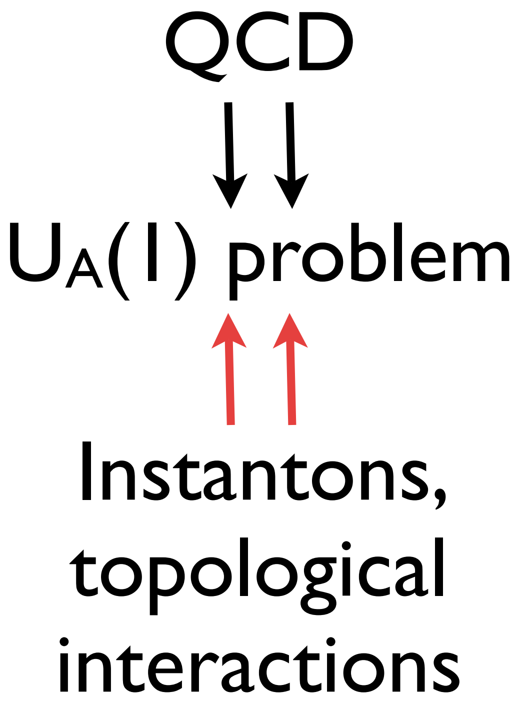
25 Years of Axion Cosmology

Frank Wilczek

The realization that axions might contribute importantly to dark matter came out of conversations at the first VEU workshop.

We’re still not sure whether they do.

Axion Basics



QCD

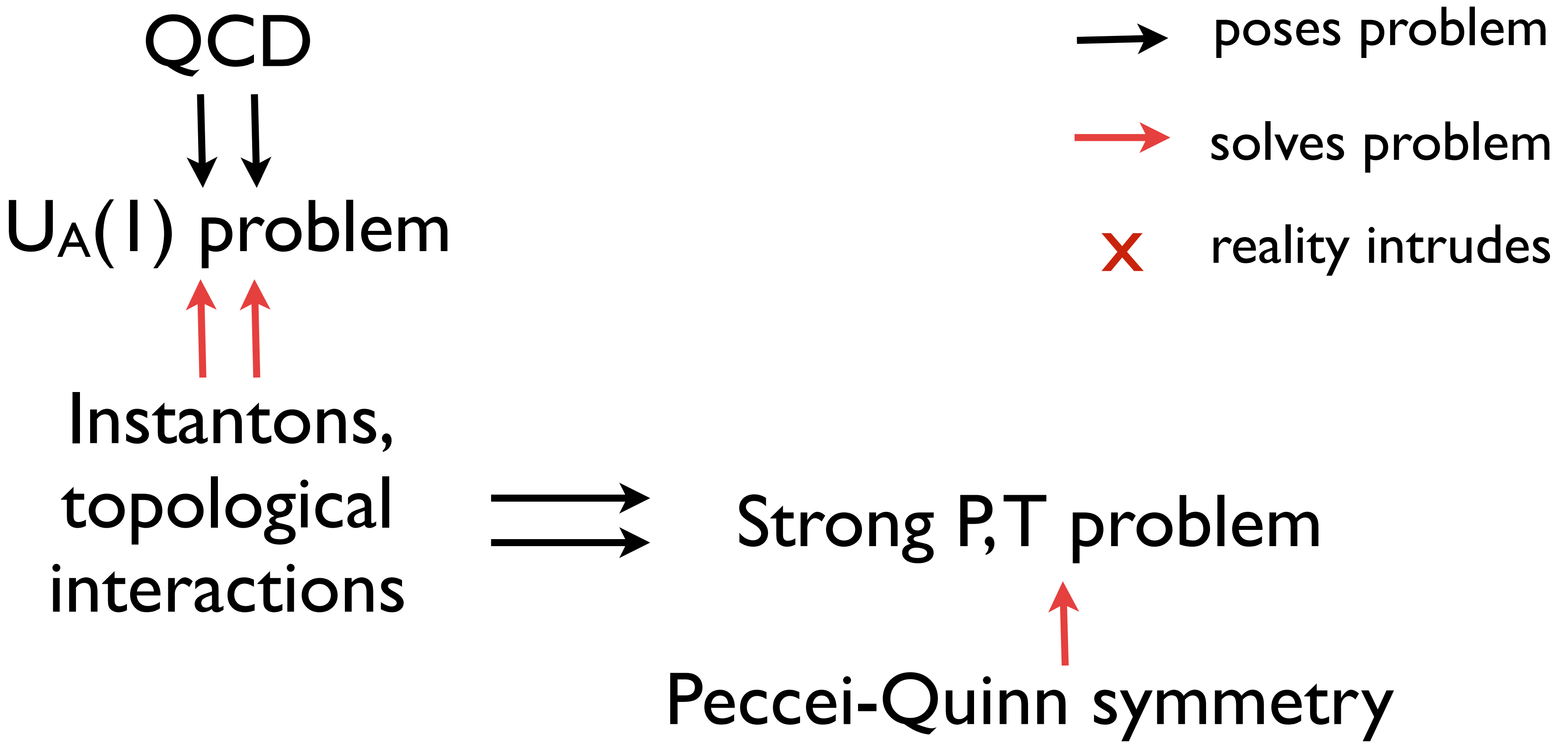
$U_A(1)$ problem

Instantons, topological interactions

→ poses problem

→ solves problem

✗ reality intrudes



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Strong P,T problem

Peccei-Quinn symmetry

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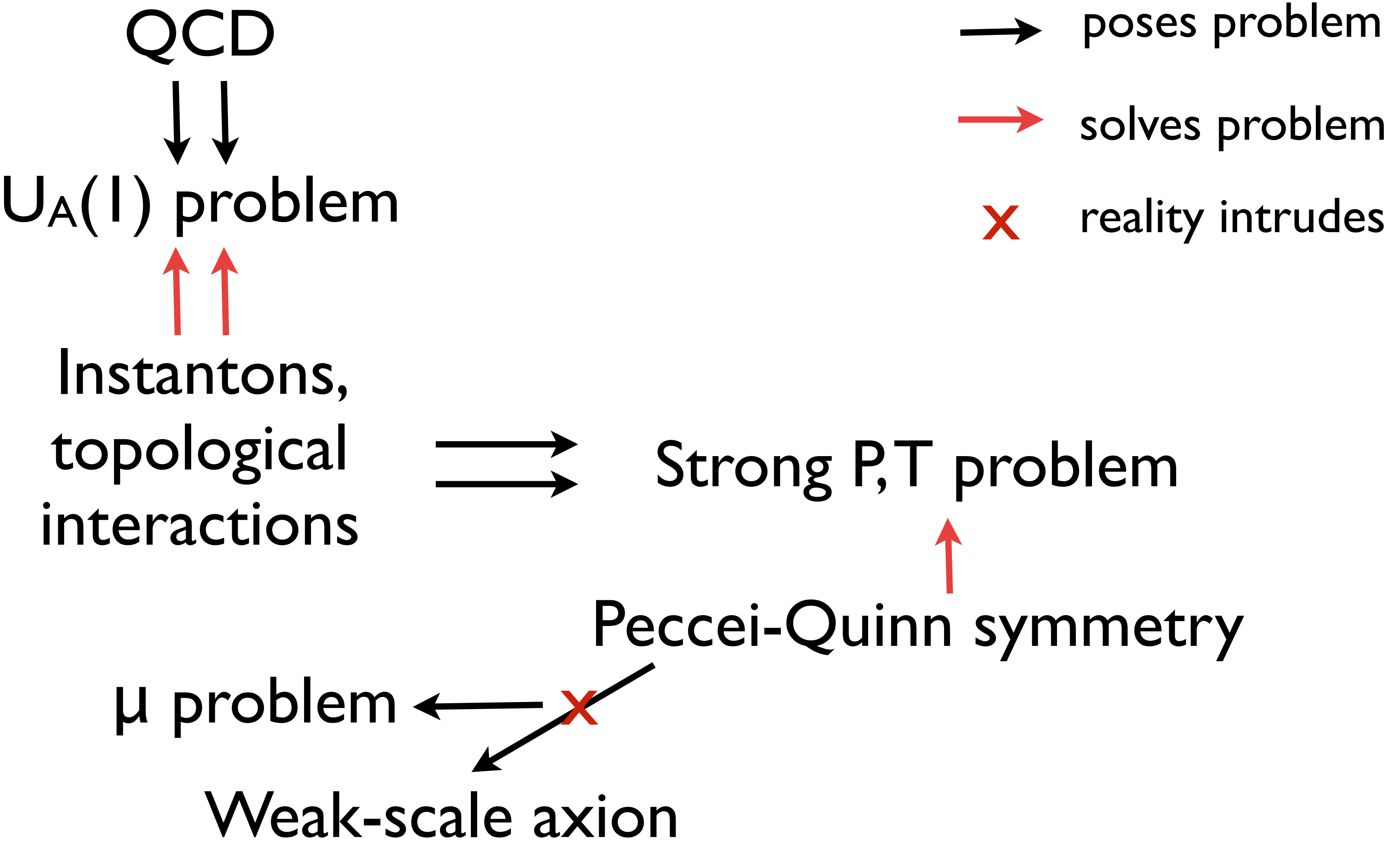

→ poses problem

→ solves problem

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QCD

$U_A(1)$ problem

Instantons, topological interactions

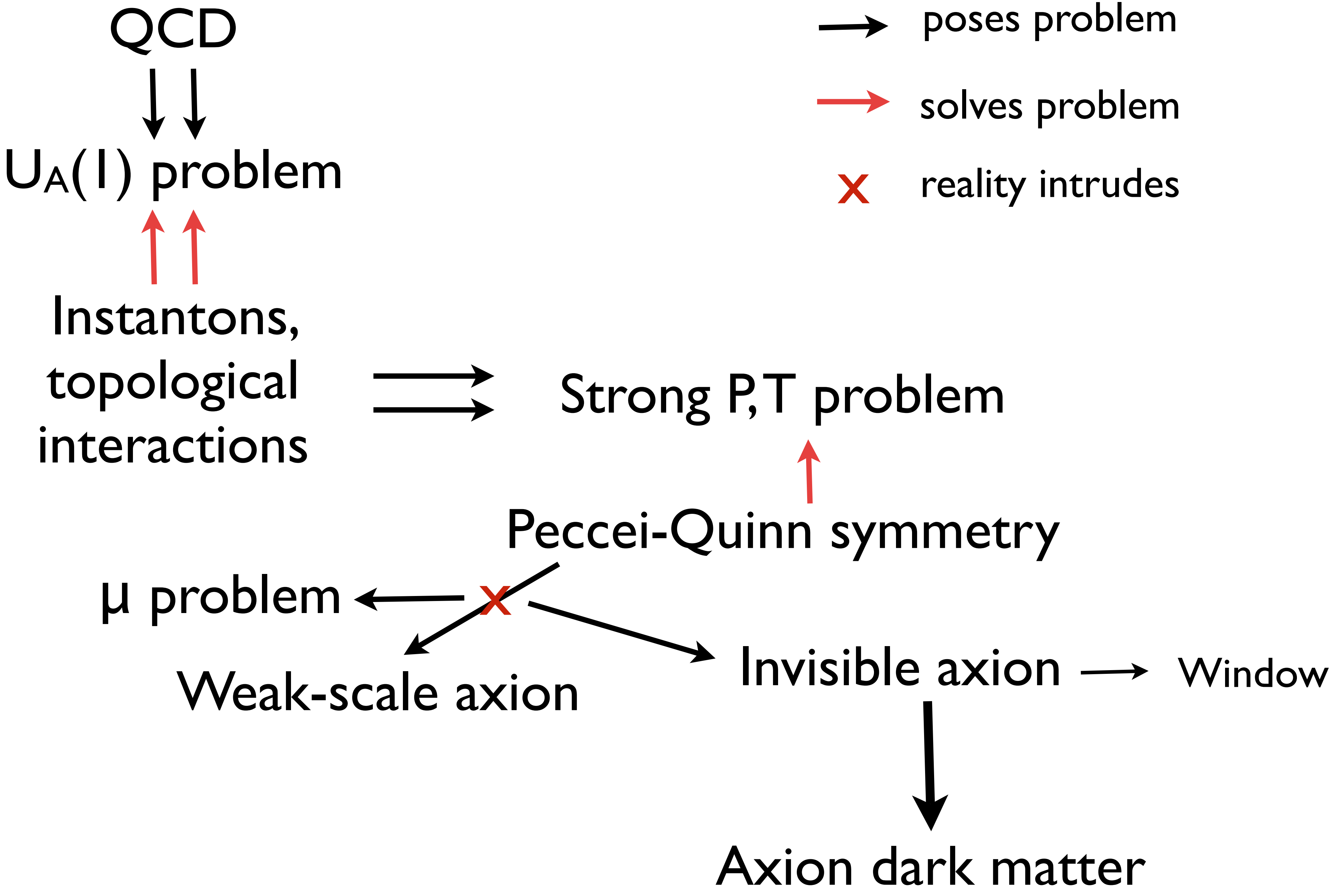
Strong P,T problem

Peccei-Quinn symmetry

μ problem

✗

Weak-scale axion



QCD

$U_A(1)$ problem

Instantons, topological interactions

Strong P,T problem

Peccei-Quinn symmetry

μ problem

Weak-scale axion

Invisible axion

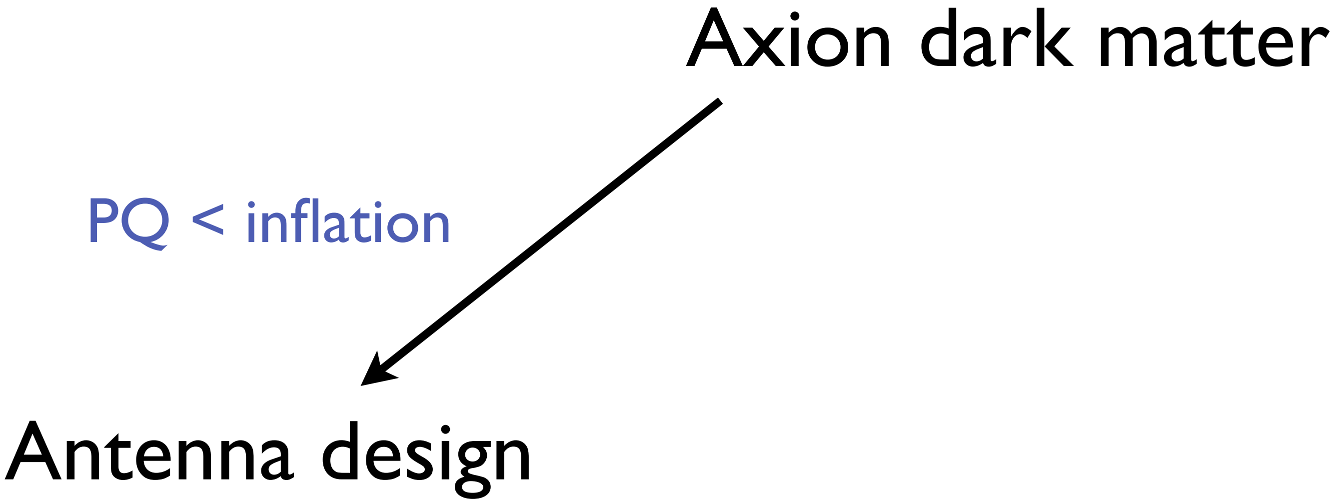
Window

Axion dark matter

→ poses problem

→ solves problem

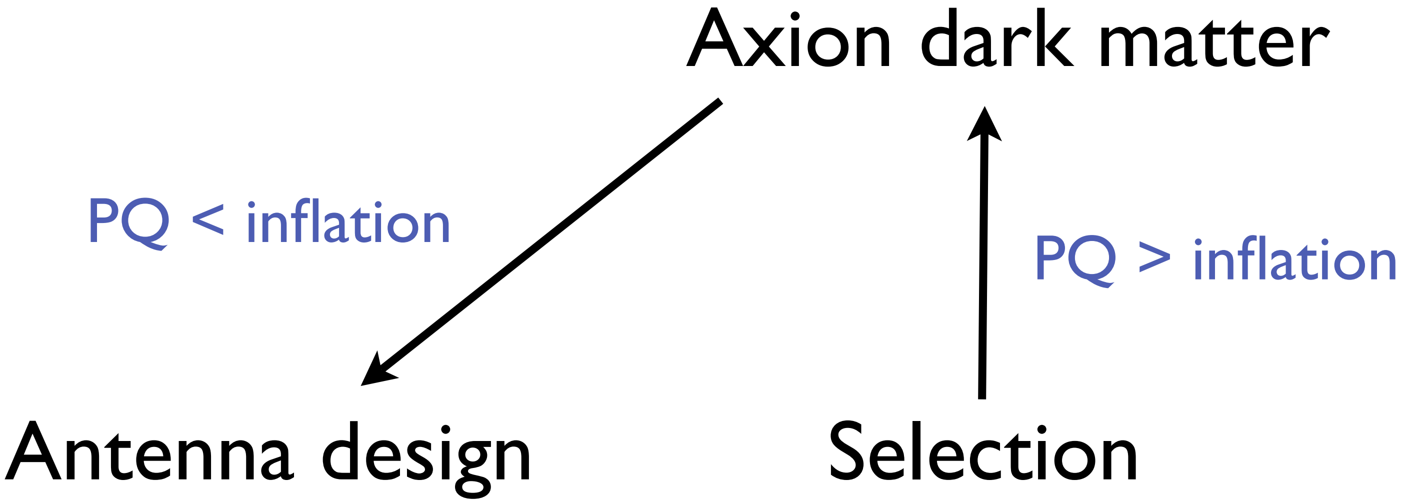
x reality intrudes



Axion dark matter

PQ < inflation

Antenna design



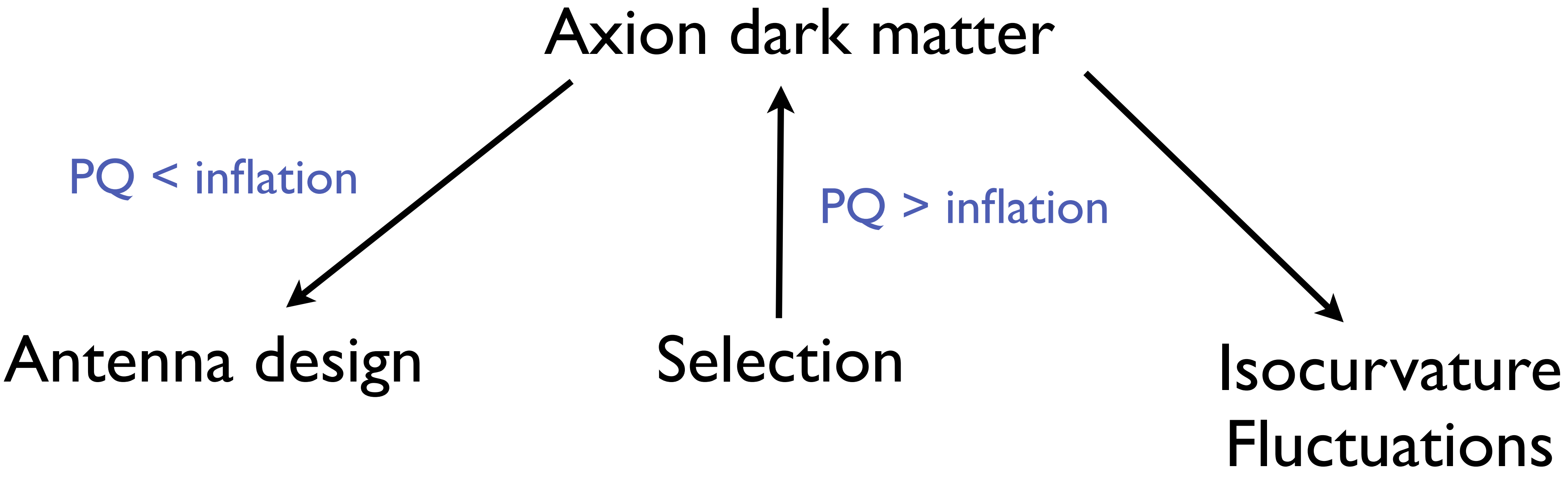
Axion dark matter

PQ < inflation

Antenna design

PQ > inflation

Selection



Axion dark matter

PQ < inflation

PQ > inflation

Antenna design

Selection

Isocurvature Fluctuations

The theory of the strong interaction (QCD) admits a parameter, θ, that is observed to be unnaturally small: $\theta < 10^{-9}$.

This can be understood by promoting translation of θ to an asymptotic symmetry that is spontaneously broken: Peccei-Quinn symmetry.

The axion field is established at the PQ transition, $\langle \Phi \rangle = F\, e^{i\theta}$.

It stores energy, due to its initial misalignment, roughly proportional to $F \sin^2\theta_0$.

$$\overset{\substack{\rho\text{ in play}\\ \searrow}}{\Lambda^4}\;\overset{\substack{\text{dilution of “particles”}\\ \downarrow}}{\left(\frac{T}{\Lambda}\right)^3}\;\overset{\substack{\text{adiabatic damping}\\ \swarrow}}{\frac{H_\Lambda}{m_a}}$$

$$\frac{H_\Lambda}{m_a} = \frac{\frac{\Lambda^2}{M_{\rm Pl}}}{\frac{\Lambda^2}{F}} = \frac{F}{M_{\rm Pl}}$$

If no inflation occurs after the PQ transition, then the correlation length, which is no larger than the horizon at the transition, corresponds to a very small length in the present universe.

We therefore average over $\sin^2\theta_0$.

F ~ 10^{12} GeV corresponds to the observed dark matter density.

This has usually been the regarded as the default axion cosmology. A cosmic axion background with F = 10^{12} GeV might be detectable, in difficult experiments.

Searches are ongoing.

Inflationary Axion Cosmology

If inflation occurs after the PQ transition, things are very different.

Then the correlated volume inflates to include the entire presently observed universe, so we shouldn't average.

F > 10^{12} GeV can be accommodated, with "atypically" small $\sin^2\theta_0$.

In this scenario, most of the multiverse is overwhelmingly axion-dominated, and inhospitable for the emergence of complex structure, let alone observers.

Selection effects must be considered.
(Linde, 1988)

θ_0 controls the dark matter density, but it has little or no effect on anything else. So we know what the prior measure is. (Namely $d\theta_0$ for θ_0, $\sin^2\theta_0\, d\theta_0$ for ρ_{DM}/ρ_b.)

We do not have to get embroiled in questions of baby universe nucleation ...

... nor, for that matter, unification, supersymmetry, string theory landscape artistry ... ...

The theory may be right, or it may be wrong, but it is hard to imagine a clearer case for applying anthropic reasoning.

Tegmark, Aguirre, Rees, FW astro-ph/0511774

Making User-Friendly Structures From Gas Clouds

The Fragility of Life

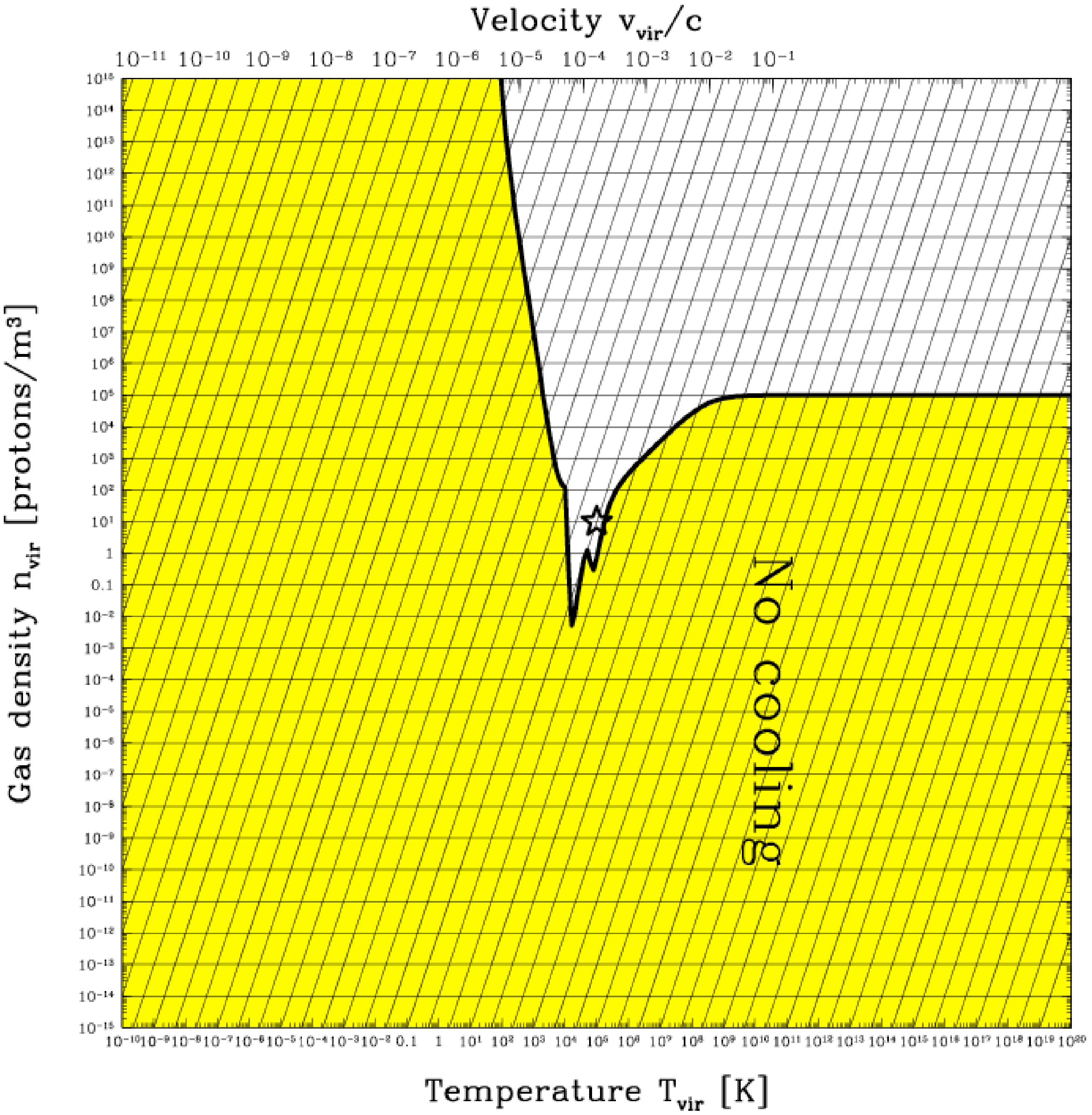
Lots of things can go wrong when you try to make nice solar systems, starting with small seed fluctuations:

The (ordinary) matter might fail to cool,
so it sloshes around and remains diffuse:

density ↑

time ↓

size ↓



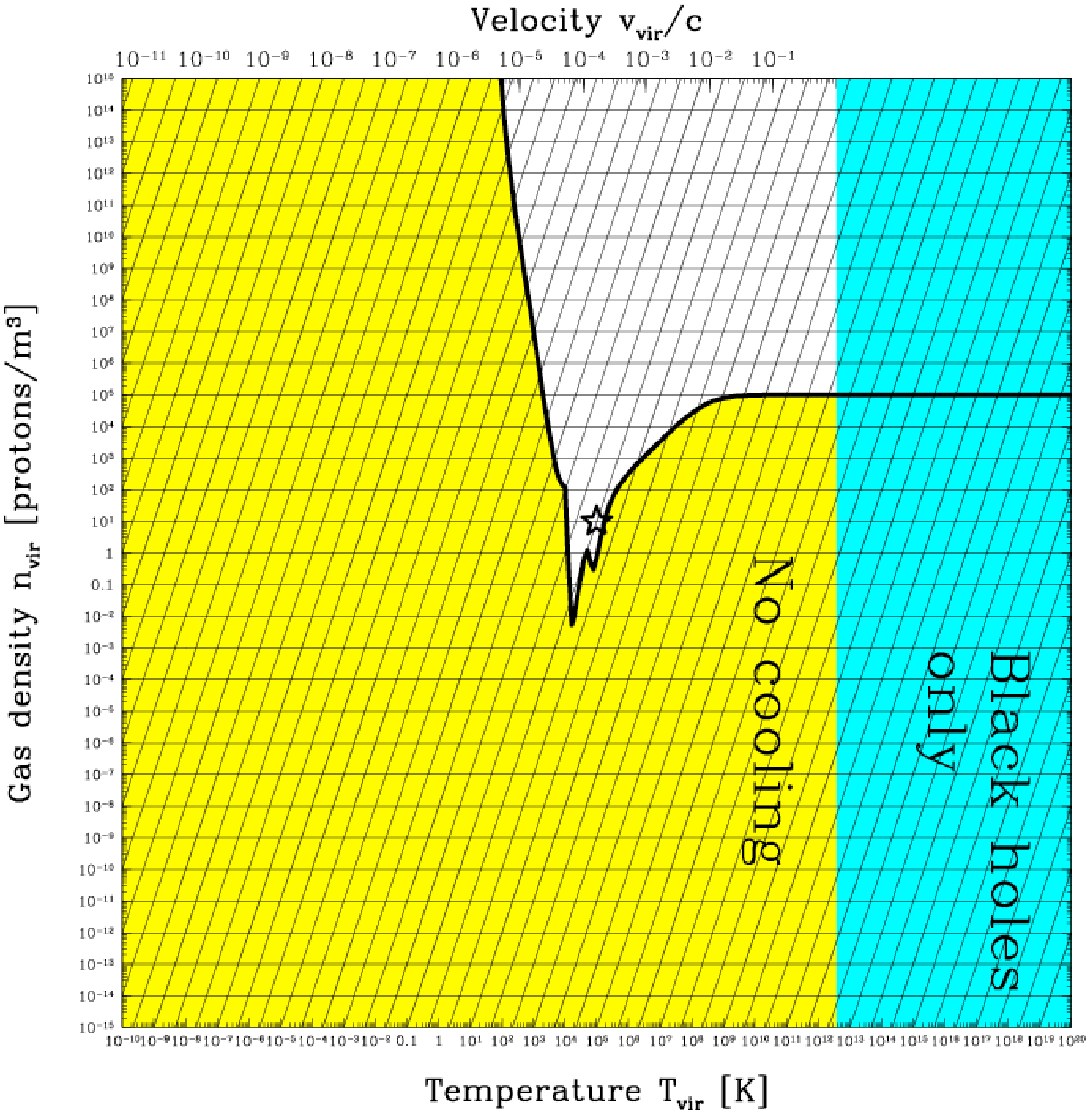
contrast →

Your fluctuations might collapse into black holes:

density ↑

time ↓

size ↓



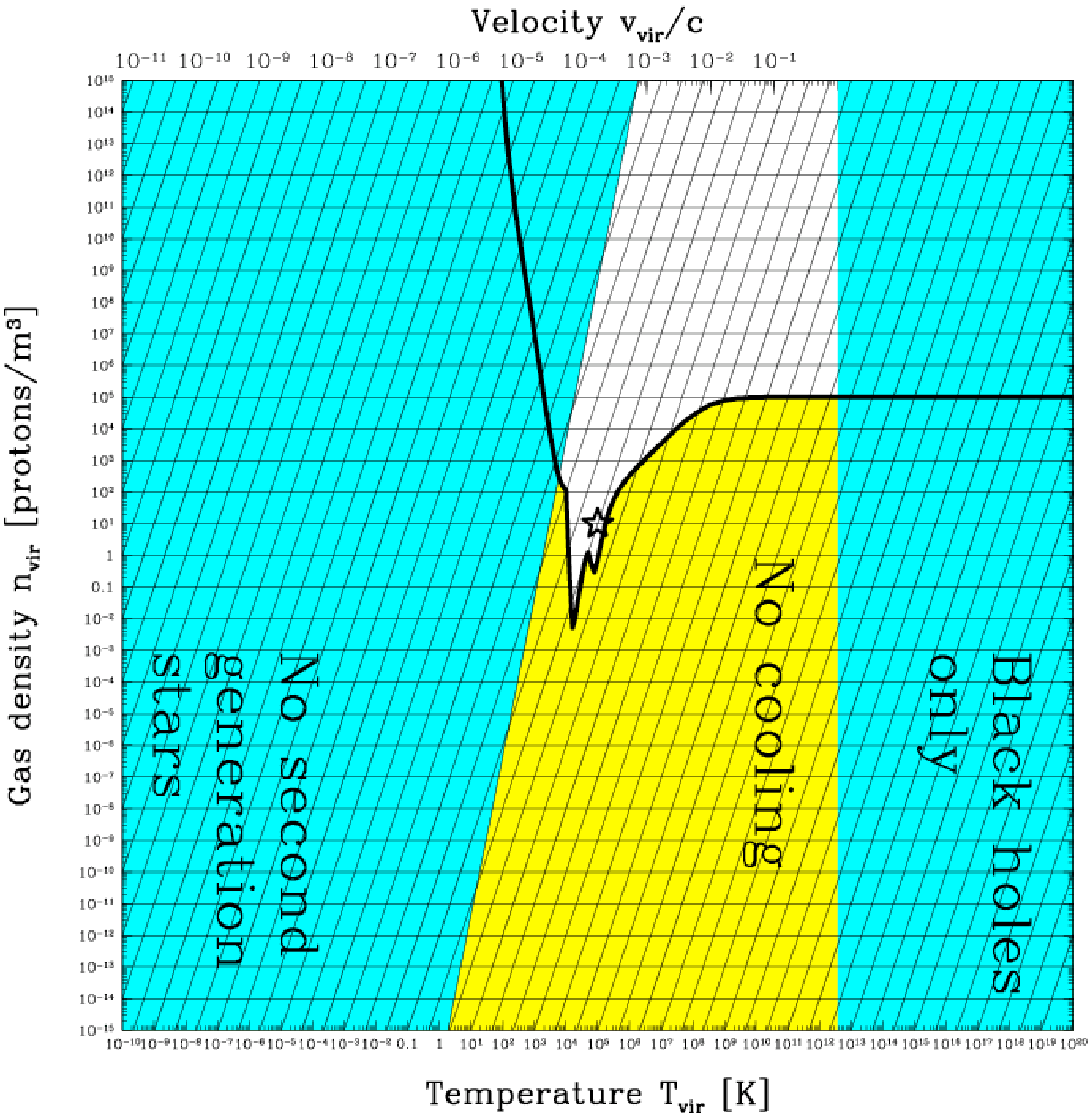
contrast →

The matter might get swept out by the first supernovae:

density ↑

time ↓

size ↓



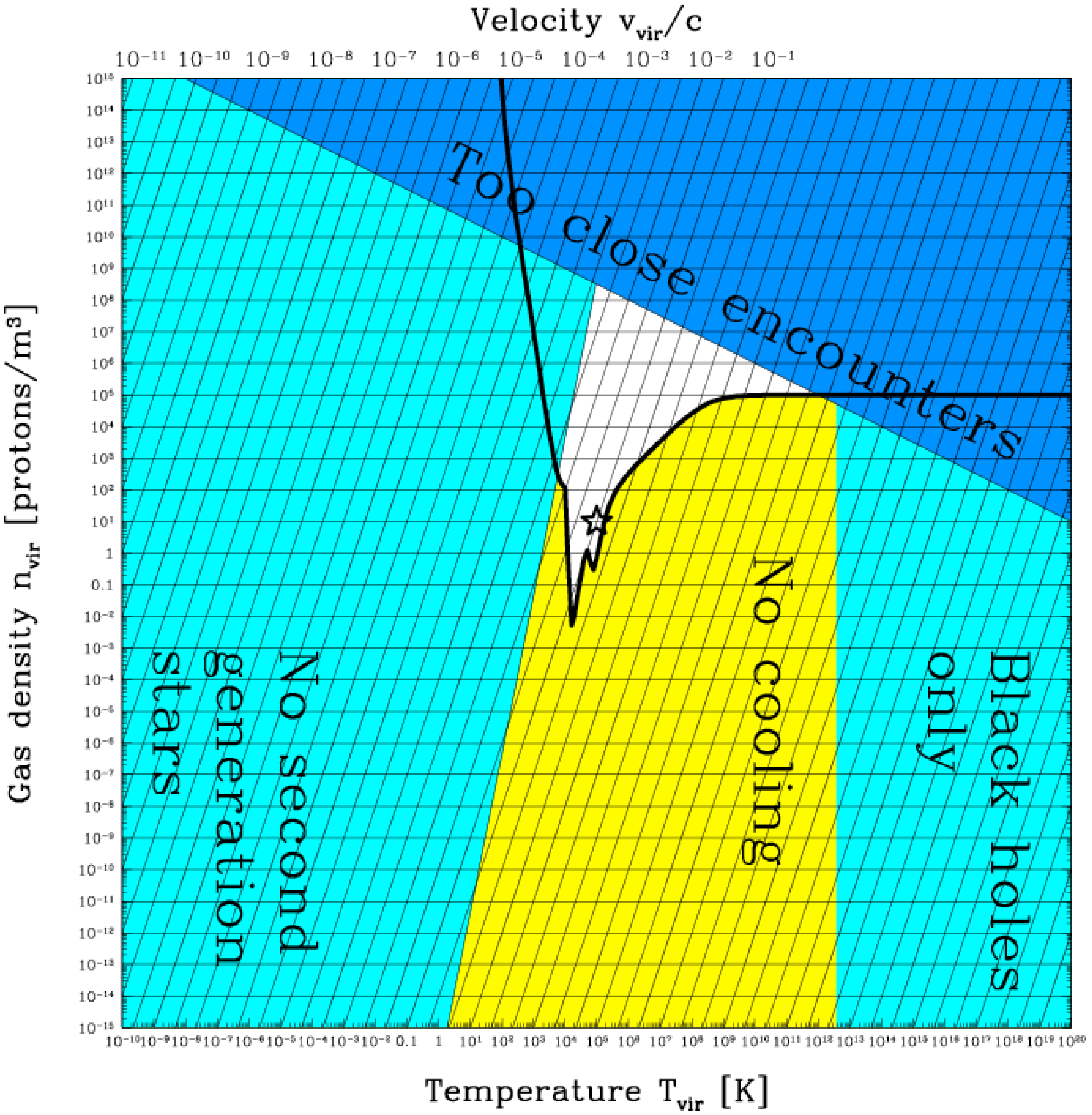
contrast →

There might be no safe haven from disruptive encounters:

density ↑

time ↓

size ↓

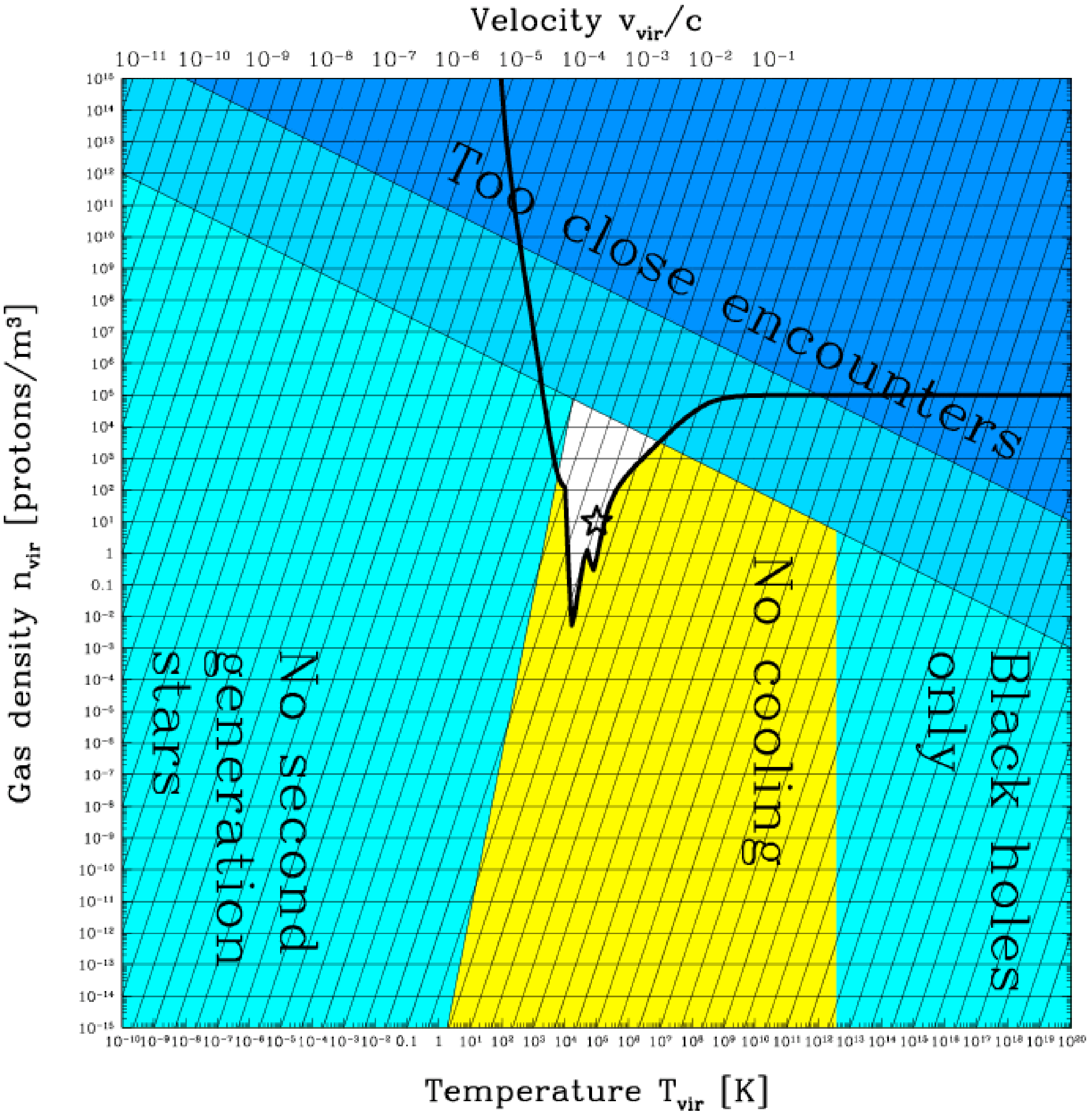


contrast →

density ↑

time ↓

size ↓

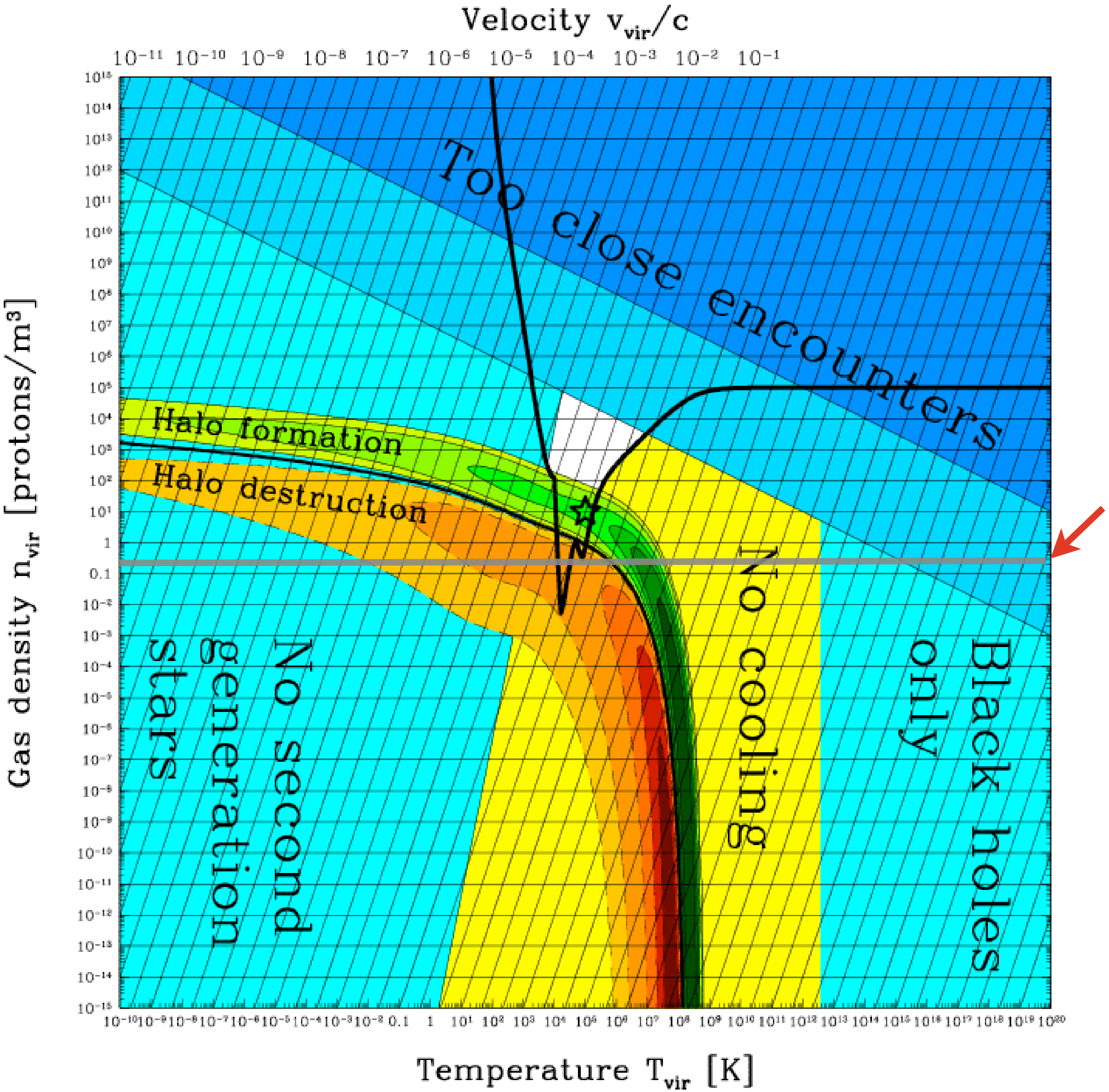


contrast →

Making Structures From Primordial Fluctuations

We can compare the seeds with what we get from primordial fluctuations.

Here is what we get with the standard fluctuation spectrum and the observed dark matter density:



Λ cutoff

density ↑

time ↓

size ↓

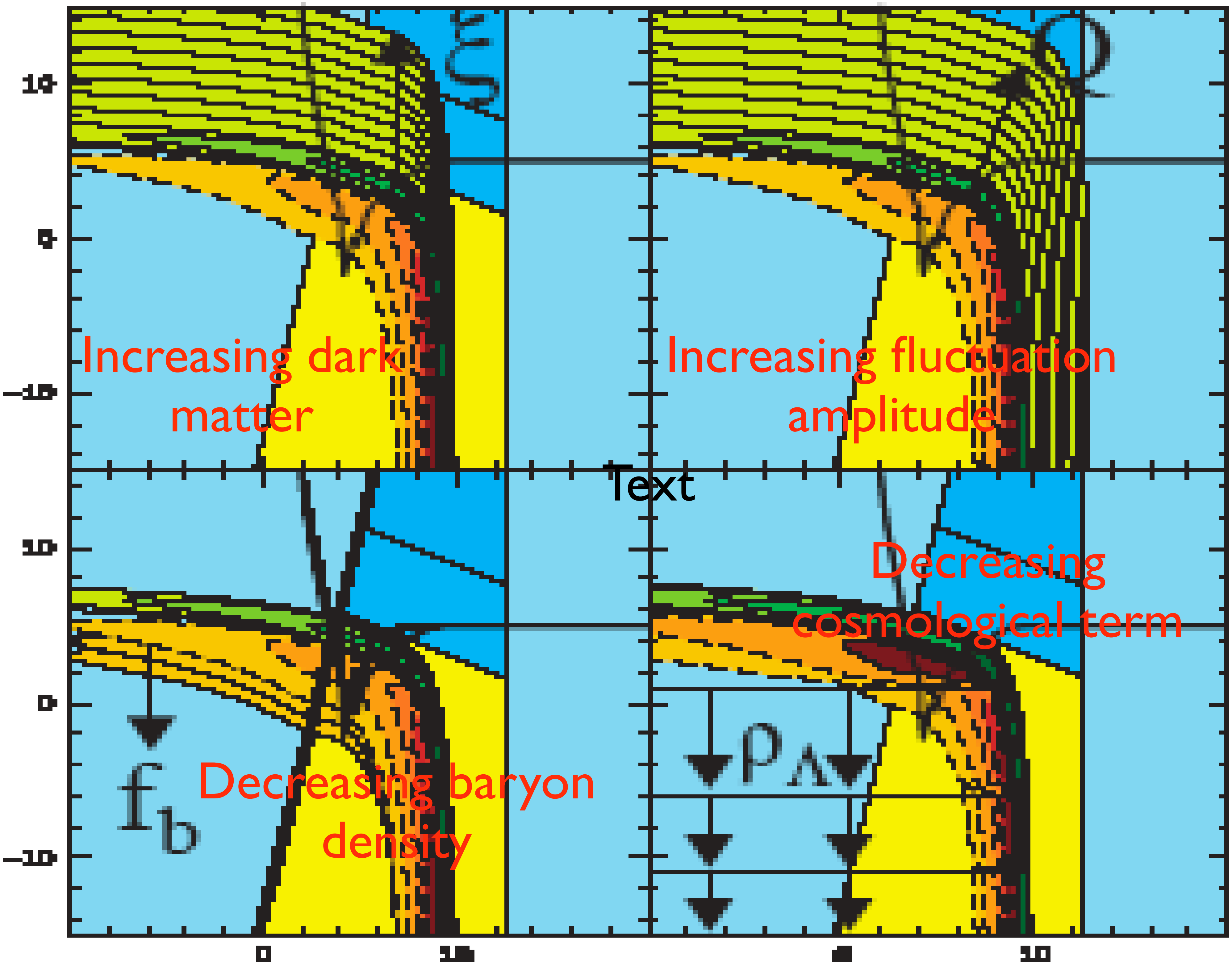
contrast →

When dark energy starts to dominate, and exponential expansion kicks in, growth of new structure is inhibited. This give the Λ_{cosmo} cutoff.

These calculations give a semi-quantitative explanation of the characteristic size of galaxies.

So far what we’ve done is entirely conventional astrophysics.

With our confidence reinforced, we can now consider the effect of changing parameters governing the primordial fluctuations:



Increasing dark matter

Increasing fluctuation amplitude

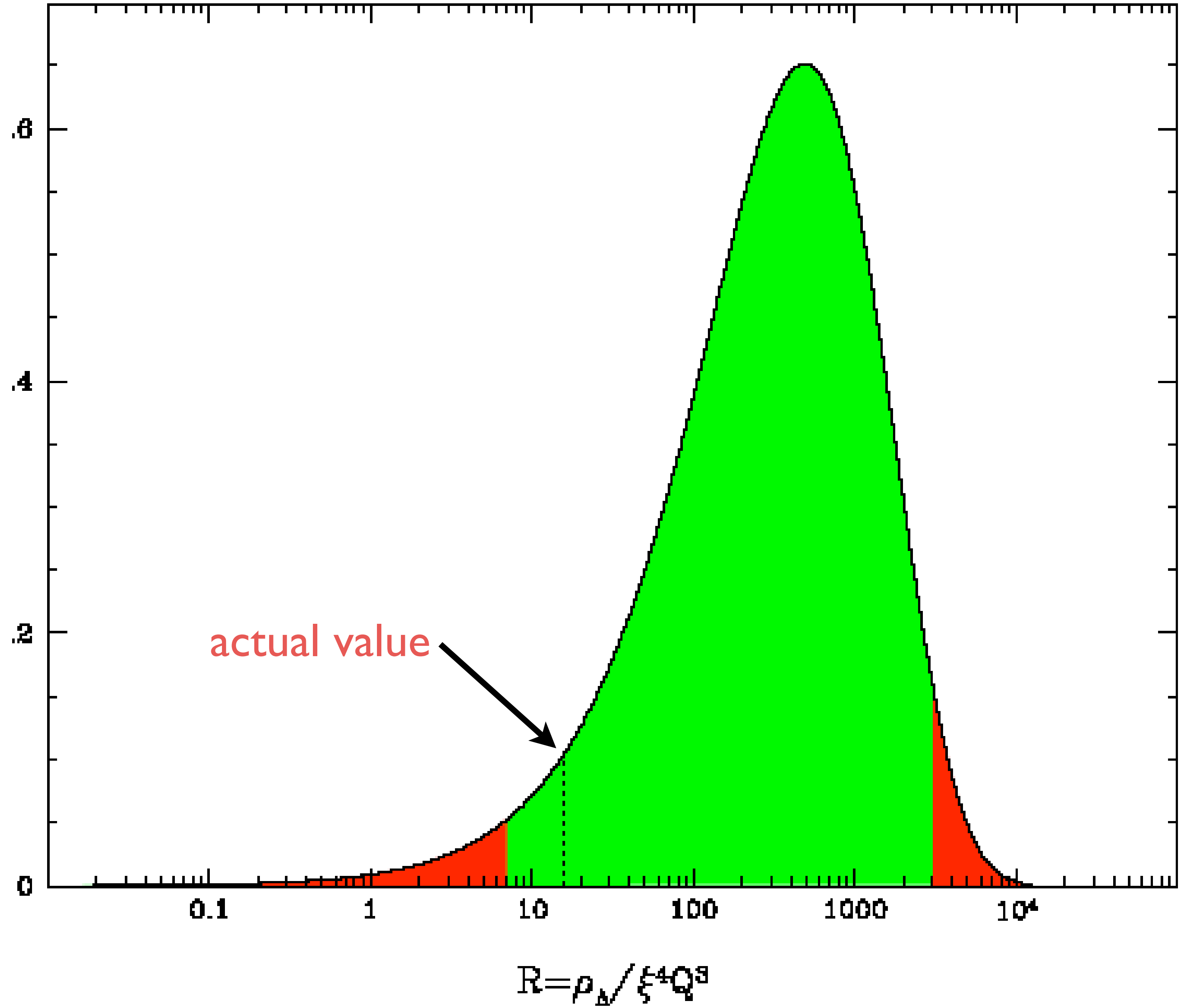
Text

Decreasing cosmological term

Decreasing baryon density

We implement selection bias by calculating probability distributions *per baryon in the user-friendly region*.

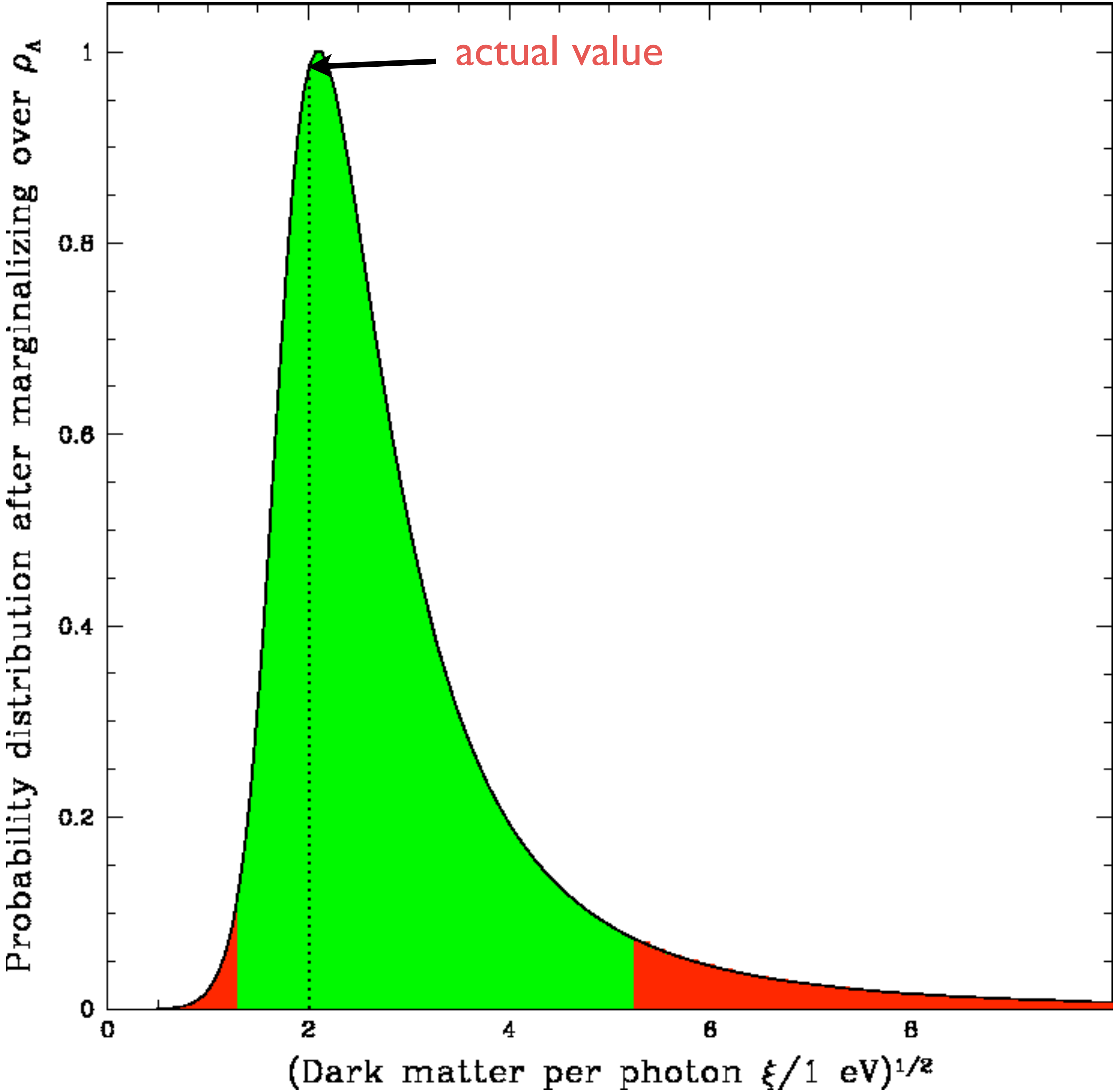
Here is the ρ_λ distribution, given a flat prior, and **holding everything else fixed**:



actual value

The result is not bad, but its foundation is very insecure.

Here is the θ_0 distribution near 0,
translated into dark matter density:


actual value

This is a striking result, I think.

The scenario with inflation after the PQ transition also removes some annoying difficulties of the traditional alternative, including the need to introduce a new scale. (For experts: axion string and domain wall issues also disappear.)

Fluctuations

An Important Test

A canonically normalized massless boson field - graviton or axion - acquires fluctuations of amplitude $T_{GH} \sim \Lambda_{infl.}^2 / M_{Pl}$.

For axions, this translates into jitter in θ_0, and thus ultimately into isocurvature density perturbations.

Constraints on isocurvature fluctuations translate into constraints on Λ_{infl} and thus on the gravity wave background.

$$\begin{aligned}\alpha_a &\equiv \frac{\langle (\delta T/T)^2_{\rm iso}\rangle}{\langle (\delta T/T)^2_{\rm tot}\rangle} \\ &\approx \frac{8}{25}\frac{(\xi_a/\xi_m)^2}{\langle (\delta T/T)^2_{\rm tot}\rangle}\sigma_\theta^2\frac{2\theta_i^2+\sigma_\theta^2}{(\theta_i^2+\sigma_\theta^2)^2}\end{aligned}$$

$$\sigma_\theta = \frac{H_I}{2\pi(f_a/N)}$$

For axions as dark matter

$$(f_a/N)\theta_i^2 \ \sim \ 10^{12} \ \mathrm{GeV}$$

$$\alpha_a \ \approx \ \frac{2}{25\pi^2} \, \frac{H_I^2}{\bar{f} \cdot 10^{12} \ \mathrm{GeV}}$$

or, using $H_I \ = \ 5\pi \, Q_t \, \bar{m}_{\mathrm{Pl}}$,

$$\alpha_a \ \approx \ Q_t^2 \, \frac{2 \, \bar{m}_{\mathrm{Pl}}^2}{\bar{f} \cdot 10^{12} \ \mathrm{GeV}}$$

So the inflationary axion cosmology could be falsified if we see a significant gravitational wave background, without a larger isocurvature background.

It could be "truthified" if we still have a dark matter problem after LHC (+ ILC), through details of the dark matter distribution, or if we discover isocurvature fluctuations.

If SUSY, and a dark matter candidate, are found at LHC, it will be important to pin its properties down and calculate its cosmological production. If that is too small, axions will happily (and naturally) supply the deficit.

Summary

Axions remain an attractive proposal for improving the standard model.

Conceptually, axion cosmology looks better than ever. It can solve the dark matter problem.

Though there is no difficulty of principle, existing ideas to test the theory are thin.

Isocurvature fluctuations are a crucial observational issue.