I. Brief Overview of Inflationary non-Gaussianity

II. Developments in string cosmology

I. Bispectrum $\langle \Phi(k_1)\Phi(k_2)\Phi(k_3)\rangle$

Φ ← $-$(Newtonian pot.)

(Komatsu Spergel Wandelt Banday Gorski + WMAP
Babich Creminelli Zaldarriaga Senatore Tegmark
Yadav Fergusson Shellard ...)

contains information about interactions of the field(s) involved in inflation.

$$\langle \Phi(k_1,t)\Phi(k_2,t)\Phi(k_3,t)\rangle$$

$$= \langle U_{int}^{-1}\,\Phi\Phi\Phi\,\underbrace{U_{int}(t,t_0)}_{Te^{-i\int_{t_0}^{t} H_{int}(t')dt'}}\rangle$$

$$\simeq -i\int_{t_0}^{t} dt' \left[\Phi^3(t),\, H_{int}(t')\right] + \cdots$$

(Maldacena; Acquaviva Bartolo Matarrese Riotto; Allen Grinstein Wise; Falk Rangarajan Srednicki; Gangui ...)

$$\langle \Phi(\vec{k}_1,t)\,\Phi(\vec{k}_2,t)\,\Phi(\vec{k}_3,t)\rangle \text{ depends on}$$

$$9 - \underset{\substack{\text{translations}\\ \sum \vec{k}=0}}{3} - \underset{\text{rotations}}{3} - \underset{\substack{\text{dimensional analysis}\\ (\Phi(x,t)\text{ dim. } 0)}}{1} = 2 \text{ variables}$$

$$\langle \Phi(\vec{k}_1,t)\,\Phi(\vec{k}_2,t)\,\Phi(\vec{k}_3,t)\rangle$$

$$= (2\pi)^3\,\delta\!\left(\sum \vec{k}\right)\underbrace{F(\vec{k}_1,\vec{k}_2,\vec{k}_3)}_{\text{homogeneous, } \sim k^{-6}}$$

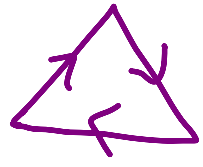
2 Special Cases:

$$F_{\text{local}} = 2(2\pi)^4\left(-\frac{3}{5}\,\underline{\underline{f_{NL}^{\text{local}}}}\,\mathcal{P}_{\mathcal{R}}^2\right)\frac{\sum_i k_i^3}{\prod_j k_j^3}$$

from $\Phi = \Phi_g + f_{NL}\Phi_g^2$

$$F_{\text{equilateral}} = f_{NL}^{\text{eq.}}\cdot 6\,\mathcal{P}_{\mathcal{R}}^2\left(\frac{1}{k_1^3 k_2^3} + (2\text{ perm.}) - \frac{2}{(k_1k_2k_3)^2} + \frac{1}{k_1 k_2^2 k_3^3} + (5\text{ perm.})\right)$$

+ tilt



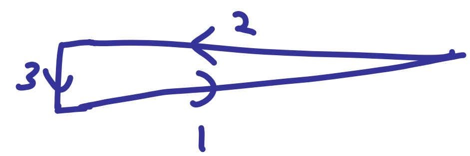
In single-field, slow roll inflation, the fields interact too weakly to produce $\langle \Phi^3 \rangle$ at a level accessible to CMBR observations.

- $\mathcal{L} = (\partial_\mu \varphi)^2 - V(\varphi)$

$\varepsilon = M_4 \frac{V'}{V} \qquad \eta = M_4^2 \frac{V''}{V} \qquad \ll 1$ ⇒ flat potential

$$V(\varphi) = V(\varphi_0) + V' \delta\varphi + \underbrace{V'' \delta\varphi^2 + \cdots}_{\text{must be small}}$$

- e.g. $k_1 \ll k_{2,3}$



$$\langle \Phi(\vec{k}_1) \Phi(\vec{k}_2) \Phi(\vec{k}_3) \rangle \propto$$

$$(2\pi)^3 \delta^{(3)}(\Sigma \vec{k}) (\eta - 3\varepsilon) \frac{1}{k_1^3} \frac{1}{k_2^3} \underbrace{P_R^2}_{\text{Gaussian power}}$$

$$\Rightarrow f_{NL} \sim (n_s - 1) \ll 1$$

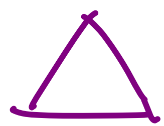
In a nutshell –

Slow Roll inflation requires flat potential $V(\varphi) \Rightarrow$ small self-interactions.

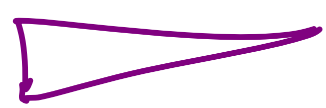
In contrast – Inflationary non-Gaussianity can be naturally large when ...

(1) • Self-interactions of the field slow it down e.g. DBI, ghost, k infl.

• General $\mathcal{L}\left((\partial\varphi)^2, \varphi\right)$

↪ equilateral shape 

(2) Other light fields, which can have large self-interactions, generate $\delta\rho/\rho$ multifield, curvaton → Linde, Bond, Lyth Talks
modulated reheating

↪ squeezed/local shape 

(1) Consider

Armendariz-Picon, Damour, Mukhanov
Garriga Gruzinov, Seery Lidsey
Chen Huang Kachru Shiu, Cheung et al

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_p^2 R + \mathcal{L}_\phi\left((\partial\phi)^2, \phi\right)\right]$$

Inflation arises given

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \ll 1 \qquad \tilde{\eta} \equiv \frac{\dot{\varepsilon}}{\varepsilon H} \ll 1$$

$$s = \frac{\dot{c}_s}{c_s H} \ll 1$$

Potential $V(\phi)$ need not be flat, and interactions can be substantial consistently with inflation.

↳ * also can help slow down the inflaton

In e.g. DBI inflation,

ES/Tong '03; Alishahiha/S/T '04; Chen

$$\mathcal{L}_\phi(\phi, \partial\phi) = -f(\phi)^{-1}\sqrt{1-(\partial\phi)^2 f(\phi)} - V(\phi)$$

$$\boxed{\dot\phi^2 < f(\phi)^{-1} \to \cdots \to \text{inflation on } \underline{\text{steep}}\ V(\phi)}$$

enforced by the <u>interactions</u>

[cf relativistic particle $\dot x^2 < 1$

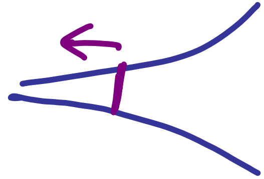
$$S = -m\int dt\sqrt{1-\dot x^2}$$

$$\delta S = 0 \Rightarrow \frac{d}{dt}\left(\frac{m\dot x}{\sqrt{1-\dot x^2}}\right) = 0 \Rightarrow \frac{m\dot x}{\sqrt{1-\dot x^2}} = E$$
]

e.g. $f(\phi) \sim \frac{\lambda}{\phi^4} \Rightarrow \dot\phi^2 < \frac{\phi^4}{\lambda}$

$$ds^2 = \frac{\phi^2}{\sqrt\lambda}dx^2 + \frac{\sqrt\lambda}{\phi^2}d\phi^2 + \ldots$$

AdS/CFT: takes forever to reach horizon.



In e.g. DBI inflation,

$$\mathcal{L}_\varphi(\varphi,\partial\varphi) = -f(\varphi)^{-1}\sqrt{1-(\partial\varphi)^2 f(\varphi)} - V(\varphi)$$

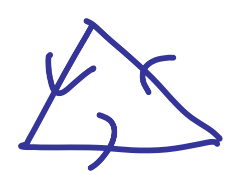
$\dot\varphi^2 < f(\varphi)^{-1}$ is enforced by the dynamics ...

And this <u>same</u> dynamics

$\Rightarrow$ Large non-Gaussianity:

$$\delta_\varphi S \sim \gamma \sim \frac{1}{\sqrt{1-f\dot\varphi^2}}$$

$$\left.\begin{aligned}\delta_\varphi^2 S &\sim \gamma^3\\ \delta_\varphi^3 S &\sim \gamma^5\end{aligned}\right\} \frac{\mathcal{L}_3}{\mathcal{L}_2} \propto \gamma^2$$

Full 3-pt Fn $F(k_1,k_2,k_3) = \dots$ 

Alishahiha, ES, Tong '04

↪ falsifiable as single-field model.

<u>Note</u> – In general, one has higher dimension operators

$$\mathcal{L} = \mathcal{L}_0 + \frac{(\partial \varphi)^4}{M_*^4} + \cdots$$

(Creminelli)

in the effective theory, from physics at scale M_*.

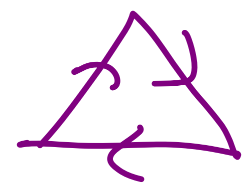
☆ $\langle \varphi \rangle$ <u>itself</u> determines masses of other fields, in general.

$$\varphi^2 \chi^2 \Rightarrow \text{series} \quad \frac{(\partial \varphi)^2}{\varphi^4} \cdot g \cdot N_\chi$$

$m_\chi = \varphi$

The shape of the 3-point function in single-field models has been computed in general

Babich Creminelli Zaldarriaga   Gruzinov
Chen Huang   Kachru Shiu   Cheung et al
next talk   Lidsey Seery

Lesson: In models with derivative interactions, $F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ peaks at equilateral shape 

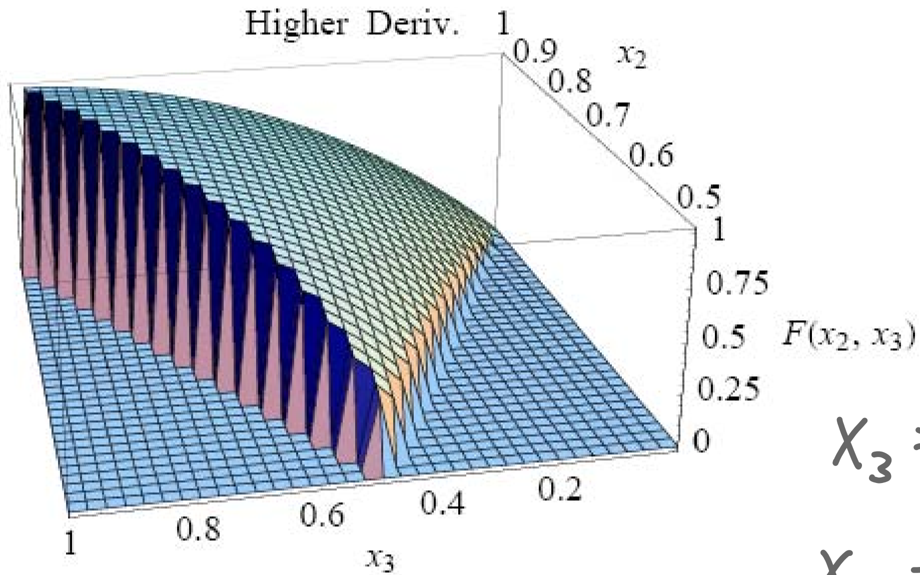
Data: WMAP; Creminelli, Senatore, Zaldarriaga, Tegmark

$$-256 < f_{NL}^{equil} < 332 \qquad 95\% \text{ C.L.}$$

DBI as source of inflation and* of $\delta\rho/\rho$* requires $|f_{NL}^{eq}| \sim \left|(-4)\left(\frac{20}{216}\gamma^2\right)\right| >> 1$

* multifield models, e.g. with curvatons or modulated reheating, much less constrained

Babich, Creminelli, Zaldarriaga



$X_3 = k_3/k_1$

$X_2 = k_2/k_1$

Figure 3: Plot of the function $F(1, x_2, x_3)\, x_2^2 x_3^2$ for non-Gaussianities generated by higher derivative interactions (12) and in the DBI model of inflation [20, 21]. The figure is normalized to have value 1 for equilateral configurations $x_2 = x_3 = 1$ and set to zero outside the region $1 - x_2 \leq x_3 \leq x_2$.

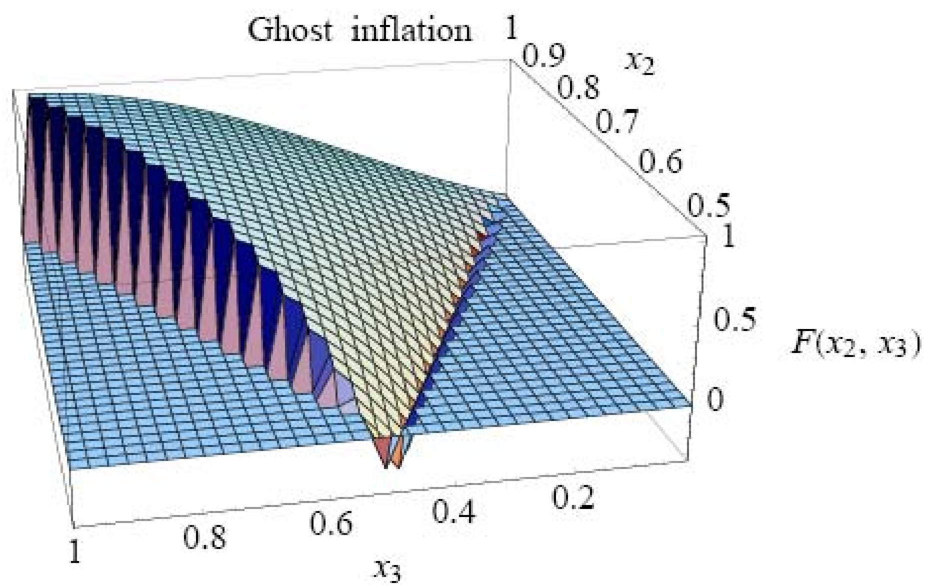


Figure 4: Plot of the function $F(1, x_2, x_3)\, x_2^2 x_3^2$ for ghost inflation (13). The figure is normalized to have value 1 for equilateral configurations $x_2 = x_3 = 1$ and set to zero outside the region $1 - x_2 \leq x_3 \leq x_2$.

New analyses (Yadav & Wandelt (preprint)
Komatsu (talk))

suggest $f_{NL}^{local} > 27$

primordial ?? cf Spergel talk here

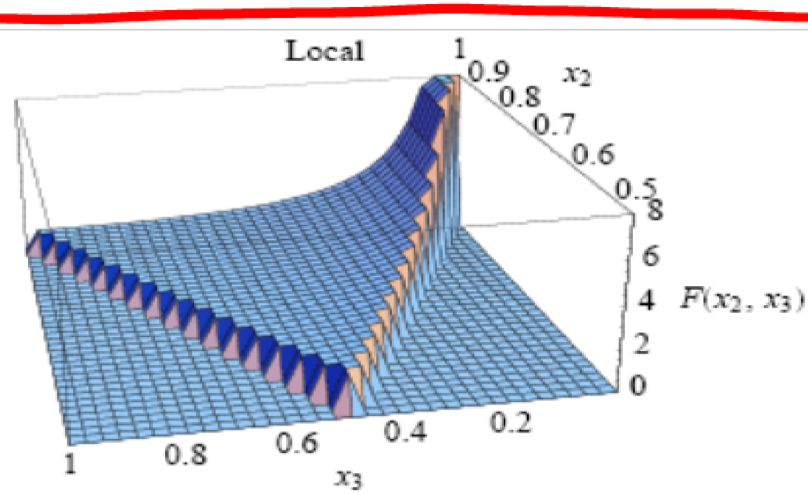


Figure 1: Plot of the function $F(1, x_2, x_3)\, x_2^2 x_3^2$ for the local distribution (6). The figure is normalized to have value 1 for equilateral configurations $x_2 = x_3 = 1$ and set to zero outside the region $1 - x_2 \le x_3 \le x_2$.

So let's review inflationary mechanisms for this.

multifield : Salopek & Bond, Bernardeau & Uzan, Bartolo, Matarrese, Riotto, ..

curvaton : Linde / Mukhanov, Lyth Wands ...

modulated reheating : Dvali, Gruzinov, Zaldarriaga
Kofman ...

Start with any inflationary sector ($\underset{\text{slow roll, higher deriv, } \vdots}{\underline{\underline{X}}}$ inflation)

slow roll
higher deriv
$\vdots$

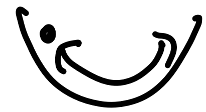
$\left.\begin{matrix}\text{slow roll} \\ \text{higher deriv} \\ \vdots\end{matrix}\right\}$ Hidden sector

& assume one or more extra <u>light</u> scalar fields $\chi_\perp$

$$m_{\chi_\perp} \ll H$$

- $\chi_\perp$ fluctuates during inflation
- Its interactions are not constrained by slow-roll conditions
- The relation between $\delta\chi$ and curvature perturbation is also nonlinear in general

$\to f_{NL}^{local}$ can be large

↪ mode evolution outside horizon dominates $\longrightarrow$ "local" shape e.g. • "Curvaton" : ρ_χ oscillations 

eventually compete with radiation

↪ χ contributes to curvature perturbation.

$$\rho_\chi \sim m_\chi^2 \tilde{\chi}^2$$

↖ amplitude of oscillation

$$\Rightarrow \frac{\delta\rho_\chi}{\rho_\chi} = 2\frac{\delta\chi}{\chi} + \frac{(\delta\chi)^2}{\chi^2}$$

cf A. Linde, Mukhanov talks

Lyth Ungarelli Wands

e.g. "modulated reheating":

decay rate Γ of inflaton depends on χ (nonlinearly)

$\Gamma(\chi(x)) \Rightarrow$ different ratio of radiation vs matter in different regions during reheating.

$$\Rightarrow ds^2 = -dt^2 + g^2\underbrace{(\Gamma(x))}_{\text{nonlinear}}\, t\, dx^2$$

after reheating.

$$\Phi \sim \frac{\delta g}{g} \sim A\,\delta\chi + B\,\delta\chi^2 + \cdots$$

$$\langle \delta\chi^3 \rangle = \cdots + N_{k_t}\left(\frac{\partial^3 V}{\partial \chi^3}\right) F_{local}$$

efoldings b/w horizon crossing & reheating

To get large f_{NL}, at fixed amplitude of $\langle \Phi^2 \rangle$,
one requires lower efficiency of the curvaton or mod. reheating mechanism (so $\delta\chi$ larger

$$\Rightarrow \uparrow \frac{\frac{\delta^3 \mathcal{L}}{\delta \chi^3}\delta\chi^3}{\frac{\delta^2 \mathcal{L}}{\delta \chi^2}\delta\chi^2} \Bigg)$$

$\rightarrow$ Need $m_\chi \ll H$ field
<u>and</u> appropriate couplings to ensure this.

Inflation works well in effective field thy

<u>However</u> : a UV completion (e.g. string theory) <u>is</u> potentially important :

Reviews: McAllister & E.S. '07; Tye, Kallosh, Burgess lectures

<u>Singularities</u>

- winding strings
- AdS/CFT & matrix models

Inflationary model building $\mathcal{L}(\varphi, (\partial\varphi)^2)$

- Planck-suppressed terms matter (η prob.)
- New mechanisms { warped throats Giddings Kachru Polchinski; KKLT, KKLMMT, ...
  for brane inflation Dvali Tye, Alexander, Burgess et al
  DBI AST

(need top-down model)

Slow roll: Baumann et al
cf Chen, Bean et al, Becker LeBlond Shandera, Kobayashi et al ..

Two recent developments

(1) A relation between Hubble Expansion (an IR effect) and the UV density of States of perturbative string theory.

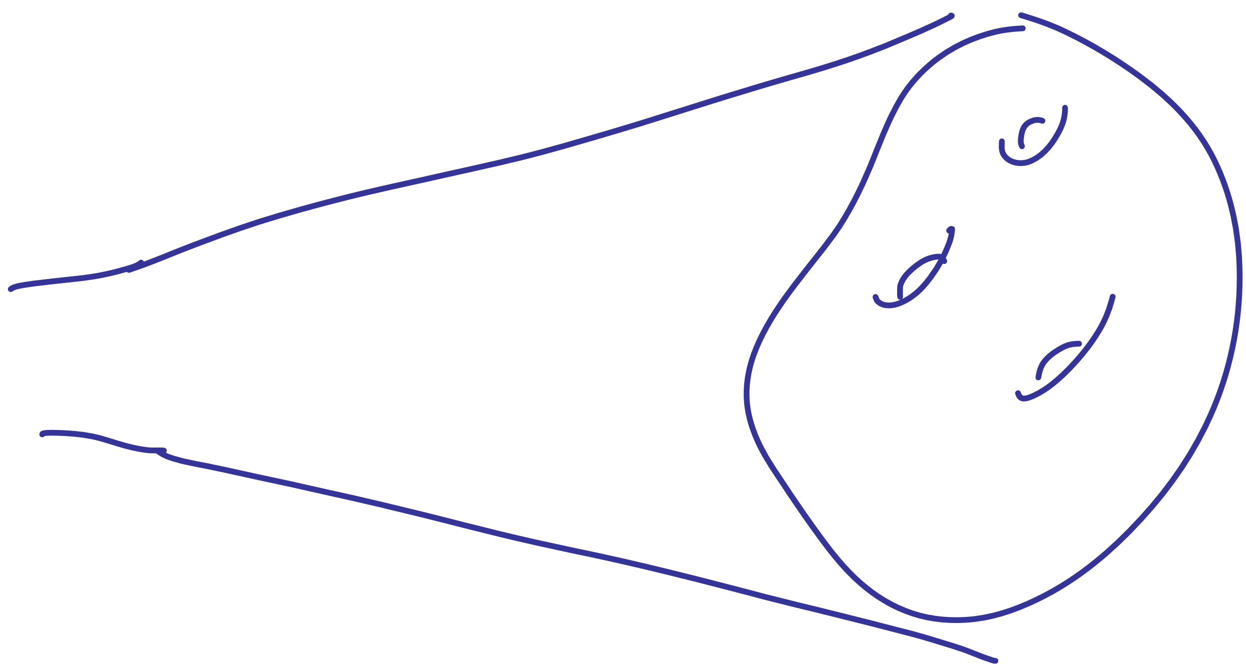
ES '05, Aharony ES '06
McGreevy Starr ES '06
Green Lawrence McGreevy Morrison ES
Maloney '07

(2) A (relatively) simple and explicit construction of metastable de Sitter space in string theory.

'07

Basic Question - what are the degrees of freedom appropriate to describe spacetime?

- At long distances: GR & QFT

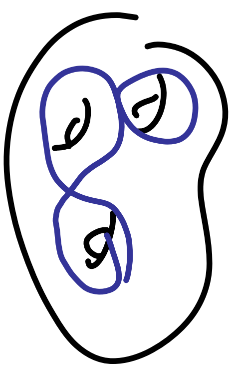


- As the universe shrinks, this question becomes UV-sensitive.

Consider compact 3d spatial slices M_3. These are now classified (up to group theory questions): Thurston Hamilton ...

They are built out of 8 elements:

$$S^3 \dots T^3 \dots \mathrm{Nil} \quad \mathrm{Sol} \quad \dots \quad \mathbb{H}_3/\Gamma$$



all but one have nontrivial $\pi_1(M_3)$

Most are hyperbolic

In the context of perturbative string theory, winding strings arise and must be taken into account.

In most spaces, $\pi_1(M)$ is of <u>exponential growth</u>:

(Milnor, Margulis, Selberg,)...

$$\rho(\ell) \sim \frac{e^{\frac{\ell}{\ell_0}}}{\ell^{\#}}$$

e.g. hyperbolic

Number of non-contractible periodic geodesics as a function of length ℓ.

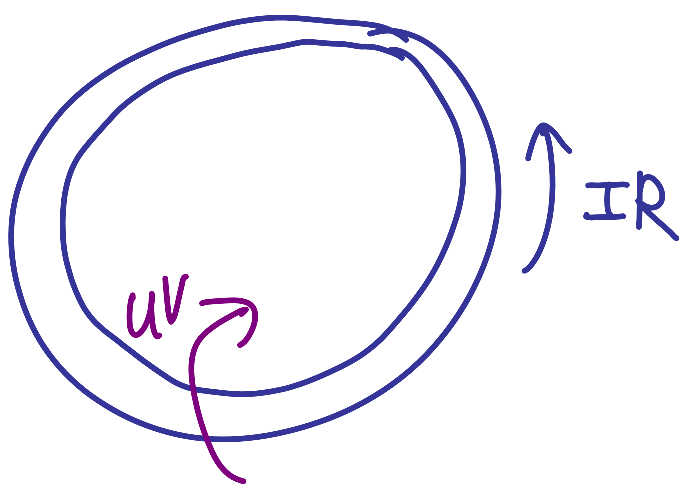
$\Rightarrow$ Since $m_{\text{winding}} = \frac{\ell}{\alpha'}$, winding strings contribute to <u>Hagedorn</u> spectrum

$$\rho(m) \sim e^{m\sqrt{\alpha'}\,\pi\sqrt{D_{\text{eff}}}}$$

(In flat space, the Hagedorn density depends on # directions of oscillation)

This enhancement to the UV density of states is related to the IR physics of the expanding space.

One-loop partition function



We analyzed this in detail for $\mathcal{M}_n \equiv \mathbb{H}_n/\Gamma$

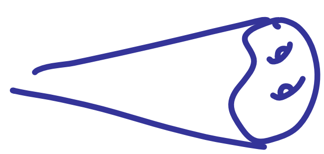
(compact hyperbolic)

simplest background solving vacuum equations (candidate worldsheet CFT)

$$ds^2 = -dt^2 + t^2\, ds^2_{\begin{cases}\mathbb{H}_n \\ \mathbb{H}_n/\Gamma\end{cases}}$$

covering space = Minkowski space

compact hyperbolic $\mathcal{M}_n$



Late times $t \gg \sqrt{\alpha'}$: $H^2 = \left(\frac{\dot a}{a}\right)^2 \ll \frac{1}{\alpha'}$

semiclassical control

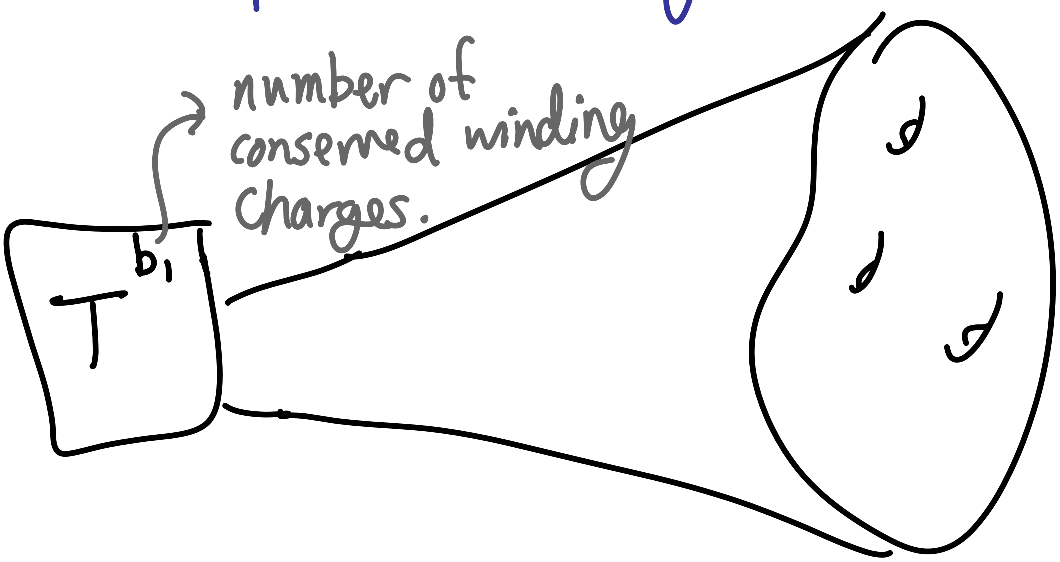
$\underline{\text{UV}}$ $\leftrightarrow$ $\underline{\text{IR}}$

$\mathbb{H}_n$: $D_{eff} = D_{eff}^{crit}$ $\quad \rho(\ell) = 0$ $\quad m^2_{min} = 0$ spatial gap

$\mathbb{H}_n/\Gamma$ $\quad D_{eff} = D_{eff}^{crit} + \frac{\alpha'(n-1)^2}{t^2}$ $\quad \rho(\ell) \sim e^{\ell/\ell_0}$ $\quad m^2_{min} = -\frac{(n-1)^2}{4t^2}$ no gap

What happens at small radius?

In simplest (most symmetric) case



the space evolves from a higher-dimensional torus at early times to a 3-dimensional hyperbolic space at late times.

<u>Note</u> - dimensionality not fixed in string theory (10d is special in having low energy SUSY, but is connected to backgrounds with $D \neq 10$.)

In general, the expansion and density of states are correlated

	$\rho(\ell)$	$a_{max}(t)$	$Vol(t)$	$R(t)$
T^3	ℓ^3	t^0	t^0	0
Nil	ℓ^4	$t^{3/4}$	t	$\frac{1}{t^4}$
Sol	e^{ℓ/ℓ_0}	$t \log^{\frac{1}{2}} t$	$t \log t$	$\frac{1}{t^2 \log t}$
$\Sigma \times S^1$	e^{ℓ/ℓ_0}	t	t^2	$\frac{1}{t^2}$
$\mathbb{H}_3/\Gamma$	$e^{\frac{\ell}{\ell_0}}$	t	t^3	$\frac{1}{t^2}$

anisotropic: Nil, Sol, $\Sigma \times S^1$

- This makes precise the naive intuition that in order for space to expand, we need extra degrees of freedom
- Extrapolating to inflation/dS — this pattern would suggest super-Hagedorn growth of the density of states...

(2) Simple de Sitter Solutions

To obtain accelerated expansion from string theory, one must stabilize the moduli. This has been outlined in several corners of the theory

cf Bousso/Polchinski

- $D > 10$ — Maloney ES Strominger '01/2
- Type IIB — Giddings Kachru Polchinski '01

Kachru Kallosh Linde Trivedi '03

Conlon Quevedo   Saltman ES

...

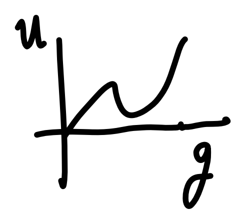
- Simplest example of moduli stabilization was an explicit compactification to AdS_4 by De Wolfe, Giryavets, Kachru, Taylor '05 in type IIA (Camara et all, Villadoro/Zwirner, ..)

→ dS and inflation ?

All sources of potential energy $\to 0$ at weak string coupling.

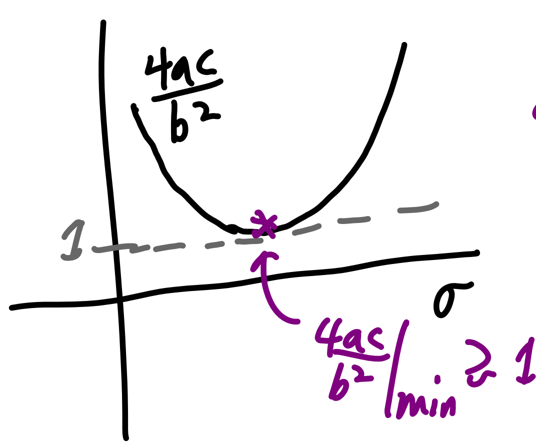
potential $U = a(\sigma)g^2 - b(\sigma)g^3 + c(\sigma)g^4 + \ldots$

For $1 < \frac{4ac}{b^2} < \frac{9}{8}$, the potential in the g direction can give dS



(need different orders to compete $\Rightarrow$ large ratios $\frac{b}{a}$ & $\frac{c}{b}$)

Obtaining



as a function of σ, for all σ

leads to dS minimum

We find concrete dS solutions using

D=10 IIA string theory on

Nil manifold *

(twisted torus: KK u(1) magnetic field on T^2)

curvature $\int R < 0 \Rightarrow$ {
- positive potential
- masses for would-be moduli

topology

supports fluxes with finely spaced quanta (fractional Chern-Simons invariants)

↳ small, tuneable coefficients

supports wrapped 5-branes *

Orientifold (for negative term)

↳ & fluxes (H m_0) cancelling its charge

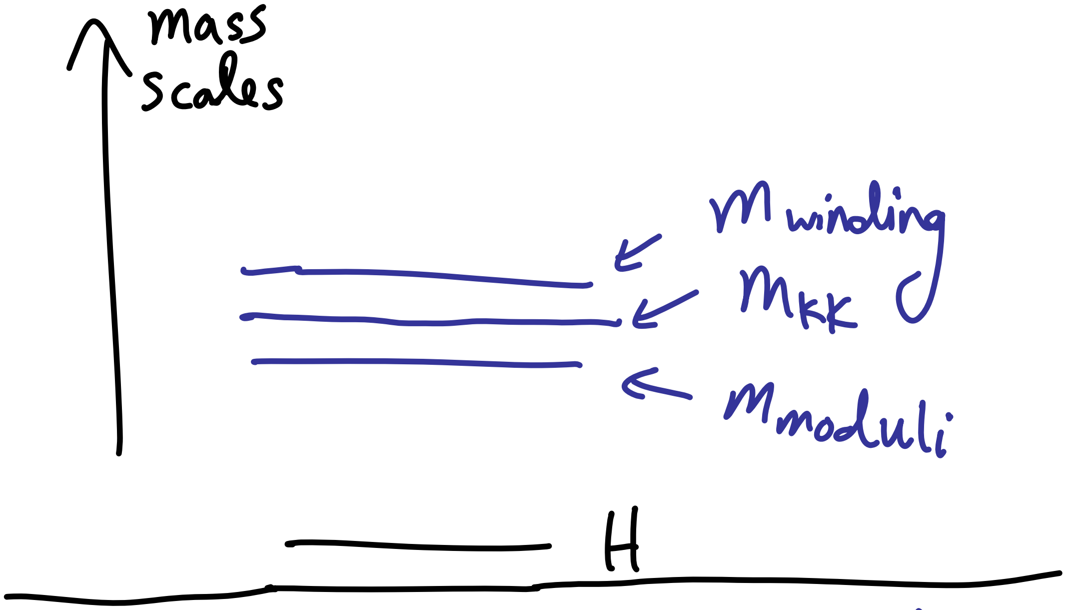
* Contains ingredients going beyond contemporaneous no-go theorem Hertzberg, Kachru, Taylor, Tegmark '07

$\rightarrow$ Scales

$$L_u^2\left(du_1^2+du_2^2\right)+L_x^2\left(dx+\frac{M}{2}\left(u_1du_2-u_2du_1\right)\right)^2$$

$$\left(\frac{r^{loc}_{SB}}{r_{SB0}}\right)\sim\left(\frac{M}{k}\right)^{\frac{1}{4}} \qquad L\sim k^{1/6} \qquad \underbrace{L_x\sim\frac{1}{M^{1/2}}}_{\text{small circle fiber, but weakly curved}}$$

$\ll 1$ to separate scales: $m_W \gg m_{KK} \gg m_{moduli}$



Angular moduli – curvature helps lift them, and other sectors of branes or orbifolding suffices.