AdS/CFT Dual Description of Cosmological Singularities

Cambridge

Dec 2007

Thomas Hertog
(APC-Paris)

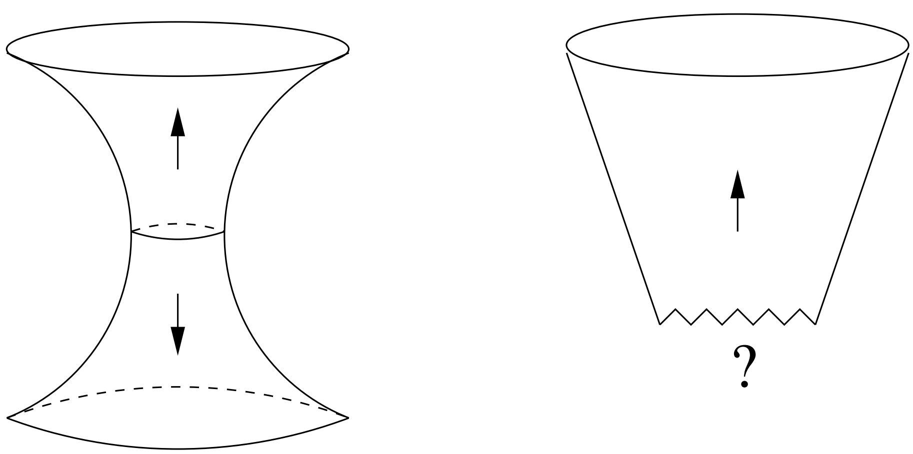
w/ Ben Craps, Neil Turok
arXiv:0711.1824[hep-th]

w/ Gary Horowitz
hep-th/0503071

Introduction

A predictive framework for cosmology must ultimately involve a quantum state of the universe.

24 years on we finally understand the predictions of the no-boundary quantum state [Hartle,Hawking,TH '07].



It selects an ensemble of inflating universes, some are approximately time-symmetric around a regular bounce, others are initially singular.

→ no-boundary resolves a specific class of cosmological singularities, in the sense that these are no longer an obstacle to prediction.

Introduction

Are there qualitatively different predictive cosmologies?

Introduction

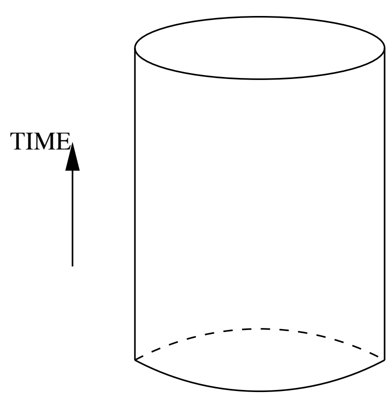
Are there qualitatively different predictive cosmologies?

String theory

$\longrightarrow$ provides a radically new viewpoint on old problems in big bang cosmology.

$\longrightarrow$ may lead to a new proposal for the quantum state of the universe in which a different class of cosmological singularities is resolved.

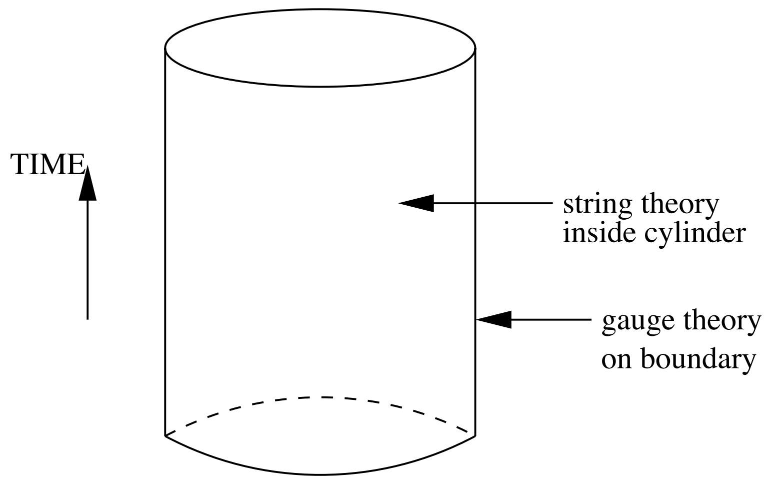
Our approach is based on holographic principle:



$\longrightarrow$ Use anti-de Sitter cylinder to model a smooth homogeneous patch of the universe.

AdS/CFT duality

String theory with anti-de Sitter boundary conditions is equivalent to certain gauge theories living on the boundary of the AdS cylinder. [Maldacena '97]



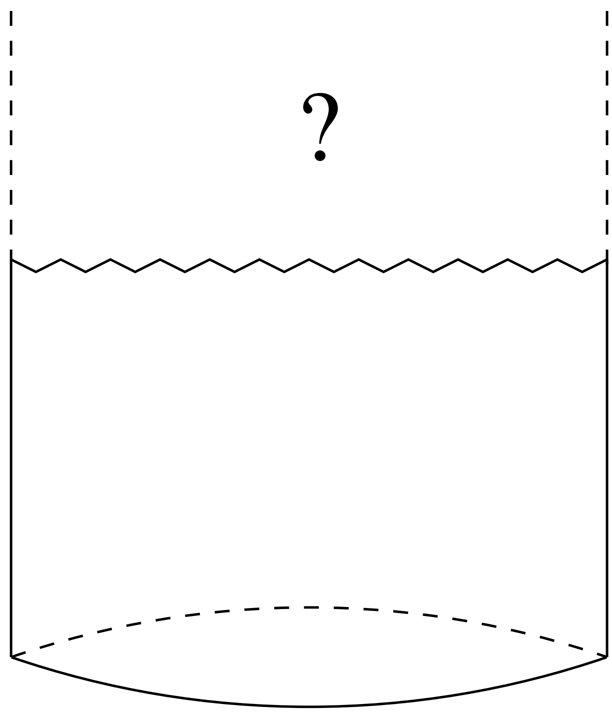
e.g. String theory on $AdS_5 \times S^5$ is dual to $\mathcal{N} = 4$ super Yang-Mills theory on $\mathbb{R} \times S^3$ with $SU(N)$, where

$$R^4/l_s^4 \leftrightarrow g_{YM}^2 N = \lambda, \qquad R^8/l_p^8 \leftrightarrow N^2$$

The finite N gauge theory is viewed as a *nonperturbative definition* of string theory with AdS boundary conditions.

Holographic Cosmology

Generalization: SUGRA solutions where smooth asymptotically AdS initial data evolve to a big crunch in the future.



1. Does the dual gauge theory evolution "resolve" the singularity in the bulk?

2. If so, what does it predict for cosmology?

Bulk Setup

Consider a consistent truncation of string theory with $AdS_5 \times S^5$ boundary conditions, at low energies, involving gravity and a single scalar field with potential

$$V = -\frac{15}{4}e^{2\gamma\varphi} - \frac{5}{2}e^{-4\gamma\varphi} + \frac{1}{4}e^{-10\gamma\varphi}$$

The scalar φ has $\quad m^2 = -4 = m^2_{BF}$

Near the boundary (at large radius r) of the anti-de Sitter cylinder φ decays as

$$\varphi(t, r, \Omega) = \frac{\alpha(t,\Omega)\ln r}{r^2} + \frac{\beta(t,\Omega)}{r^2}$$

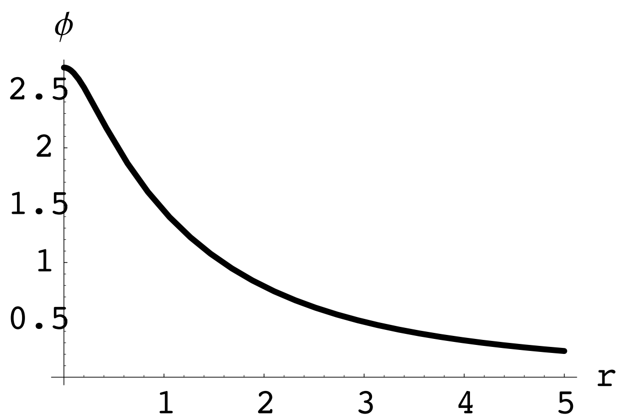
Dynamics Depends on Boundary Conditions $\alpha(\beta)$

AdS Cosmology

Take boundary conditions $\alpha = f\beta$, $\qquad f > 0$

Conserved charges remain finite but asymptotic conformal invariance is logarithmically broken.

For $f > 0$ there are smooth $M \approx 0$ initial data that evolve to a singularity which extends to the boundary of AdS in finite global time.



Asymptotically (at large r) one has

$$\varphi \approx \frac{\beta(t)}{r^2}(1 + f \ln r) + \mathcal{O}(r^{-3}), \qquad \beta(t) = \frac{\beta(0)}{\sqrt{\cos t}}$$

Boundary Field Theory

String theory with $AdS_5 \times S^5$ boundary conditions is dual to $\mathcal{N}$=4 super Yang-Mills theory in $D = 4$.

- For $\alpha = 0$, $\varphi \sim \beta/r^2$ is dual to $\Delta = 2$ operator $\mathcal{O}$,

$$\mathcal{O} = \frac{1}{N} Tr \left[\phi^2 - \frac{1}{5} \sum_{i=2}^{6} \phi_i^2 \right]$$

  and

$$\beta \leftrightarrow \langle \mathcal{O} \rangle$$

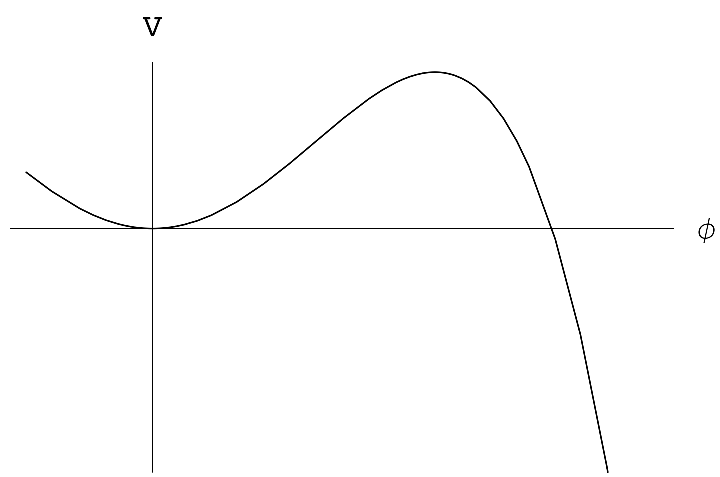
- Taking $\alpha(\beta) \neq 0$ corresponds to adding a multitrace interaction $\int W(\mathcal{O})$ to the CFT, such that [Witten '02, Berkooz et al. '02]

$$\alpha = -\frac{\delta W}{\delta \beta}$$

Instability

With $\alpha = f\beta$,

$$S = S_{YM} + \frac{f}{2}\int \mathcal{O}^2$$



$$\mathcal{V}(\phi) \sim +R^{-2}\phi^2 - f\phi^4$$

The dual description of AdS cosmologies involves field theories with effective potentials $\mathcal{V}(\langle\mathcal{O}\rangle)$ *that are unbounded from below in certain directions.*

Instability

This is a universal feature of the dual description of AdS cosmologies (in contrast to the dual description of null singularities): the field theory directly "sees" the gravitational instability to form singularities.

In particular this is also a feature of analogous cosmologies in four dimensions.
[T.H. & Horowitz '04]

Instability

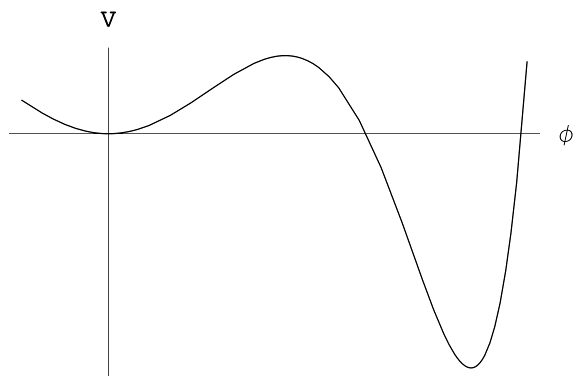
This is a universal feature of the dual description of AdS cosmologies (in contrast to the dual description of null singularities): the field theory directly "sees" the gravitational instability to form singularities.

In particular this is also a feature of analogous cosmologies in four dimensions.
[T.H. & Horowitz '04]

What are the principles?

Dynamics: Qualitatively

Consider first a regularized version of the field theory:



$$V(\phi) \sim +R^{-2}\phi^2 - f\phi^4 + \epsilon\phi^8$$

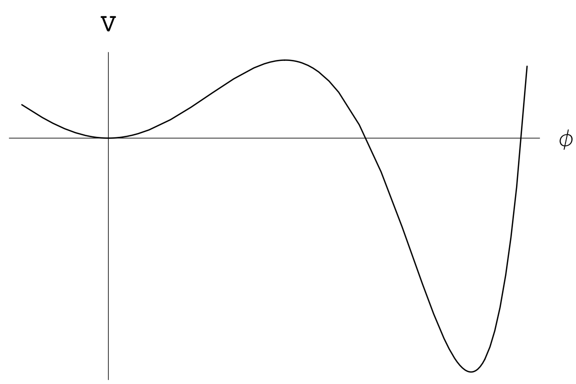
When a wave packet rolls down the evolution becomes ultralocal:

$$\delta\ddot{\phi} = -V_{,\phi\phi}\delta\phi - k^2\delta\phi$$

$\longrightarrow$ inhomogeneities become dynamically unimportant when $k^2 \leq |V_{,\phi\phi}|$, and different spatial points decouple

Dynamics: Qualitatively

Consider first a regularized version of the field theory:



$$V(\phi) \sim +R^{-2}\phi^2 - f\phi^4 + \epsilon\phi^8$$

When a wave packet rolls down the evolution becomes ultralocal:

$$\delta\ddot{\phi} = -V_{,\phi\phi}\delta\phi - k^2\delta\phi$$

$\longrightarrow$ inhomogeneities become dynamically unimportant when $k^2 \leq |V_{,\phi\phi}|$, and different spatial points decouple

Stationary endstate in this model, but maybe evolution at intermediate times is more relevant for cosmology??

Effective Potential

$$S = S_{YM} + \frac{f}{2} \int \mathcal{O}^2$$

This is renormalizable and asymptotically free in f, with one-loop exact effective potential (at large $\mathcal{O}$)

$$\mathcal{V}(\mathcal{O}) = -\frac{\mathcal{O}^2}{\ln(\mathcal{O}/M^2)} \quad \rightarrow \quad -\frac{\phi^4}{N^2 \ln(\phi/M)}$$

Hence $\mathcal{V}(\mathcal{O}) \rightarrow -\infty$ for $\mathcal{O} \rightarrow \infty$

Note: logarithmic running consistent with asymptotic behavior of bulk scalar [Witten '02]

Ultralocality implies different spatial points decouple at large ϕ

$\rightarrow$ describe quantum field by set of independent quantum mechanical systems.

$\rightarrow$ quantum mechanics with unbounded potentials.

Quantum Mechanics

A right-moving wave packet in $V(x)$ follows essentially a WKB trajectory and reaches infinity in finite time.

To ensure probability is not lost at infinity one constructs a self-adjoint extension of the Hamiltonian, by carefully specifying its domain [Reed & Simon 70's]

$\longrightarrow$ unitary evolution for all time.

Physically self-adjoint boundary conditions mean a right-moving wavepacket is always accompanied by a left-moving "reflected" wavepacket. Schematically,

$$\Psi \sim \left(e^{iS_1} + e^{i\theta} e^{iS_2}\right) \sim \frac{1}{x} \cos(x^3 + \theta)$$

$$|\Psi|^2 \sim x^{-2} \quad \text{at large } x$$

Quantum Mechanics

A right-moving wave packet in $V(x)$ follows essentially a WKB trajectory and reaches infinity in finite time.

To ensure probability is not lost at infinity one constructs a self-adjoint extension of the Hamiltonian, by carefully specifying its domain [Reed & Simon 70's]

$\rightarrow$ unitary evolution for all time.

Physically self-adjoint boundary conditions mean a right-moving wavepacket is always accompanied by a left-moving "reflected" wavepacket. Schematically,

$$\Psi \sim \left(e^{iS_1} + e^{i\theta} e^{iS_2}\right) \sim \frac{1}{x}\cos(x^3 + \theta)$$

$$|\Psi|^2 \sim x^{-2} \quad \text{at large } x$$

We implement self-adjoint extension point by point.

Is this self-consistent?

Field Theory Evolution

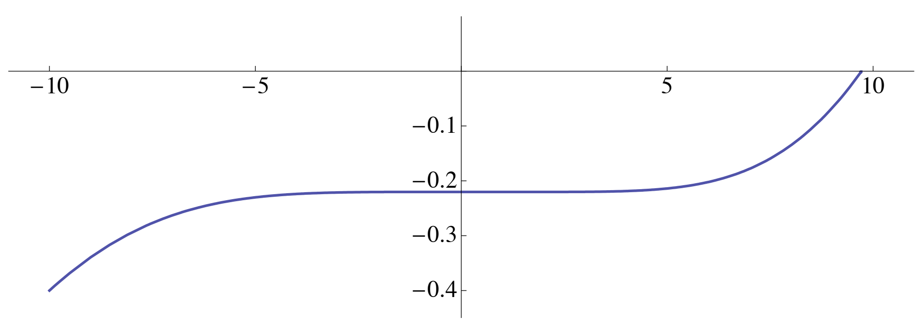
$$\Psi(t_f, \bar{\phi}_f, \delta\phi) \sim \left(e^{iS_1} + e^{i\theta}e^{iS_2}\right)$$

Initial conditions:

1. Gaussian wavepacket for homogeneous background on S^3: $\phi + 2i\hbar^{-1}\pi_\phi(\Delta\phi)^2 = -\phi_c$

2. Fluctuations $\delta\phi$ in ground state

$\longrightarrow$ *quantum spread leads to complex classical background solution* (shown here is $\phi^{-1}(t)$)



$$\delta\ddot{\phi} = -V_{,\phi\phi}\delta\phi - k^2\delta\phi$$

$\longrightarrow$ high energy particle production suppressed for most $\bar{\phi}$ indicating backreaction is under control.

Conclusion

- A dual 'holographic' description of (AdS) cosmologies involves unstable conformal field theories.
- The ultralocality of the field theory evolution near the singularity means one can specify consistent unitary quantum evolution on the boundary by imposing a self-adjoint extension point by point.
- The quantum spread of the unstable homogeneous mode provides a UV cutoff on particle creation.
- For a certain range of parameters this causes the homogeneous field to roll back up.
- It is natural to interpret this in the bulk as a quantum transition from big crunch to big bang.

Reasonable Speculation

- This suggests that it should be possible to define a quantum state of the universe holographically, as a state in a dual field theory. In this quantum state, the dominant cosmology would bounce and have an arrow of time pointing in the same direction everywhere.