Anomalies and Gaps in HEE

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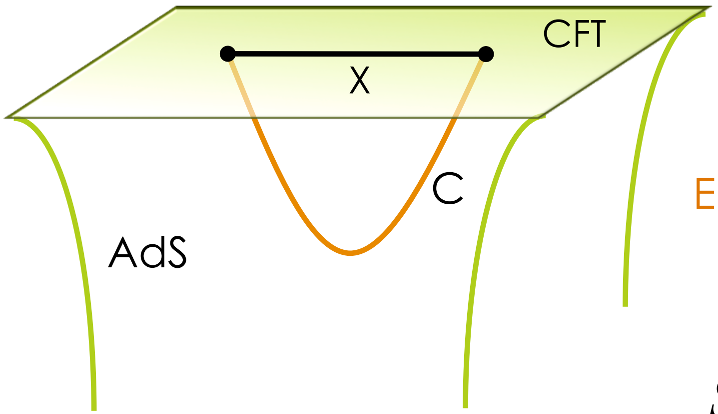
DAMTP, Cambridge

Holographic Entanglement Entropy



Conformal
Field theory

Anti-de Sitter
Gravity

Entanglement Entropy → Minimal areas

$$S_{\rm EE} = -{\rm Tr}\rho_X \log \rho_X$$

Ryu-Takayanagi formula

Long list of features …

- Leading contribution yields area law.
- In a pure state, $S(X)=S(X^c)$.
- Strong subadditivity and other inequalities.
- Recover known results.
- Generalizations: susy, higher spins, …
- There is a derivation from first principles in holography.

Long list of applications …

- It characterizes d.o.f. of the theory and anomalies.
- Ties with renormalization group flows.
- Probe of phase transitions (e.g. confinement/decofinement).
- Useful characterization of excited states (quenches).

GOAL for TODAY

- Interplay between **anomalies** and **entanglement entropy**
- Anomalies **probe** other aspects of the dual bulk geometry

FOCUS

- Gravitational anomalies in AdS_3/CFT_2
- Generalization of Ryu-Takayanagi
- Test in a dynamical setup

OUTLINE

1. CFT_2 :
EE & anomalies revisited

with Detournay,
Iqbal & Perlmutter.
arXiv:1405.2792

2. AdS_3:
Spinning probes

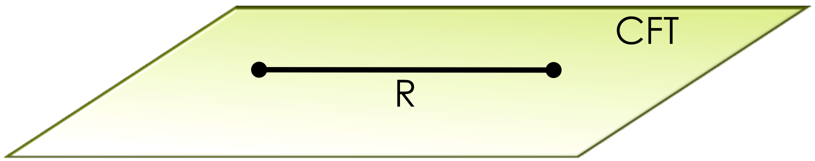
3. Dynamics:
Gapped systems in AdS_3

with Belin & Hung.
arXiv: soon!

CFT_2

Entanglement entropy and anomalies revisited

The relationship between entanglement and anomalies is well known. For example, single interval on the vacuum state:



$$S_{\mathrm{EE}} = \frac{c}{3}\log\left(\frac{R}{\varepsilon}\right)$$

[Holzhey, Larsen & Wilczek]

Explicit relation between **conformal anomaly** and **entanglement entropy**

Important assumption here: $c = \dfrac{c_L + c_R}{2}$ $\quad c_L = c_R$

Is entanglement entropy sensitive to **diffeormorphism anomalies**?

Diffeomorphism anomaly

In the OPE expansion of the stress tensor:

$$T(z)T(0) \sim \frac{c_L/2}{z^4} + \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \dots$$

$$\bar{T}(\bar{z})\bar{T}(0) \sim \frac{c_R/2}{\bar{z}^4} + \frac{\bar{T}(0)}{\bar{z}^2} + \frac{\bar{\partial}\bar{T}(0)}{\bar{z}} + \dots$$

$$\boxed{c_L \neq c_R}$$

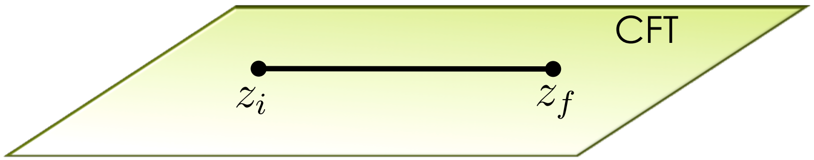
Simple examples: chiral matter or holomorphic CFTs.

The anomaly can be presented in two ways:

- Stress tensor is symmetric, but not conserved
- Stress tensor is conserved, but not symmetric

Twist fields & EE

$$S_{\mathrm{EE}} = \lim_{n\to 1} S_n = -\mathrm{Tr}\rho\log\rho$$



$$S_n = \frac{1}{1-n}\log\mathrm{Tr}\rho^n$$

$$\mathrm{Tr}\rho^n = \langle\Phi_+(z_i)\Phi_-(z_f)\rangle_{\mathcal{C}^n/\mathbb{Z}_n}$$

Twist field. For simplicity only consider a single interval

[Cardy, Castro-Alvaredo & Doyon; Cardy &Calabrese]

Include diff anomaly: Conformal dimensions of the twist fields are

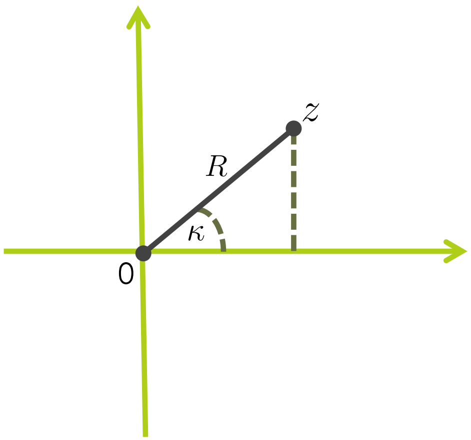
$$\boxed{\begin{aligned}\Delta &= h+\bar{h}\\ &= \frac{c_L+c_R}{24}\left(n-\frac{1}{n}\right)\end{aligned}}$$

$$\boxed{\begin{aligned}s &= h-\bar{h}\\ &= \frac{c_L-c_R}{24}\left(n-\frac{1}{n}\right)\end{aligned}}$$

Results

Vacuum state & boosted interval

$$S_{\mathrm{EE}} = \frac{c_L}{12}\log\left(\frac{z}{\varepsilon}\right) + \frac{c_R}{12}\log\left(\frac{\bar{z}}{\varepsilon}\right)$$
$$= \frac{c_L + c_R}{6}\log\left(\frac{R}{\varepsilon}\right) - \frac{c_L - c_R}{6}\,\kappa$$



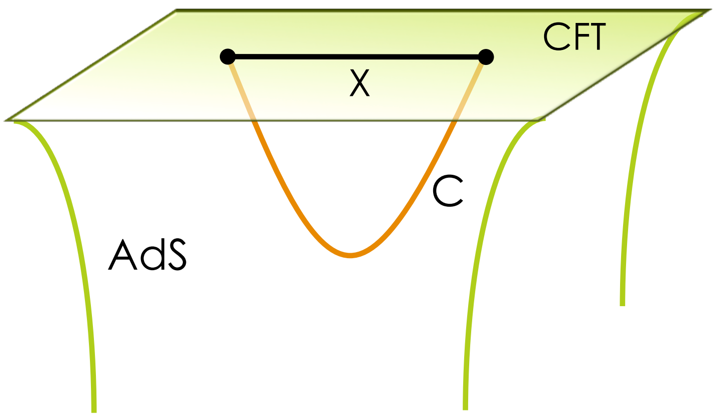
Thermal state: high temperature, finite angular velocity

$$S_{\mathrm{EE}} = \frac{c_L}{6}\log\left(\frac{\beta_L}{\pi\varepsilon}\sinh\frac{\pi R}{\beta_L}\right) + \frac{c_R}{6}\log\left(\frac{\beta_R}{\pi\varepsilon}\sinh\frac{\pi R}{\beta_R}\right)$$

AdS_3

Spinning particles and anomalies

Einstein Hilbert gravity



Ryu-Takayanagi formula

$$S_{\mathrm{EE}} = -\mathrm{Tr}(\rho_X \log \rho_X) = \frac{L_C}{4G_3}$$

It cannot capture the diffeormorphism anomaly: not enough data to have unbalanced left/right central charges.

Topological massive gravity

$$
\begin{aligned}
S_{\mathrm{TMG}} \quad &= \quad \frac{1}{16\pi G_3}\int d^3x\sqrt{-g}\left(R+\frac{2}{\ell^2}\right) \\
&\quad +\frac{1}{32\pi G_3\mu}\int d^3x\sqrt{-g}\,\underbrace{\epsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu}+\frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right)}_{\text{Gravitational Chern-Simons term}}
\end{aligned}
$$

Asymptotically **AdS$_3$** spacetime:

$$
c_L=\frac{3\ell}{2G_3}\left(1-\frac{1}{\mu\ell}\right)\qquad c_R=\frac{3\ell}{2G_3}\left(1+\frac{1}{\mu\ell}\right)
$$

HEE revisited

Task: find the generalization of R-T in the presence of diff anomalies

Two routes (that complement each other):

- Conical singularity method applied to TMG
  [Lewkowycz & Maldacena; Dong; Camps]
- Design a bulk probe that captures the right CFT physics

Spinning probes

Hint: Recall, twist field has quantum numbers

$$\Delta = \frac{c_L + c_R}{12}(n-1) + O(n-1)^2 \qquad c_L = \frac{3\ell}{2G_3}\left(1 - \frac{1}{\mu\ell}\right)$$

$$s = \frac{c_L - c_R}{12}(n-1) + O(n-1)^2 \qquad c_R = \frac{3\ell}{2G_3}\left(1 + \frac{1}{\mu\ell}\right)$$

- **Ryu-Takayanagi formula**

Geodesic equation $\rightarrow$ Massive probe $\rightarrow$ $\mathfrak{m} = \frac{\Delta}{\ell} = \frac{1}{4G_3}$

- **TMG EE formula**

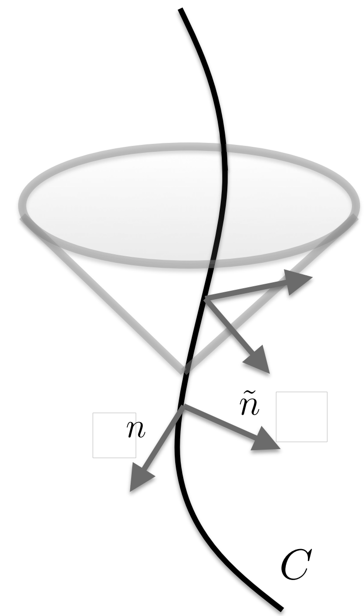
Spinning probe $\rightarrow$ $\mathfrak{m} = \frac{\Delta}{\ell} = \frac{1}{4G_3}$ $\qquad \mathfrak{s} = s = \frac{1}{4\mu G_3}$

MPD equation

Motion of a **spinning particle** is described by the Mathisson-Papapetrou-Dixon equations:

$$\boxed{\nabla\left[\mathfrak{m} v^\mu + v_\rho \nabla s^{\mu\rho}\right] = -\frac{1}{2} v^\nu s^{\rho\sigma} R^\mu{}_{\nu\rho\sigma}}$$

$$s^{\mu\nu} = \mathfrak{s}\left(n^\mu \tilde{n}^\nu - \tilde{n}^\mu n^\nu\right)$$



Action principle

$$S_{\text{probe}} = \int_C ds\left(\mathfrak{m}\sqrt{g_{\mu\nu} v^\mu v^\nu} + \mathfrak{s}\,\tilde{n}\cdot\nabla n\right) + \underbrace{S_{\text{constraints}}}_{\substack{\text{Orthogonality of}\\ \text{normal vectors.}\\ \text{Vanishes on-shell.}}}$$

The Proposal for HEE

$$S_{\mathrm{EE}} = \int_C ds \left(\mathfrak{m} \sqrt{g_{\mu\nu} v^\mu v^\nu} + \mathfrak{s}\, \tilde{n} \cdot \nabla n \right)$$

$$\mathfrak{m} = \frac{\Delta}{\ell} = \frac{1}{4G_3} \qquad \mathfrak{s} = s = \frac{1}{4\mu G_3}$$

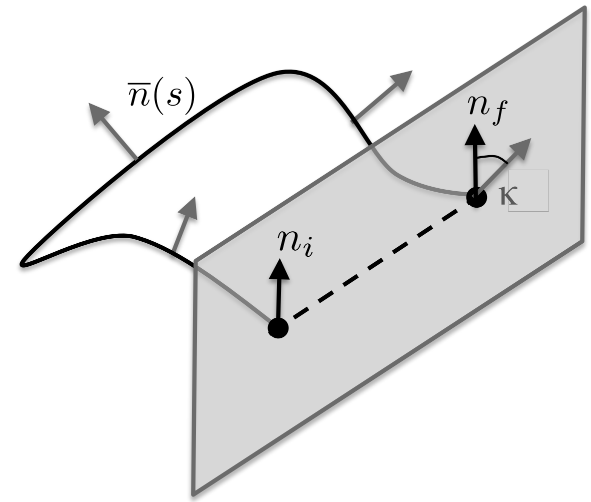
Features:

- We reproduced it from the cone action. However the dynamics is less obvious.
- Spin does not add a new bulk d.o.f. We gave life to a pure gauge mode to make the answer look covariant.
- The action now measures the **torsion** of the curve. It is related the **framing anomaly** in Chern-Simons theory as well.

Sanity checks

Poincare AdS_3 & boosted interval

$$S_{\mathrm{EE}} = \frac{c_L + c_R}{6} \log\left(\frac{R}{\varepsilon}\right) - \frac{c_L - c_R}{6}\,\kappa$$



BTZ BH: high temperature, finite angular velocity

$$S_{\mathrm{EE}} = \frac{c_L}{6} \log\left(\frac{\beta_L}{\pi\varepsilon} \sinh\frac{\pi R}{\beta_L}\right) + \frac{c_R}{6} \log\left(\frac{\beta_R}{\pi\varepsilon} \sinh\frac{\pi R}{\beta_R}\right)$$

Dynamics

Gapped systems in the presence of anomalies

Holographic gapped systems

Two basic properties that we seek in a gapped system are:

Spectrum $\Delta E = \Delta_0 + \cdots$

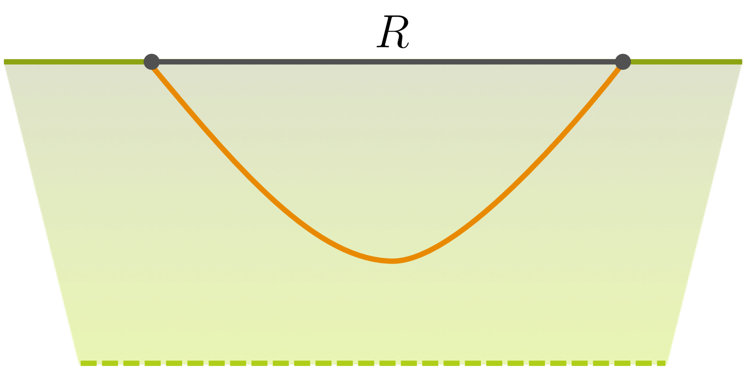
Entanglement $S_{\mathrm{EE}} = S_0 + \cdots$

Finite and independent of the size of the system as we uncompactify

Holographic gapped systems

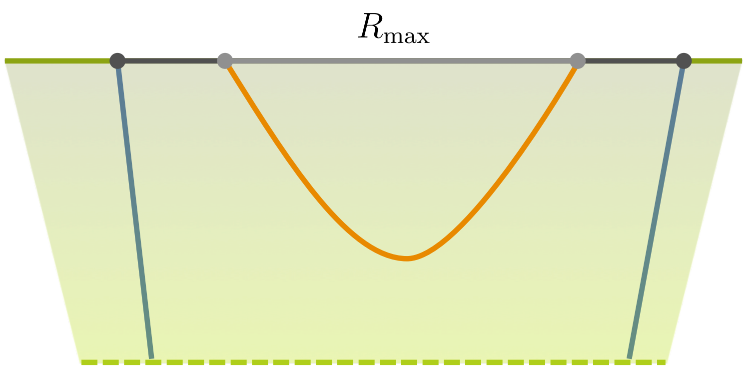
How to tell if a systems is gapped in the IR using HEE?

[Klebanov, Kutasov, Murugan; Liu, Mezei]



UV

IR



NOT GAPPED:
Connected solution exists for any size R and dominates.

GAPPED:
Connected path ceases to exists above a max length. Disconnected solution takes over.

Domain walls

Geometry of the gapped system:

$$ds^2 = \frac{\eta_{ij} dx^i dx^j}{z^2} + \frac{dz^2}{z^2 f(z)}$$

UV $\qquad f(z) \to 1$

IR $\qquad f(z) \sim z^n$

[Gubser; Freedman et al; Kiristis et al; …]

NOTE: These solutions, for arbitrary f(z), have vanishing Cotton tensor

A "gapped" domain wall is allowed in a theory with a diffeomorphism anomaly!

Gaps & Diff anomalies

Anomalies are powerful and wise.

Chiral anomalies are protected by RG.
Chiral matter will not become massive in the IR. No gaps!

2d 't Hooft anomaly matching condition: $c_L - c_R$

How does holography capture this simple and
yet robust statement!?

MPD solutions

This is a challenge to our spinning probe!
Does HEE capture anomaly matching correctly?

Functional: $$S_{\mathrm{EE}} = \int_C ds \left(\mathfrak{m}\sqrt{g_{\mu\nu}v^\mu v^\nu} + \mathfrak{s}\,\tilde{n}\cdot\nabla n\right)$$

MPD Dynamics: $$\nabla\left[\mathfrak{m}v^\mu + v_\rho \nabla s^{\mu\rho}\right] = -\frac{1}{2}v^\nu s^{\rho\sigma} R^\mu{}_{\nu\rho\sigma}$$

$$s^{\mu\nu} = -\mathfrak{s}\epsilon^{\mu\nu\lambda}v_\lambda$$

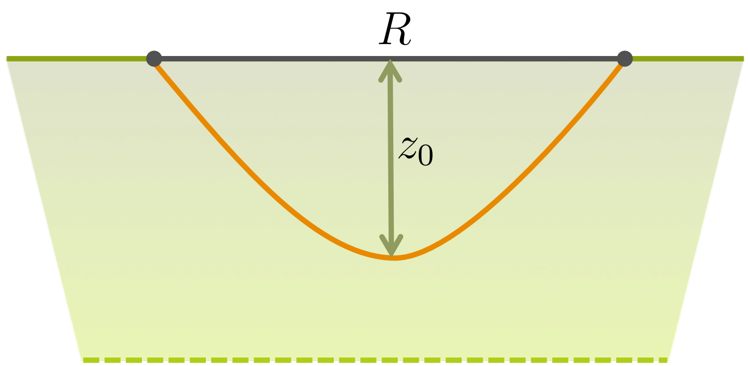
Task: Characterize solutions to MPD in the domain wall background

$$ds^2 = \frac{\eta_{ij}dx^i dx^j}{z^2} + \frac{dz^2}{z^2 f(z)}$$

$$\boxed{\nabla\left[\mathfrak{m} v^{\mu} + v_{\rho} \nabla s^{\mu\rho}\right] = -\frac{1}{2} v^{\nu} s^{\rho\sigma} R^{\mu}{}_{\nu\rho\sigma}}$$

Comments:

- Equation highly non-linear.
- Exact solutions: either disconnected or only valid in AdS_3.
- Connected solutions are **not static**. The probe wants to twist.
- Build connected solutions perturbatively in the spin.



$$R \sim z_0 + O(z_0^2)\ , \qquad z_0 \text{ small}$$

$$R \sim z_0^{1-n/2} + s^2 z_0^{1+n/2} + \cdots\ , \qquad z_0 \text{ large}$$

Connected solution exists for any size of the boundary interval.

Outlook

What is next?

Anomalies

- Generalization to higher dimensions: spinning membranes?
- Generalization in 4k+2 QFTs
- Mixed gauge-gravitational anomalies

Gaps

- Work in progress: linearized spectrum should corroborate results.
- Exact connected solutions to MPD equations.
- CFT interpretation to all MPD solutions?

Thank you!