New Ekpyrotic Cosmology and Non-Gaussianity

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hep-th/0702154, hep-th/0706.3903, hep-th/0710.5172

Related work: Lehners, McFadden, Turok & Steinhardt, hep-th/0702153
Creminelli & Senatore, hep-th/0702165
Koyama & Wands, hep-th/0703040
Finelli, hep-th/0206112, Brandenberger & Finelli,
Tolley & Wesley, hep-th/0703101
Koyama, Mizuno, Vernizzi and Wands, hep-th/0708.4321
Lehners & Steinhardt, to appear
Dear Prof. Shellard,

The title of my talk will be

"New Ekpyrotic Cosmology and Non-Gaussianity".

Looking forward to this exciting meeting!

All best wishes,

Justin
Ekpyrotic Cosmology

Before big bang, universe underwent a long phase of slow contraction

(Spiritually the opposite of inflation)

This is driven by a scalar field with steep, negative potential:

\[ V(\phi) = -V_0 e^{-\phi/\Lambda} \]

where \( \Lambda \equiv \sqrt{\epsilon} M_{Pl} \) and \( \epsilon \ll 1 \)

Leads to scaling solution, corresponding to slow contraction

\[ a(t) \sim (-t)^{2\epsilon} \quad ; \quad \phi(t) = \Lambda \log \left( \frac{V_0}{2\Lambda^2(1 - 6\epsilon)} t^2 \right) \]

\[ w = \frac{1}{3\epsilon} - 1 \gg 1 \]
Origin of scale invariance

Since $\epsilon \ll 1$, scale factor is nearly constant $\Rightarrow$ ignore gravity

$$\dot{\phi} = -\sqrt{-2V}$$

with solution

$$-t = \int_{-\infty}^{\phi} d\phi \frac{e^{\phi/2\Lambda}}{\sqrt{2V_0}} = 2\Lambda \frac{e^{\phi/2\Lambda}}{\sqrt{2V_0}} = \sqrt{\frac{2}{-V, \phi\phi}}$$

Moving on to perturbations:

$$\ddot{\delta\phi_k} + \left(k^2 + V,_{\phi\phi}\right) \delta\phi_k = 0$$

which reduces to

$$\ddot{\delta\phi_k} + \left(k^2 - \frac{2}{t^2}\right) \delta\phi_k = 0$$
However this time-delay mode exactly projects out the curvature perturbation $\zeta$ $\Rightarrow$ not scale invariant

Lyth, hep-ph/0105153
JK, Ovrut, Steinhardt and Turok, hep-th/0109050
Brandenberger and Finelli, hep-th/0109004

Problematic because $\zeta$ is constant on super-horizon scales (in the absence of entropy perturbations). This is true independent of GR

Lyth, Malik and Sasaki, astro-ph/0411220

Caveat: New (stringy) effects at the singularity could save the day by mixing the time-delay and curvature modes

Tolley, Steinhardt and Turok, hep-th/0306109
McFadden, Turok and Steinhardt, hep-th/0512123
Battefeld, Patil and Brandenberger, hep-th/0401010
Idea: Consider scale invariant entropy perturbations, convert it onto curvature pertn, so that $\zeta$ is scale invariant well before the bounce

Lehners, McFadden, Turok and Steinhardt, hep-th/0702153
Buchbinder, JK and Ovrut, hep-th/0702154
Creminelli and Senator, hep-th/0702165
New Ekpyrotic Cosmology

Consider 2 scalar fields, each with its own ekpyrotic potential:

\[ V(\phi_1, \phi_2) = -V_1 e^{-\phi_1/\Lambda_1} - V_2 e^{-\phi_2/\Lambda_2} \]

Again this leads to scaling solution

\[ a(t) \sim (-t)^{2(\epsilon_1+\epsilon_2)} ; \quad \phi_i(t) = \Lambda_i \log \left( \frac{V_i}{2\Lambda_i^2 (1 - 6(\epsilon_1 + \epsilon_2)) t^2} \right) \]

However not an attractor because of tachyonic instability

Lehners, McFadden, Turok and Steinhardt, hep-th/0702153
Buchbinder, JK and Ovrut, hep-th/0702154
Koyama and Wands, hep-th/0703040
Tachyon most easily seen by using new field variables

\[ \phi = \frac{\Lambda_1 \phi_1 + \Lambda_2 \phi_2}{\sqrt{\Lambda_1^2 + \Lambda_2^2}} ; \quad \chi = \frac{\Lambda_2 \phi_1 - \Lambda_1 \phi_2}{\sqrt{\Lambda_1^2 + \Lambda_2^2}} \]

in terms of which the potential becomes

\[ V(\phi, \chi) = -V_0 e^{-\phi/\Lambda} \left( 1 + \frac{\chi^2}{2\Lambda^2} + \ldots \right) \]

Scaling soln corresponds to rolling along \( \phi \), while \( \chi \) remains fixed.
Tachyon is good

Since $\chi$ is fixed, its fluctuations are entropy perturbations.

Key feature of potential: $V_{,\chi\chi} = V_{,\phi\phi} = -\frac{2}{t^2}$

- from form of potential
- from scaling solution

Substitute in perturbation eqn:

$$\ddot{\delta\chi_k} + \left(k^2 - \frac{2}{t^2}\right)\delta\chi_k = 0$$

which implies a scale invariant spectrum

Tachyonic direction is therefore essential in generating scale-invariant entropy perturbation spectrum.
Converting to curvature pertn

Entropy pertn sources $\zeta$ on large scales

$$\zeta \approx \frac{2H}{\dot{\phi}} \dot{\theta} \delta \chi$$

where $\tan \theta(t) = \dot{\chi}/\dot{\phi}$ describes the angle in field space.

**Sharp turn** in field trajectory imprints scale-invariant $\delta \chi$ onto curvature perturbation.

Alternative conversion mechanisms:
Koyama and Wands, hep-th/0703040
Battefeld, hep-th/0710.2540
Lehners et al., hep-th/0702153
Final Amplitude

In sharp turn approx,

\[ k^{3/2} \zeta_k \sim \Delta \theta \frac{H}{\sqrt{\epsilon M_{Pl}}} \]

where \( H \) is Hubble at the end of ekpyrotic phase, and \( \Delta \theta \) is overall change in field direction.

Nearly identical to inflationary expression

\[ k^{3/2} \zeta_k \sim \frac{H}{\sqrt{\epsilon_{\text{inf}} M_{Pl}}} \]
What about the bounce?

Using ghost condensate mechanism, can generate a non-singular bounce, without introducing ghosts or other pathologies.

Creminelli, Luty, Nicolis and Senatore, JHEP 12, 080 (2006)

Can successfully merge the ekpyrotic phase with this subsequent ghost condensation phase.

Buchbinder, JK, Ovrut, hep-th/0702154

Find that $\zeta$ goes through the bounce unscathed and emerges in the hot big bang phase with a scale-invariant spectrum.
Non-Gaussianity

\[ V(\phi, \chi) = -V_0 e^{-\phi/\Lambda} \left( 1 + \frac{\chi^2}{2\Lambda^2} + \frac{\alpha_3}{3!} \frac{\chi^3}{\Lambda^3} + \ldots \right) \]

where \( \alpha_i \) are expected to be \( O(1) \)

2 sources of non-Gaussianity:

-- Intrinsic NG because of self-interactions: \( \langle \delta \chi_{k_1} \delta \chi_{k_2} \delta \chi_{k_3} \rangle \)

(Since \( \zeta \sim \delta \chi \), curvature pertn inherits this NG.)

-- Non-linear relation between \( \zeta \) and \( \delta \chi \):

\[ \zeta(x) = a_1 \delta \chi(x) + a_2 \delta \chi^2(x) + a_3 \delta \chi^3(x) \ldots \]

(Local form)
Both contributions give NG of **local form**:

\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \left( \frac{6}{5} f_{NL} [P_{\zeta}(k_1)P_{\zeta}(k_2) + \ldots] \right)
\]

i.e. encoded in a single parameter \( f_{NL} \)

Result is generically **large**:

\[
f_{NL} = f_{NL}^{\text{int}} + f_{NL}^{\text{conv}} = \frac{5}{24(\Delta \theta)^2 \epsilon} \left( 1 - \frac{4}{3} \alpha_3 \Delta \theta \right)
\]

\[
\Rightarrow \quad f_{NL} \sim \epsilon^{-1}
\]

**WMAP 3-year**: \(-36 < f_{NL} < 100\)

**Yadav & Wandelt (2007)**: \(26.9 < f_{NL} < 146.7\) at 95%CL
NG offers distinguishing prediction from single-field, slow-roll inflation

Large inflationary NG can be obtained in DBI inflation, modulon/DGZ mechanism & curvaton

Different shape for 3-pt function

\[ f_{NL} \sim \frac{1}{\Omega \sigma} \sim O(1) \]  

* Identical shape for 3-pt  
* Distinguishable with 4-pt function
Four-point amplitude:

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = (2\pi)^3 \delta^3 \left( \sum_i \vec{k}_i \right) \left[ T(k_1, k_2, k_3, k_4) + T'(k_1, k_2, k_3, k_4) \right] \]

where

\[ T(k_1, k_2, k_3, k_4) = \frac{1}{2} \tau_{NL} \{ \mathcal{P}_{\zeta}(k_1) \mathcal{P}_{\zeta}(k_2) \mathcal{P}_{\zeta}(k_3) + 23 \text{ perm.} \} ; \]

\[ T'(k_1, k_2, k_3, k_4) = \kappa_{NL} \{ \mathcal{P}_{\zeta}(k_1) \mathcal{P}_{\zeta}(k_2) \mathcal{P}_{\zeta}(k_3) + 3 \text{ perm.} \} \]

Therefore have two shape parameters: \( \tau_{NL} \) & \( \kappa_{NL} \)

Both are generically large:

\[ \tau_{NL}, \kappa_{NL} \sim f_{NL}^2 \sim 10^4 \]

**WMAP (estimated):** \( \tau_{NL} < 10^4 \)

**Planck:** \( \tau_{NL} < 600 \)

Kogo & Komatsu (2006)
Explicitly:
\[ \tau_{NL} = \frac{1}{16(\Delta \theta)^4 \epsilon^2} \left( 1 + \frac{16}{9} \alpha_3^2(\Delta \theta)^2 \right) ; \]
\[ \kappa_{NL} = \frac{\alpha_3(2\alpha_3 \Delta \theta - 1) + \Delta \theta \alpha_4}{12(\Delta \theta)^3 \epsilon^2} . \]

where recall that \( \alpha_i \) arise from self-interactions

\[ V(\phi, \chi) = -V_0 e^{-\phi/\Lambda} \left( 1 + \frac{\chi^2}{2\Lambda^2} + \frac{\alpha_3 \chi^3}{3! \Lambda^3} + \ldots \right) \]

By contrast, simplest curvaton models rely on a \((\text{free})\) massive scalar field to generate entropy perturbations \( \Rightarrow \alpha_i \) are zero

\[ \Rightarrow \quad \tau_{NL} \sim f_{NL}^2 \quad \text{(as in our case)} \]
\[ \kappa_{NL} \sim f_{NL} \quad \text{(much smaller)} \]

Byrnes, Sasaki & Wands (2006)

Hence, shape of 4-pt is generically different

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + \frac{1}{2 \Lambda^2} (\Box \phi)^2 + \ldots \]

Naively there is a ghost:

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - \partial\mu \chi \partial^\mu \phi - \frac{1}{2} \Lambda^2 \chi^2 + \ldots \]

which reproduces original \( \mathcal{L} \) upon integrating out \( \chi \). Can diagonalize kinetic term by shifting \( \psi \equiv \phi + \chi \):

\[ \mathcal{L} = -\frac{1}{2} (\partial \psi)^2 + \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} \Lambda^2 \chi^2 + \ldots \]

However, mass of ghost is at the cutoff, so can’t trust it. New physics at scale \( \Lambda \) can remove it.

e.g.  \[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - \partial\mu \chi \partial^\mu \phi - (\partial \chi)^2 - \frac{1}{2} \Lambda^2 \chi^2 \]

\[ \mathcal{L} = -\frac{1}{2} (\partial \psi)^2 - \frac{3}{2} (\partial \chi)^2 - \frac{1}{2} \Lambda^2 \chi^2 \]

Integrate out \( \chi \), then reproduce original \( \mathcal{L} \) with correction \( \mathcal{O} \left( \left( \frac{\Box}{\Lambda^2} \right)^n \right) \).
Our model:

\[ \mathcal{L} = M^4 P(X) + V(\phi) - \frac{(\Box \phi)^2}{\Lambda^2} + \ldots \quad \text{where} \quad X = -\frac{(\partial \phi)^2}{2m^4} \]

Near the bounce, dispersion relation is

\[ \omega^2 = -\frac{2\dot{H}M_{Pl}^2}{M^4} k^2 + \frac{(\omega^2 - k^2)^2}{\Lambda^2} + \ldots \]

Ignore \( \dot{H} \) term for a moment, this has two poles:

\[ \omega^2 \approx \frac{k^4}{2\Lambda^2} \ll k^2 \quad \omega^2 > \Lambda^2 \]

Hence, in regime of validity of EFT, dispersion relation reduces to

\[ \omega^2 \approx -\frac{2\dot{H}M_{Pl}^2}{M^4} k^2 + \frac{k^4}{\Lambda^2} \]

where \( k^4 \) term is important within regime \( k^2 \ll \Lambda^2 \) and \( \omega^2 \ll \Lambda^2 \)
Summary:

New Ekpyrotic Cosmology is a candidate alternative theory of the early universe.

Predictions:

* Amplitude and spectral tilt: identical to inflation

* NG generically much larger than slow-roll inflation

* Negligible primordial gravity wave amplitude on large scales