Scattering equations, supergravity, and the worldsheet

Tim Adamo
DAMTP, University of Cambridge

EuroStrings 2015

24 March 2015

Work with E. Casali & D. Skinner [arXiv:1312.3828, 1409.5656, 1502.06826]

Motivation

Over last 10+ years, lots of advances in study of scattering amplitudes.

Many different theories/techniques...

Motivation

Over last 10+ years, lots of advances in study of scattering amplitudes.

Many different theories/techniques...

- BCFW recursion, generalized unitarity
- dual conformal symmetry, integrability (planar $\mathcal{N} = 4$ SYM in $d = 4$)
- KLT, BCJ, double copy, etc. ($\mathcal{N} = 8$ SUGRA in $d = 4$)
- pure spinor formalism (superstring theory)

Motivation

Over last 10+ years, lots of advances in study of scattering amplitudes.

Many different theories/techniques...

- BCFW recursion, generalized unitarity
- dual conformal symmetry, integrability (planar $\mathcal{N} = 4$ SYM in $d = 4$)
- KLT, BCJ, double copy, etc. ($\mathcal{N} = 8$ SUGRA in $d = 4$)
- pure spinor formalism (superstring theory)

Today: focus on one particular development – tree-level S-matrix of (super)gravity.

Why do we care?

Tree-level scattering amplitudes provide an indicator of theory's on-shell complexity ('theoretical data').

Why do we care?

Tree-level scattering amplitudes provide an indicator of theory's on-shell complexity ('theoretical data').

Naïve expectation: tree-level S-matrix of (super)gravity should be a mess!

- Perturbation theory of Einstein-Hilbert action is bad news
- Infinite number of interaction vertices

However...

Expectation undermined by litany of increasingly simple/compact/general formulae for tree-level S-matrix

[deWitt, Nguyen-Spradlin-Volovich, Hodges, Cachazo-Geyer, Cachazo-Skinner, Cachazo-He-Yuan, ...]

However...

Expectation undermined by litany of increasingly simple/compact/general formulae for tree-level S-matrix

[deWitt, Nguyen-Spradlin-Volovich, Hodges, Cachazo-Geyer, Cachazo-Skinner, Cachazo-He-Yuan, ...]

Important: many unrelated to perturbation theory of Einstein-Hilbert action

CHY Formula

Most general representation (any d) of tree-level S-matrix [Cachazo-He-Yuan] :

$$\mathcal{M}_{n,0} = \int \frac{1}{\mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})} \frac{|z_1 z_2 z_3|}{\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3} \prod_{i=4}^{n} \bar{\delta}\left(\mathrm{d}z_i \sum_{j\neq i} \frac{k_i \cdot k_j}{z_i - z_j}\right) \mathcal{I}_n$$

$\{z_i\} \subset \Sigma \cong \mathbb{CP}^1$, $\{k_i\}$ null momenta,

Integrand $\mathcal{I}_n$ encodes kinematic data.

CHY Formula

Most general representation (any d) of tree-level S-matrix [Cachazo-He-Yuan] :

$$\mathcal{M}_{n,0} = \int \frac{1}{\mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})} \frac{|z_1 z_2 z_3|}{\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}z_3} \prod_{i=4}^{n} \bar{\delta}\left(\mathrm{d}z_i \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j} \right) \mathcal{I}_n$$

$\{z_i\} \subset \Sigma \cong \mathbb{CP}^1$, $\{k_i\}$ null momenta,

Integrand $\mathcal{I}_n$ encodes kinematic data.

Integrals over positions $\{z_i\}$ fixed by delta functions, imposing the *scattering equations* [Fairlie-Roberts, Gross-Mende, Witten] :

$$i \in \{4, \ldots, n\}, \qquad \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j} = 0$$

Just so you know...

$$\mathcal{I}_n = \mathrm{Pf}'(M)\,\mathrm{Pf}'(\widetilde{M}) \in \bigotimes_{i=1}^{n} K_i^2$$

for skew-symmetric $2n \times 2n$ matrices M, $\widetilde{M}$

$$M = \begin{pmatrix} A & -C^{\mathrm{T}} \\ C & B \end{pmatrix}, \qquad \mathrm{Pf}'(M) = (-1)^{i+j}\frac{\sqrt{\mathrm{d}z_i\,\mathrm{d}z_j}}{z_i - z_j}\mathrm{Pf}(M^{ij}_{ij}),$$

$$A_{ij} = k_i \cdot k_j \frac{\sqrt{\mathrm{d}z_i\,\mathrm{d}z_j}}{z_i - z_j}, \quad B_{ij} = \epsilon_i \cdot \epsilon_j \frac{\sqrt{\mathrm{d}z_i\,\mathrm{d}z_j}}{z_i - z_j}, \quad C_{ij} = \epsilon_i \cdot k_j \frac{\sqrt{\mathrm{d}z_i\,\mathrm{d}z_j}}{z_i - z_j}$$

$$A_{ii} = B_{ii} = 0, \qquad C_{ii} = -\mathrm{d}z_i \sum_{j \neq i} \frac{C_{ij}}{\sqrt{\mathrm{d}z_i\,\mathrm{d}z_j}}$$

Questions:

- What is the origin of the CHY formula?
- Where do the scattering equations come from?
- Does the formula generalize beyond tree-level?
- What is this telling us about the underlying field theory?

Questions:

- What is the origin of the CHY formula?
- Where do the scattering equations come from?
- Does the formula generalize beyond tree-level?
- What is this telling us about the underlying field theory?

All these questions have nice answers!

Worldsheet origin of CHY

As structure suggests, CHY formula = sphere correlator of certain 2d CFT.

A (holomorphic) complexification of spinning worldline action [Mason-Skinner] :

$$S = \frac{1}{2\pi}\int_{\Sigma} P_{\mu}\,\bar{\partial}X^{\mu} + \Psi_{\mu}\bar{\partial}\Psi^{\mu} - \chi P_{\mu}\Psi^{\mu} + \widetilde{\Psi}_{\mu}\bar{\partial}\widetilde{\Psi}^{\mu} - \widetilde{\chi}P_{\mu}\widetilde{\Psi}^{\mu} - \frac{e}{2}P^{2}$$

$P_{\mu} \in \Omega^{0}(\Sigma, K)$ and $\Psi^{\mu}, \widetilde{\Psi}^{\mu} \in \Pi\Omega^{0}(\Sigma, K^{1/2})$

Worldsheet origin of CHY

As structure suggests, CHY formula = sphere correlator of certain 2d CFT.

A (holomorphic) complexification of spinning worldline action [Mason-Skinner] :

$$S = \frac{1}{2\pi}\int_{\Sigma} P_{\mu}\,\bar{\partial}X^{\mu} + \Psi_{\mu}\bar{\partial}\Psi^{\mu} - \chi P_{\mu}\Psi^{\mu} + \widetilde{\Psi}_{\mu}\bar{\partial}\widetilde{\Psi}^{\mu} - \widetilde{\chi}P_{\mu}\widetilde{\Psi}^{\mu} - \frac{e}{2}P^{2}$$

$P_{\mu} \in \Omega^{0}(\Sigma, K)$ and $\Psi^{\mu}, \widetilde{\Psi}^{\mu} \in \Pi\Omega^{0}(\Sigma, K^{1/2})$

Gauge fields $e, \chi, \widetilde{\chi}$ (non-trivial conformal weights) enforce constraints

$$P^{2} = 0\,, \qquad \Psi \cdot P = 0\,, \qquad \widetilde{\Psi} \cdot P = 0\,.$$

Worldsheet origin of CHY

As structure suggests, CHY formula = sphere correlator of certain 2d CFT.

A (holomorphic) complexification of spinning worldline action [Mason-Skinner] :

$$S = \frac{1}{2\pi}\int_{\Sigma} P_\mu \bar{\partial} X^\mu + \Psi_\mu \bar{\partial}\Psi^\mu - \chi P_\mu \Psi^\mu + \widetilde{\Psi}_\mu \bar{\partial}\widetilde{\Psi}^\mu - \tilde{\chi} P_\mu \widetilde{\Psi}^\mu - \frac{e}{2}P^2$$

$P_\mu \in \Omega^0(\Sigma, K)$ and $\Psi^\mu, \widetilde{\Psi}^\mu \in \Pi\Omega^0(\Sigma, K^{1/2})$

Gauge fields $e, \chi, \tilde{\chi}$ (non-trivial conformal weights) enforce constraints

$$P^2 = 0\,, \qquad \Psi \cdot P = 0\,, \qquad \widetilde{\Psi} \cdot P = 0\,.$$

Note: also a pure spinor version [Berkovits, Gomez-Yuan, TA-Casali]

Gauging these constraints + worldsheet gravity gives action and BRST charge:

$$S = \frac{1}{2\pi}\int_{\Sigma} P_\mu\,\bar\partial X^\mu + \Psi_\mu\bar\partial\Psi^\mu + \widetilde{\Psi}_\mu\bar\partial\widetilde{\Psi}^\mu + b\,\bar\partial c + \tilde{b}\,\bar\partial\tilde{c} + \beta\,\bar\partial\gamma + \tilde{\beta}\,\bar\partial\tilde{\gamma}$$

$$Q = \oint c\,T^{\mathrm{m}} + :bc\partial c: + \frac{\tilde{c}}{2}P^2 + \gamma P\cdot\Psi + \tilde{\gamma}P\cdot\widetilde{\Psi}\,.$$

with $Q^2 = 0$ for $d = 10$.

Gauging these constraints + worldsheet gravity gives action and BRST charge:

$$S=\frac{1}{2\pi}\int_{\Sigma}P_{\mu}\,\bar{\partial}X^{\mu}+\Psi_{\mu}\bar{\partial}\Psi^{\mu}+\widetilde{\Psi}_{\mu}\bar{\partial}\widetilde{\Psi}^{\mu}+b\,\bar{\partial}c+\tilde{b}\,\bar{\partial}\tilde{c}+\beta\,\bar{\partial}\gamma+\tilde{\beta}\,\bar{\partial}\tilde{\gamma}$$

$$Q=\oint c\,T^{\mathrm{m}}+:bc\partial c:+\frac{\tilde{c}}{2}P^{2}+\gamma P\cdot\Psi+\tilde{\gamma}P\cdot\widetilde{\Psi}\,.$$

with $Q^2=0$ for $d=10$.

Vertex operators are in the cohomology of Q.

Fact: vertex operators in 1:1-correspondence with massless spectrum of type II SUGRA in $d=10$ [TA-Casali-Skinner].

Example: Graviton vertex operators

fixed:

$$V = c\,\tilde{c}\,\delta(\gamma)\,\delta(\tilde{\gamma})\,\epsilon_{\mu\nu}\Psi^{\mu}\widetilde{\Psi}^{\nu}\,\mathrm{e}^{ik\cdot X}\,,$$

integrated:

$$\int_{\Sigma}\bar{\delta}\left(\mathrm{Res}_z P^2\right)U(z) =$$
$$\int_{\Sigma}\bar{\delta}\left(\mathrm{Res}_z P^2\right)\epsilon_{\mu\nu}\left(P^{\mu}+\Psi^{\mu}k\cdot\Psi\right)\left(P^{\nu}+\widetilde{\Psi}^{\mu}k\cdot\widetilde{\Psi}\right)\,\mathrm{e}^{ik\cdot X}$$

Example: Graviton vertex operators

fixed:

$$V = c\,\tilde{c}\,\delta(\gamma)\,\delta(\tilde{\gamma})\,\epsilon_{\mu\nu}\Psi^{\mu}\widetilde{\Psi}^{\nu}\,\mathrm{e}^{ik\cdot X}\,,$$

integrated:

$$\int_{\Sigma}\bar{\delta}\left(\mathrm{Res}_{z}P^{2}\right)U(z) = \int_{\Sigma}\bar{\delta}\left(\mathrm{Res}_{z}P^{2}\right)\epsilon_{\mu\nu}\left(P^{\mu}+\Psi^{\mu}k\cdot\Psi\right)\left(P^{\nu}+\widetilde{\Psi}^{\mu}k\cdot\widetilde{\Psi}\right)\,\mathrm{e}^{ik\cdot X}$$

Q-closure $\Leftrightarrow k^2 = 0 = \epsilon\cdot k$ (double contractions w/ P^2, $\Psi\cdot P$, $\widetilde{\Psi}\cdot P$)

Sphere correlators

Prescription

$$\left\langle c_1 \tilde{c}_1 V_1 \, c_2 \tilde{c}_2 V_2 \, c_3 \tilde{c}_3 U_3 \prod_{i=4}^{n} \int_{\Sigma} \bar{\delta}\left(\mathrm{Res}_i P^2\right) U_i \right\rangle$$

This gives the CHY formula (for NS-NS sector) [Mason-Skinner]

Sphere correlators

Prescription

$$\left\langle c_1 \tilde{c}_1 V_1 \, c_2 \tilde{c}_2 V_2 \, c_3 \tilde{c}_3 U_3 \prod_{i=4}^{n} \int_{\Sigma} \bar{\delta}\left(\mathrm{Res}_i P^2\right) U_i \right\rangle$$

This gives the CHY formula (for NS-NS sector) [Mason-Skinner]

How to see scattering equations?

Note: $X^{\mu}(z)$ only enters in the plane waves $\mathrm{e}^{ik\cdot X}$, and no XX-OPE

$\Rightarrow$ do X path integral

Result:

$$\bar{\partial} P_\mu(z) = 2\pi i \, \mathrm{d}z \wedge \mathrm{d}\bar{z} \sum_{i=1}^{n} k_{i\,\mu} \, \delta^2(z - z_i)$$

Result:

$$\bar{\partial} P_\mu(z) = 2\pi i \, \mathrm{d}z \wedge \mathrm{d}\bar{z} \sum_{i=1}^{n} k_{i\,\mu} \, \delta^2(z - z_i)$$

Implies that $P^2(z)$ is a meromorphic quadratic differential:

$$\mathrm{Res}_{z=z_i} P^2(z) = k_i \cdot P(z_i) = \mathrm{d}z_i \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j}$$

Result:

$$\bar{\partial} P_\mu(z) = 2\pi i \, \mathrm{d}z \wedge \mathrm{d}\bar{z} \sum_{i=1}^{n} k_{i\,\mu} \, \delta^2(z - z_i)$$

Implies that $P^2(z)$ is a meromorphic quadratic differential:

$$\mathrm{Res}_{z=z_i} P^2(z) = k_i \cdot P(z_i) = \mathrm{d}z_i \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j}$$

So each insertion of $\bar{\delta}\left(\mathrm{Res}_i P^2\right)$ in correlator gives one of the scattering equations!

Scattering equations from worldsheet

Setting $n-3$ residues of P^2 to zero gives scattering equations

$$\Leftrightarrow$$

Setting $P^2 = 0$ globally on $\Sigma \cong \mathbb{CP}^1$

(recall $P^2 = 0$ constraint from gauge-fixing)

Scattering equations from worldsheet

Setting $n-3$ residues of P^2 to zero gives scattering equations

$$\Leftrightarrow$$

Setting $P^2 = 0$ globally on $\Sigma \cong \mathbb{CP}^1$

(recall $P^2 = 0$ constraint from gauge-fixing)

Claim

$P^2(z) = 0$ globally on Σ defines the scattering equations for *any* genus

Explicitly, the required delta functions are:

$$
\begin{aligned}
g = 0 \quad & (n-3) \times \operatorname{Res}_{z=z_i} P^2(z) = 0 \\
g = 1 \quad & (n-1) \times \operatorname{Res}_{z=z_i} P^2(z) = 0\,, \quad P^2(z_1) = 0 \\
g \geq 2 \quad & n \times \operatorname{Res}_{z=z_i} P^2(z) = 0\,, \qquad (3g-3) \times P^2(z_r) = 0
\end{aligned}
$$

Explicitly, the required delta functions are:

$$\begin{aligned}
g = 0 &\quad (n-3) \times \ \mathrm{Res}_{z=z_i} P^2(z) = 0 \\
g = 1 &\quad (n-1) \times \ \mathrm{Res}_{z=z_i} P^2(z) = 0\,, \quad P^2(z_1) = 0 \\
g \geq 2 &\quad \quad n \times \ \mathrm{Res}_{z=z_i} P^2(z) = 0\,, \quad \quad (3g-3) \times P^2(z_r) = 0
\end{aligned}$$

What about the correlators?

Correlators at higher genus

For $g > 0$, field $P_\mu(z)$ acquires zero modes:

$$P_\mu(z) = \sum_{I=1}^{g} \ell^I_\mu \, \omega_I(z) + \sum_{i=1}^{n} k_{i\,\mu} \, \tilde{S}_g(z, z_i|\Omega)$$

Correlators at higher genus

For $g > 0$, field $P_\mu(z)$ acquires zero modes:

$$P_\mu(z) = \sum_{I=1}^{g} \ell_\mu^I \, \omega_I(z) + \sum_{i=1}^{n} k_{i\,\mu} \, \tilde{S}_g(z, z_i|\Omega)$$

Correlators take the general form:

$$\mathcal{M}_{n,g} = \int \prod_{I=1}^{g} \mathrm{d}^{10}\ell^I \, \mathfrak{M}_{n,g}(k_i, \epsilon_i)$$

Conjecture

$\mathfrak{M}_{n,g}$ is the g-loop *integrand* of type II supergravity

Evidence in favor:

- Partition functions in $\mathfrak{M}_{n,g}$ modular invariant [TA-Casali-Skinner]
- Factorizes in moduli space onto rational function of kinematics [TA-Casali-Skinner]
- Explicit checks of IR behavior for $n = 4$, $g = 1, 2$ [Casali-Tourkine, TA-Casali]

Obstruction to general proof:

- Solving scattering equations for $g > 0$...*hard*!

So...

- CHY formula = sphere correlator of 2d CFT
- Scattering equations have geometric interpretation ($P^2 = 0$)
- Scattering eqns and CHY formula have natural $g > 0$ extensions

So...

- CHY formula = sphere correlator of 2d CFT
- Scattering equations have geometric interpretation ($P^2 = 0$)
- Scattering eqns and CHY formula have natural $g > 0$ extensions

Still want to know:

What is this telling us about the underlying field theory (i.e., classical GR/supergravity)?

Analogy

Compare and contrast with string theory

String theory

- Sphere amps $\xrightarrow{\alpha' \to 0}$ SUGRA tree-level S-matrix
- linearized EFEs $\leftrightarrow$ anomalous conformal weights

Worldsheet theory

- Sphere amps $=$ SUGRA tree-level S-matrix
- linearized EFEs $\leftrightarrow$ anomalies w/ currents P^2, $\Psi \cdot P$, $\widetilde{\Psi} \cdot P$

How to get non-linear statement in string theory?

- Formulate non-linear sigma model on curved target space
- Demand worldsheet conformal invariance $\rightarrow$ compute β-functions
- Conformal anomaly vanishes as $\alpha' \rightarrow 0 \Leftrightarrow$ non-linear field eqns.

[Callan-Martinec-Perry-Friedan, Banks-Nemeschansky-Sen]

How to get non-linear statement in string theory?

- Formulate non-linear sigma model on curved target space
- Demand worldsheet conformal invariance $\rightarrow$ compute β-functions
- Conformal anomaly vanishes as $\alpha' \rightarrow 0 \Leftrightarrow$ non-linear field eqns.

[Callan-Martinec-Perry-Friedan, Banks-Nemeschansky-Sen]

- Must work perturbatively in α' (background field expansion)
- Higher powers of $\alpha' \leftrightarrow$ higher-curvature corrections to field equations [Gross-Witten, Grisaru-van de Ven-Zanon]

Wish List

So based on contrast with string theory, we want

- Formulate the worldsheet theory on a curved target space
- Do it so theory is solveable (no background field/perturbative expansion required)
- See non-linear field equations as some anomaly cancellation condition

Put CFT on a curved background $(g_{\mu\nu},\, B_{\mu\nu},\, \Phi)$:

$$S = \frac{1}{2\pi}\int_{\Sigma} P_{\mu}\,\bar{\partial}X^{\mu} + \bar{\psi}_{\mu}\,\bar{\partial}\psi^{\mu} + \bar{\psi}_{\mu}\,\Gamma^{\mu}_{\nu\rho}\bar{\partial}X^{\rho}\,\psi^{\nu} + \frac{1}{4}R_{\Sigma}\,\log\left(\mathrm{e}^{-2\Phi}\sqrt{g}\right)$$

Put CFT on a curved background $(g_{\mu\nu},\, B_{\mu\nu},\, \Phi)$:

$$S = \frac{1}{2\pi}\int_{\Sigma} P_{\mu}\,\bar{\partial}X^{\mu} + \bar{\psi}_{\mu}\,\bar{\partial}\psi^{\mu} + \bar{\psi}_{\mu}\,\Gamma^{\mu}_{\nu\rho}\bar{\partial}X^{\rho}\,\psi^{\nu} + \frac{1}{4}R_{\Sigma}\,\log\left(\mathrm{e}^{-2\Phi}\sqrt{g}\right)$$

$$\xrightarrow{\text{field redef'n}} \frac{1}{2\pi}\int_{\Sigma} \Pi_{\mu}\,\bar{\partial}X^{\mu} + \bar{\psi}_{\mu}\,\bar{\partial}\psi^{\mu} + \frac{1}{4}R_{\Sigma}\,\log\left(\mathrm{e}^{-2\Phi}\sqrt{g}\right)$$

So free worldsheet OPEs! (crucial difference with non-linear sigma model)

Put CFT on a curved background $(g_{\mu\nu}, B_{\mu\nu}, \Phi)$:

$$S = \frac{1}{2\pi} \int_{\Sigma} P_{\mu}\, \bar{\partial} X^{\mu} + \bar{\psi}_{\mu}\, \bar{\partial}\psi^{\mu} + \bar{\psi}_{\mu}\, \Gamma^{\mu}_{\nu\rho} \bar{\partial} X^{\rho}\, \psi^{\nu} + \frac{1}{4} R_{\Sigma} \log\left(\mathrm{e}^{-2\Phi}\sqrt{g}\right)$$

$$\xrightarrow{\text{field redef'n}} \frac{1}{2\pi} \int_{\Sigma} \Pi_{\mu}\, \bar{\partial} X^{\mu} + \bar{\psi}_{\mu}\, \bar{\partial}\psi^{\mu} + \frac{1}{4} R_{\Sigma} \log\left(\mathrm{e}^{-2\Phi}\sqrt{g}\right)$$

So free worldsheet OPEs! (crucial difference with non-linear sigma model)

Curved background currents: $\psi \cdot P \rightarrow \mathcal{G}$, $\bar{\psi} \cdot P \rightarrow \bar{\mathcal{G}}$, $P^2 \rightarrow \mathcal{H}$

BRST charge

$$Q = \oint c\, T^{\mathrm{m}} + :bc\partial c: + \frac{\tilde{c}}{2}\, \mathcal{H} + \bar{\gamma}\, \mathcal{G} + \gamma\, \bar{\mathcal{G}}$$

Anomalies

After carefully checking (quantum) diffeomorphism invariance (space-time & Σ), find $Q^2 = 0$ iff

- $d = 10$ (conformal anomaly)
- Other symmetry currents obey

$$\mathcal{G}(z)\,\mathcal{G}(w) \sim 0 \sim \bar{\mathcal{G}}(z)\,\bar{\mathcal{G}}(w)\,, \qquad \mathcal{G}(z)\,\bar{\mathcal{G}}(w) \sim \frac{\mathcal{H}}{z-w}$$

Anomalies

After carefully checking (quantum) diffeomorphism invariance (space-time & Σ), find $Q^2 = 0$ iff

- $d = 10$ (conformal anomaly)
- Other symmetry currents obey

$$\mathcal{G}(z)\,\mathcal{G}(w) \sim 0 \sim \bar{\mathcal{G}}(z)\,\bar{\mathcal{G}}(w)\,, \qquad \mathcal{G}(z)\,\bar{\mathcal{G}}(w) \sim \frac{\mathcal{H}}{z-w}$$

Remarkably, the only obstructions are [TA-Casali-Skinner] :

$$\begin{aligned} R_{\mu\nu} - \frac{1}{4}H_{\mu\rho\sigma}\,H_{\nu}^{\rho\sigma} + 2\nabla_{\mu}\nabla_{\nu}\Phi &= 0\,, \\ \nabla_{\rho}H^{\rho}_{\mu\nu} - 2H^{\rho}_{\mu\nu}\nabla_{\rho}\Phi &= 0\,, \\ R + 4\nabla_{\mu}\nabla^{\mu}\Phi - 4\nabla_{\mu}\Phi\,\nabla^{\mu}\Phi - \frac{H^2}{12} &= 0\,. \end{aligned}$$

Summary

So we've answered our last question:

Scattering eqns, CHY formula, etc. $\leftrightarrow$ alternative formulation of supergravity as a 2d CFT with free OPEs

Summary

So we've answered our last question:

Scattering eqns, CHY formula, etc. $\leftrightarrow$ alternative formulation of supergravity as a 2d CFT with free OPEs

Where's it all going?

- 'S-matrices' on non-flat space-times?
- Ramond-Ramond backgrounds ($\Rightarrow$ pure spinor story)?
- String theory with $\alpha' \to 0$ manifest? [pipe-dream]
- Other field theories? [Next talk!]