_0 \equiv |\phi(0)|. \)

Parameter matching:

\( (\phi_0, \gamma, X, Y) \leftrightarrow (b, \chi, 0, 0) \)
Finding Solutions

- For each $\phi_0$ tune remaining parameters to find curves in $(b, \chi)$ for which $I_R$ approaches a constant at large $b$.

- Those are the Lorentzian histories.

- Extrapolate backwards using the Lorentizan equations to find their behavior at earlier times.

- The result is a one-parameter family of classical histories whose probabilities are

$$p(\phi_0) \propto \exp(-2I_R)$$
There is a significant probability that the universe
never reached the Planck scale in its entire evolution.

\[ \mu = 3 \]

\[ \mu \equiv (3/\Lambda)^{1/2} m \]
Classicality Constraint ---Pert. Th.

Small field perturbations on deSitter space.

\[ \mu < \frac{3}{2} \]

Classical \quad \mu \equiv \left( \frac{3}{\Lambda} \right)^{1/2} m \quad \text{Not-classical}

This is a simple consequence of two decaying modes for \( \mu < \frac{3}{2} \), and two oscillatory modes for \( \mu > \frac{3}{2} \).
No nearly empty models for $\mu > 3/2$, a minimum amount of matter is needed for classicality.
Arrows of Time

- It is likely that the NBWF will predict growing fluctuations immediately away from the bounce.
- The thermodynamic arrow points away from the bounce on both sides.
- Events on one side will have little effect on events on the other. They would have to propagate their influence backward in time to do so.
There is scalar field driven inflation for all histories allowed by the classicality constraint, but a small number of efoldings $N$ for the most probable of them.
Probabilities for Our Data

- The NBWF predicts probabilities for entire classical histories.
- Our observations are restricted to a part of a light cone extending over a Hubble volume and located somewhere in spacetime.
- To get the probabilities for our observations we must sum over the probabilities for the classical spacetimes that contain our data at least once, and then sum over the possible locations of our light cone in them.
- This defines the probability of our data in a way that is gauge invariant and dependent only on data on our past light cone.
Volume Factors Favor Inflation

Hawking 07

- In homogeneous, isotropic models the sum over spacetimes multiples the probability of each classical history by the number of Hubble volumes in the present volume, roughly $\exp(3N - 2I_R)$ where $N$ is the number of efolds.

- This favors larger universes and more inflation. In a larger universe there are more places for our Hubble volume to be.

- For the quadratic potential models this is not significant, but for more realistic potentials it may be.
Suppose the NBWF requires $V(\phi_0) > V(\phi_0^c)$ for classicality, and favors $\phi_0 \approx \phi_0^c$ (low inflation) as in the quadratic potential case.

The broad maximum with a great many efoldings may turn the probability distribution around to favor long inflation.
The Main Points Again

Homogeneous, isotropic, scalar field in a quadratic potential, $\mu > 3/2$

- Only special states in quantum gravity predict classical spacetime.

- The NBWF predicts probabilities for a restricted set of entire classical histories that may bounce or be singular in the past. All of them inflate.

- The classicality constraint requires a minimum amount of scalar field (no big empty U’s).

- Probabilities of the past conditioned on limited present data favor inflation.

- The classicality constraint could be an important part of a vacuum selection principle.
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- The classicality constraint could be an important part of a vacuum selection principle.
If the universe is a quantum mechanical system it has a quantum state. What is it?