Black holes in the 1/D expansion

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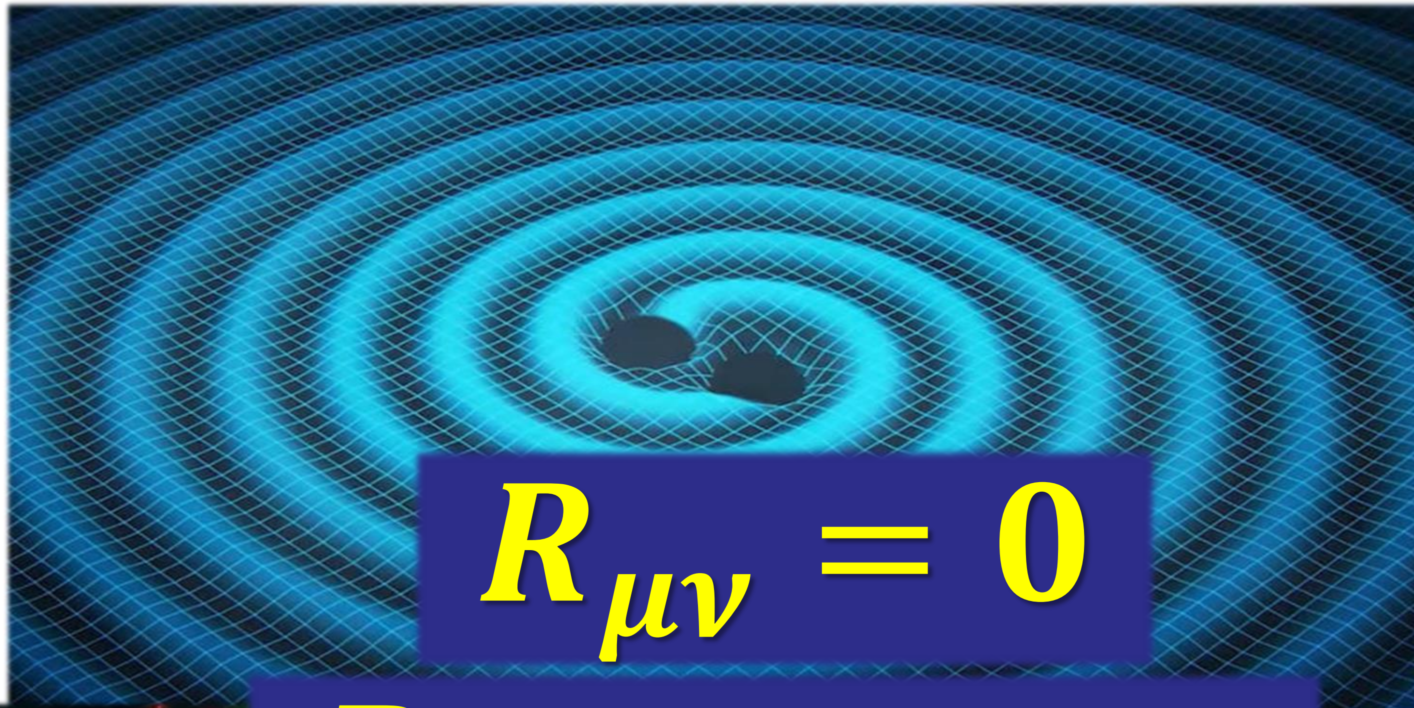
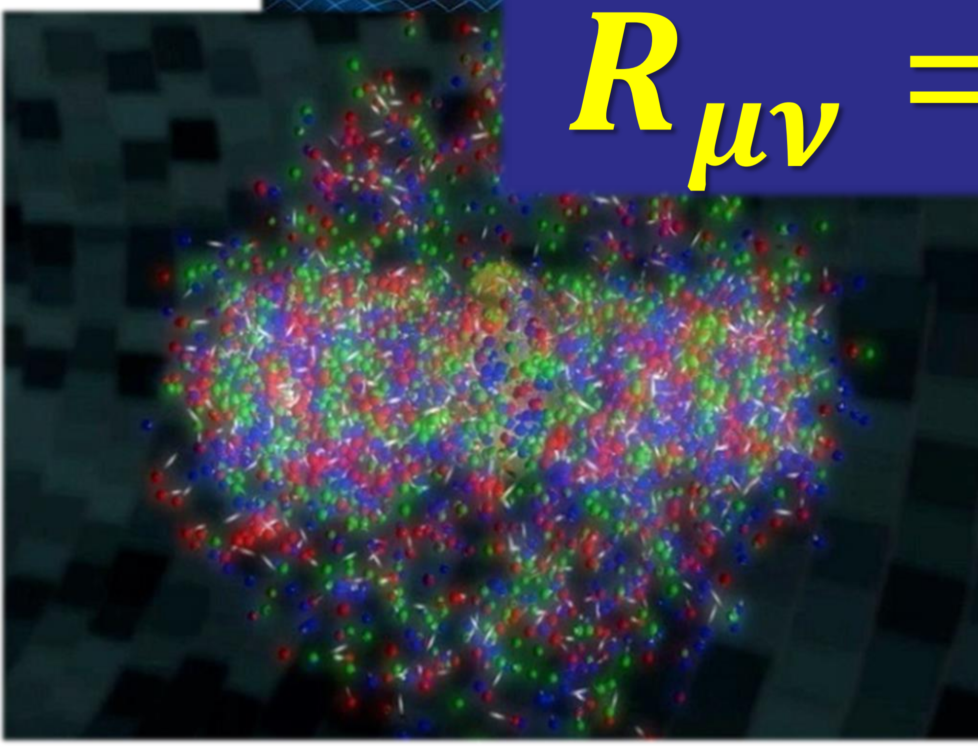
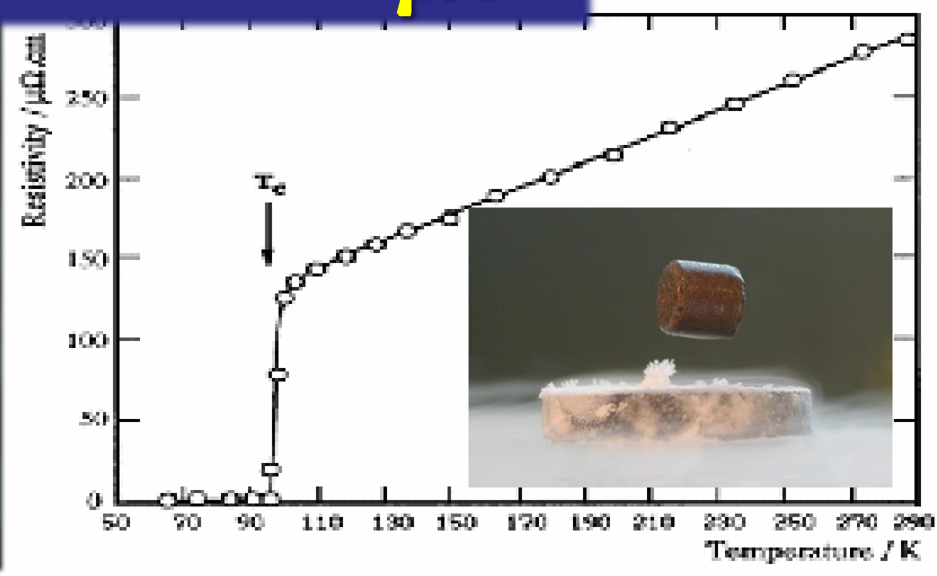


$$R_{\mu\nu} = 0$$

$$R_{\mu\nu} = -\Lambda g_{\mu\nu}$$





Black holes are very important objects in **GR**,

but they do not appear in the fundamental formulation of the theory

They're non-linear, extended field configurations with complicated dynamics

Strings are very important in **YM** theories,

but they do not appear in the fundamental formulation of the theory

They're non-linear, extended field configurations with complicated dynamics

Strings *become fundamental* objects in the **large N** limit of SU(**N**) YM

In this limit, they are still extended objects, but their dynamics simplifies drastically

Is there a limit of GR in which Black Hole dynamics simplifies a lot?

Yes, the limit of **large D**

any other parameter?

Is there a limit in which GR can be formulated with black holes as the fundamental (extended) objects?

Maybe, the limit of large D

BH in *D* dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

Localization of interactions

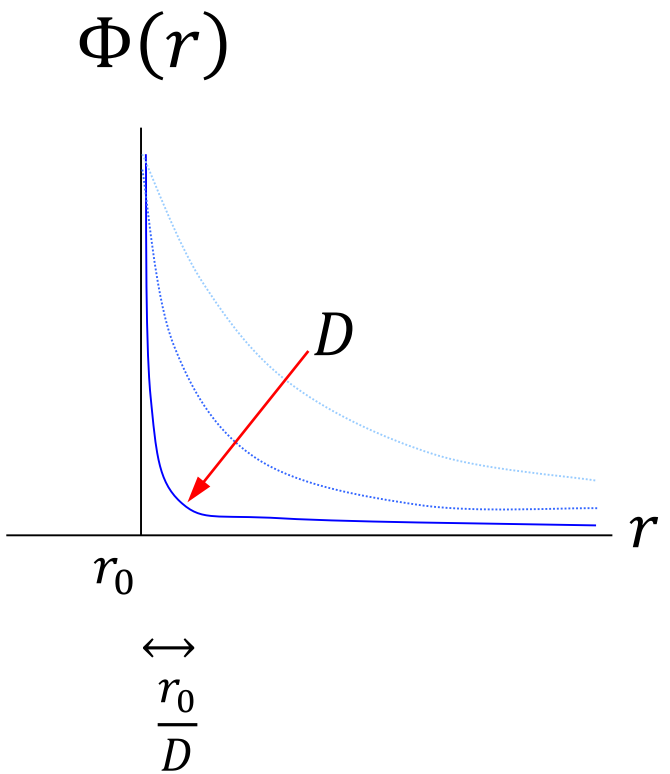
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

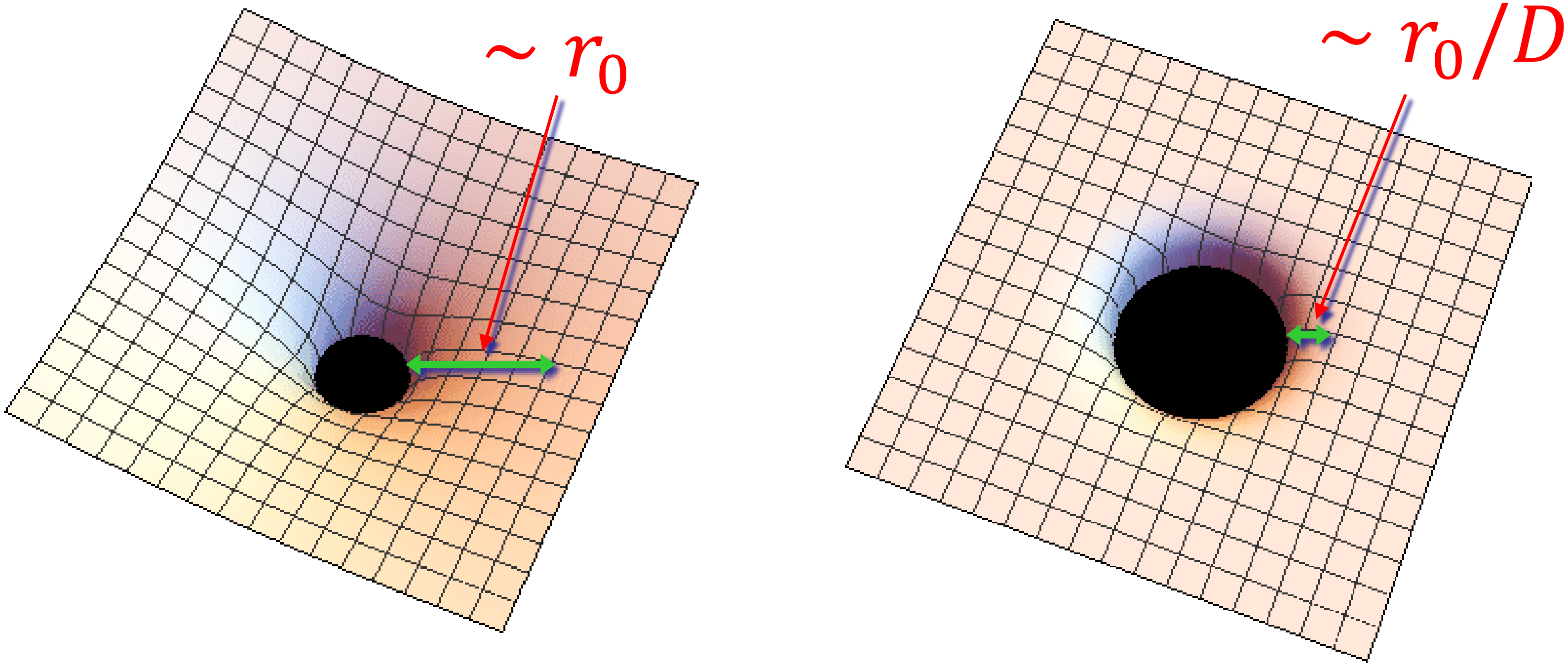
$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\boxed{\frac{r_0}{D} \ll r_0}$$

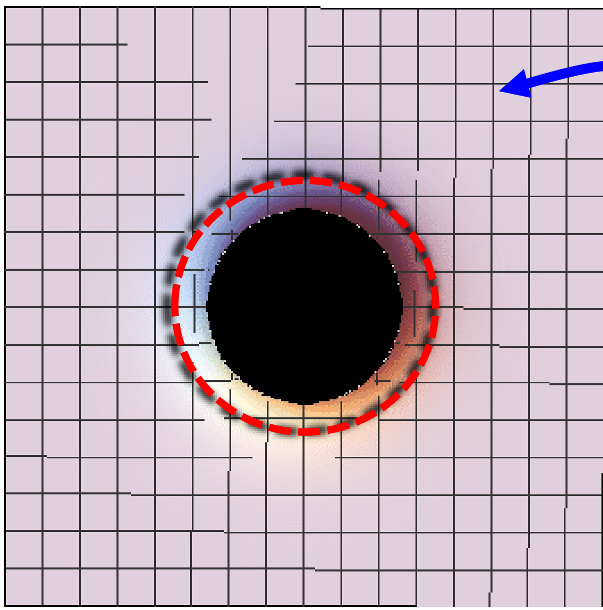


Large-D ⇒ neat separation bh/background



$D = 4$ $\qquad$ $D \gg 4$

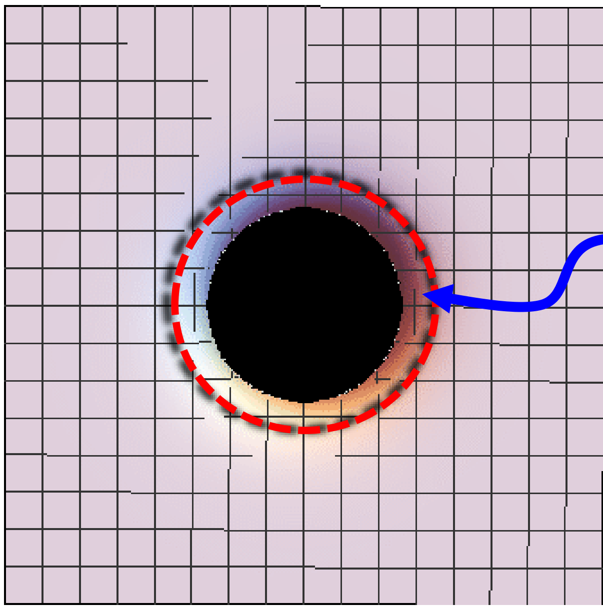
$$r \gg \frac{r_0}{D} \Rightarrow \left(\frac{r_0}{r}\right)^{D-3} \to 0$$



Flat space

“**Far**” region

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \iff r - r_0 \lesssim \frac{r_0}{D}$$



$r - r_0 \sim \frac{r_0}{D}$

$r \ll r_0$: “**Near-horizon**” region

Near-horizon geometry

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0}t \end{aligned}\right\} \begin{array}{l}\text{finite}\\ \text{as } D \to \infty\end{array}$$

Near-horizon geometry

2d string bh

$$ds_{nh}^2 \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho\ dt_{near}^2 + d\rho^2\right)$$

$$+\, r_0^2\,(\cosh\rho)^{4/D}\, d\Omega_{D-2}^2$$

2d dilaton

Soda 1993
Grumiller et al 2002
RE+Grumiller+Tanabe 2013

‘string length’ $\ell_s \sim \frac{r_0}{D}$

Near-horizon universality

2d string bh = near-horizon geometry of **all neutral non-extremal bhs**

rotation = local boost

(along horizon)

cosmo const = 2d bh mass-shift

Does this help understand/solve bh dynamics?

Quasinormal modes

capture interesting perturbative dynamics:

-possible instabilities

-hydrodynamic behavior

but, w/out a small parameter, these modes are not easily distinguished from other more boring quasinormal modes

Large D introduces a generic small parameter

It isolates the 'interesting' quasinormal modes from the 'boring' modes

The distinction comes from whether the modes are normalizable or non-normalizable in the near-horizon region

‘Boring’ modes

Non-normalizable in near-zone

Not decoupled from the far zone

High frequency: $\omega \sim D/r_0$

Universal spectrum: only sensitive to bh radius

Almost **featureless oscillations of a hole in flat space**

‘Interesting’ modes

Normalizable in near zone

Decoupled from the far zone

Low frequency: $\omega \sim D^0/r_0$

Sensitive to bh geometry beyond the leading order

Capture instabilities and hydro

Efficient calculation to high orders in $\frac{1}{D}$

Black hole perturbations

Quasinormal modes of Schw-(A)dS bhs

Gregory-Laflamme instability

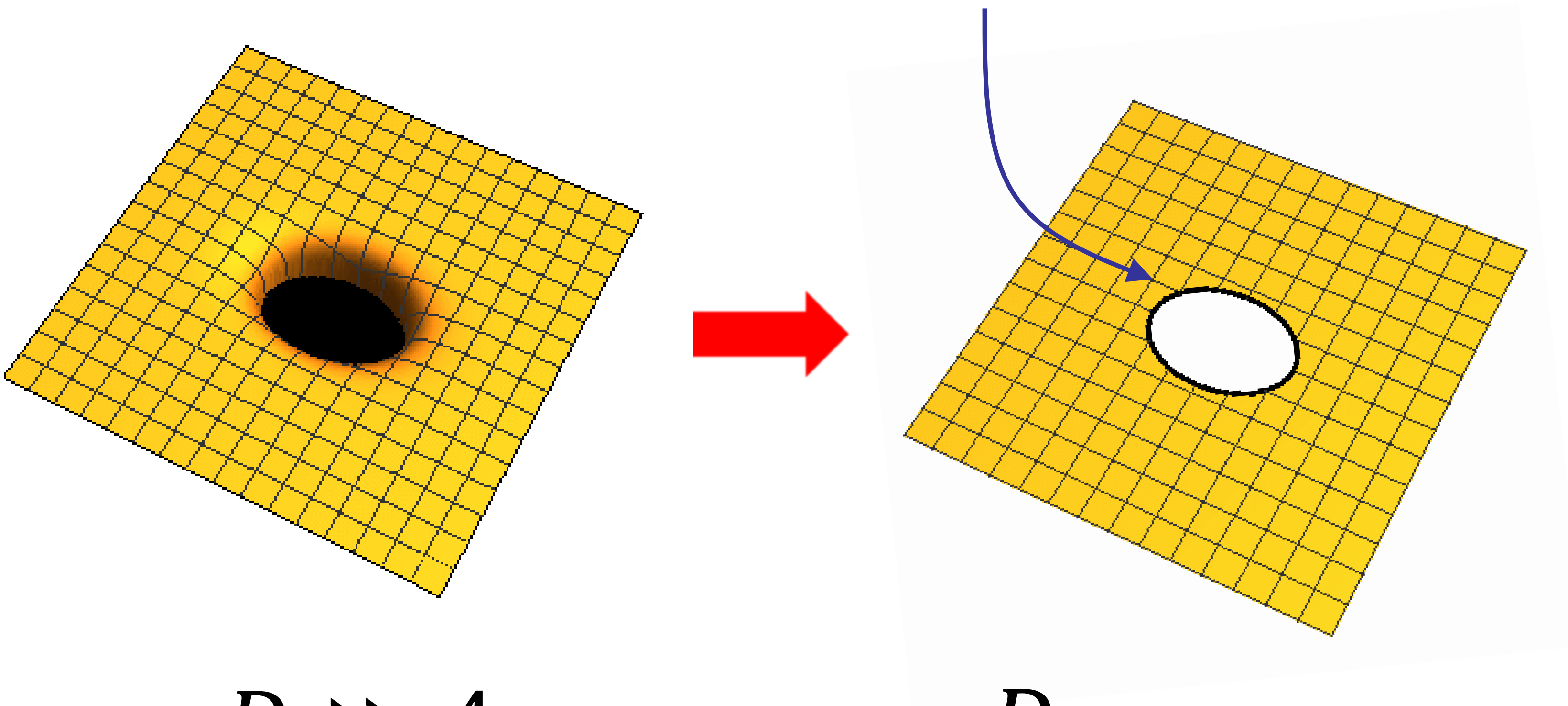
Ultraspinning instability

All solved analytically

Fully non-linear GR @ large D

Replace bh ⟶ Surface in background

What's the dynamics of this surface?



$D \gg 4$

$D \to \infty$

Large D Effective Theory

Solve near-horizon equations

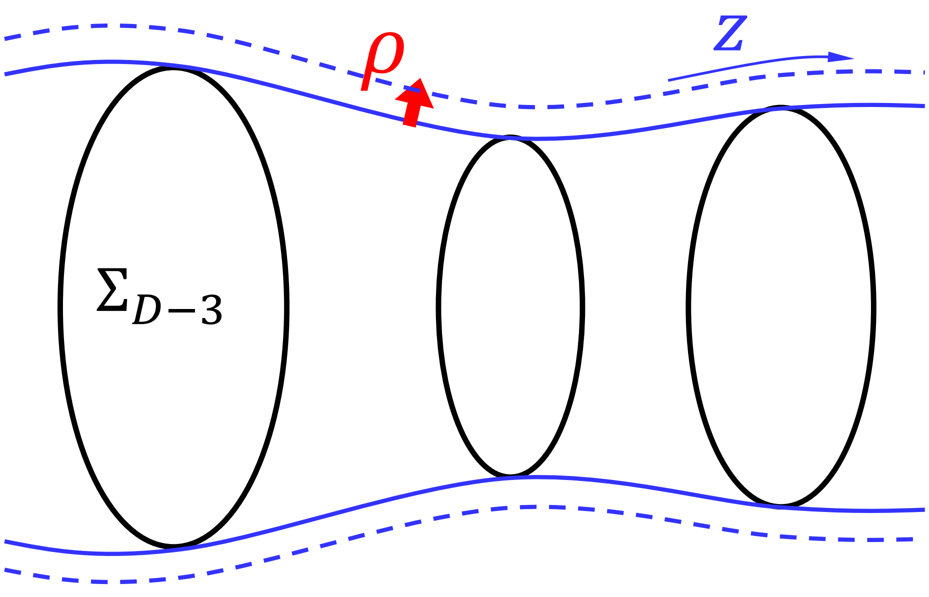
→ *Effective theory*

for the 'slow' decoupling modes

Gradient hierarchy

$\perp$ Horizon: $\partial_\rho \sim D$

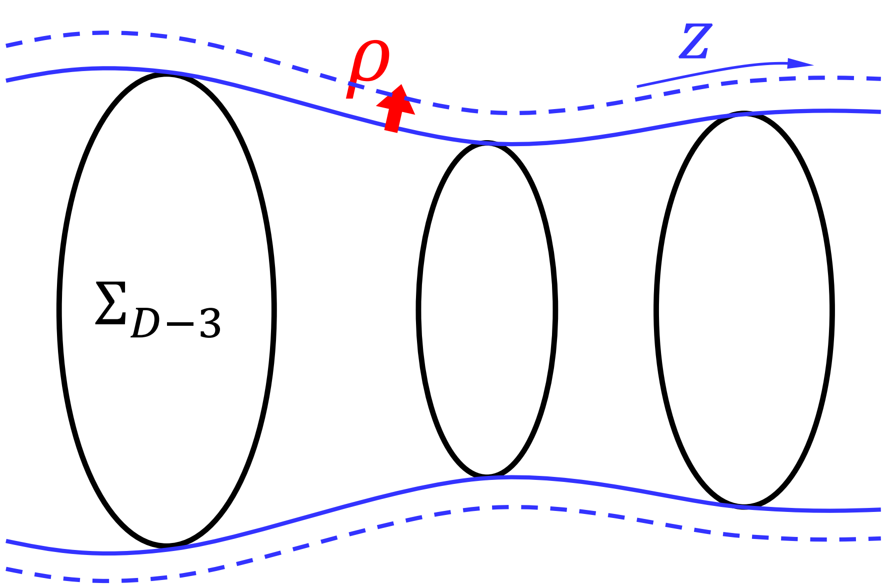
$\parallel$ Horizon: $\partial_z \sim 1$



Static geometry

$$ds^2 = N^2(z)\frac{d\rho^2}{D^2} + g_{\Omega\Omega}(\rho,z)d\Sigma_{D-3}$$

$$+g_{tt}(\rho,z)dt^2 + g_{zz}(\rho,z)dz^2$$



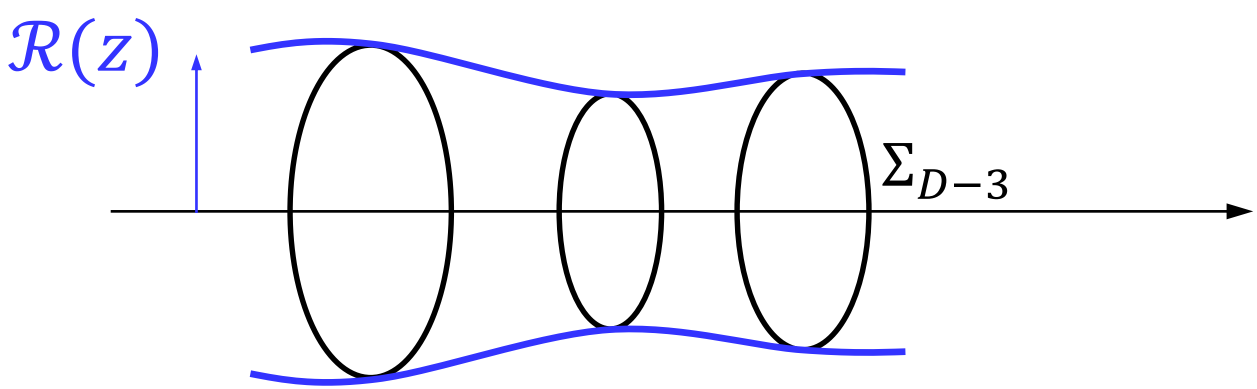
Einstein ‘momentum-constraint’ in ρ:

$$\sqrt{-g_{tt}}K = \text{const}$$

K = mean curvature of ‘horizon surface’

$$ds^2|_h = g_{tt}(z)dt^2 + dz^2 + \mathcal{R}^2(z)d\Sigma_{D-3}$$

embedded in background



Large D static black holes:

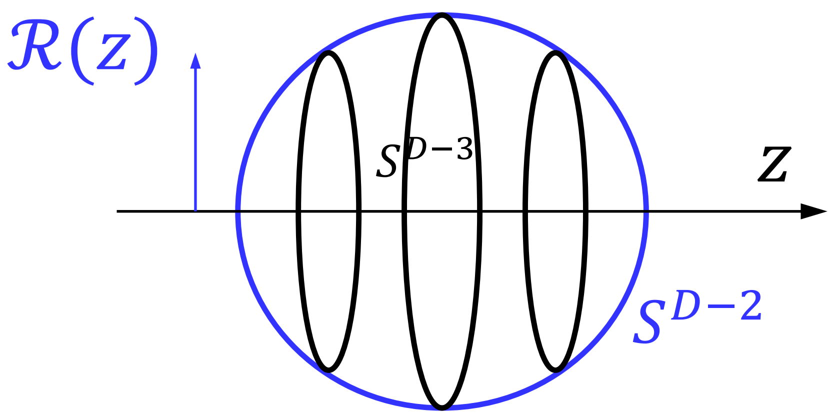
Soap-film equation (redshifted)

$$\sqrt{-g_{tt}}K = \mathrm{const}$$

Some applications

Soap bubble in Minkowski = Schw BH

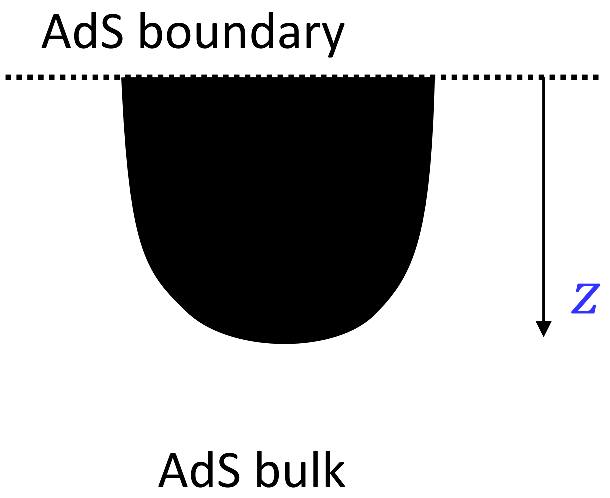
$$\sqrt{-g_{tt}}K = \text{const} \;\Rightarrow\; \mathcal{R}'^2 + \mathcal{R}^2 = 1$$



$$\Rightarrow \mathcal{R}(z) = \sin z$$

Black droplets

Black hole at boundary of AdS



dual to CFT in BH background

Numerical solution:

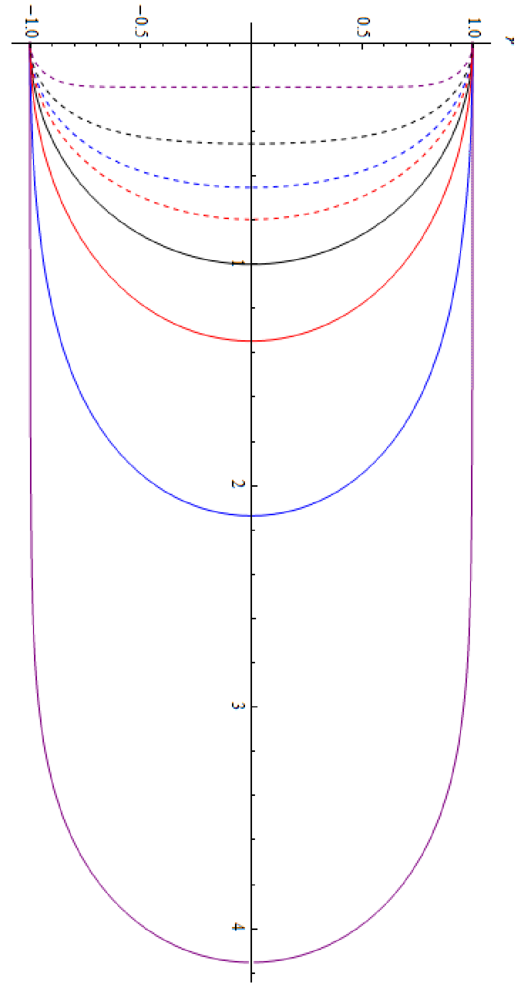
Figueras+Lucietti+Wiseman

Our numerical code

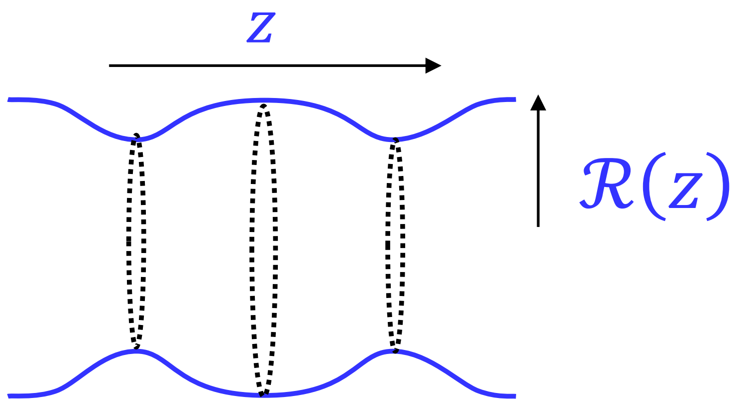
```
zmin = 0.000001;
zmax = 0.67;
r0 = .5;
NDSolve[{r'[z] == -z/r[z] (1 - Sqrt[r[z]^2 + z^2 (1 - r[z]^2)])/(1 - z^2), r[zmin] == r0}, r, {z, zmin, zmax}]
```

$$\texttt{NDSolve}\left[\left\{r'[z] == -\frac{z}{r[z]}\,\frac{1-\sqrt{r[z]^2+z^2\left(1-r[z]^2\right)}}{1-z^2},\ r[\texttt{zmin}] == \texttt{r0}\right\},\ r,\ \{z,\texttt{zmin},\texttt{zmax}\}\right]$$

Black droplets

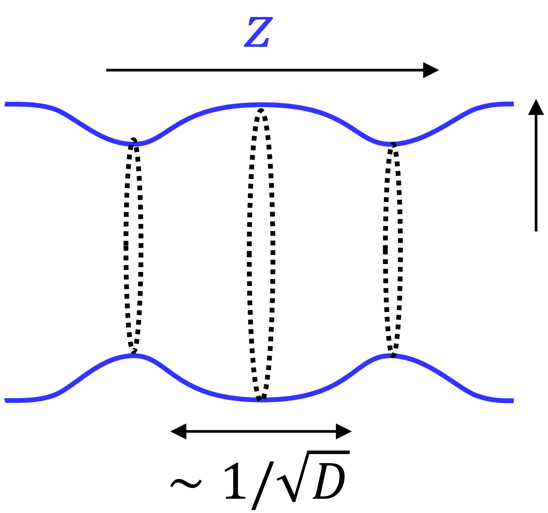


Non-uniform black strings



Numerical solution: *Wiseman*

Non-uniform black strings



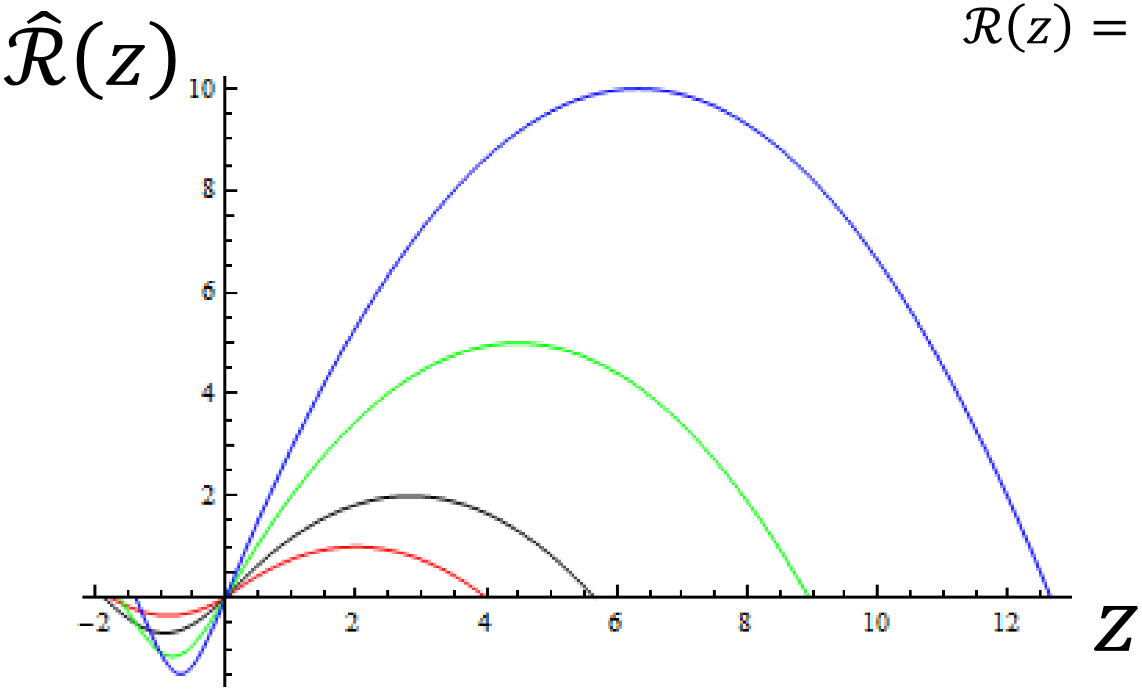
$$\mathcal{R}(z) = 1 + \frac{2\hat{\mathcal{R}}(z)}{D}$$

$$K = \text{const}$$

$$\Longrightarrow\ \hat{\mathcal{R}}'' + \hat{\mathcal{R}}'^{2} + \hat{\mathcal{R}} = \text{const}$$

Non-uniform black strings

$$\mathcal{R}(z) = 1 + \frac{2\hat{\mathcal{R}}\,(z)}{D}$$



At NLO we find a critical dimension D^* for black strings (from 2nd order to 1st order)

at $D^* = 13$

Numerical value

$D^* \simeq 13.5$ *E Sorkin 2004*

We've also solved for **stationary black holes**

Ultraspinning bifurcations of
(single-spin) Myers-Perry black holes at

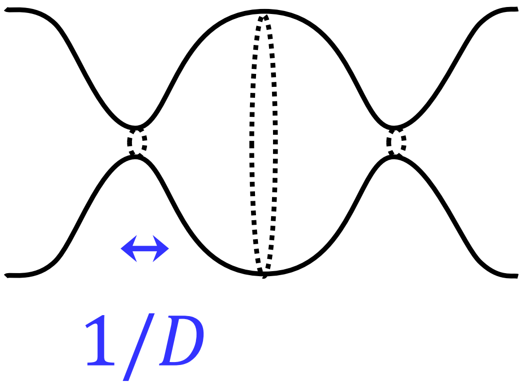
$$\frac{a}{r_+} = \sqrt{3}, \sqrt{5}, \sqrt{7}, \ldots$$

Numerical (D=8): $\frac{a}{r_+} = 1.77, 2.27, 2.72 \ldots$ *Dias et al*

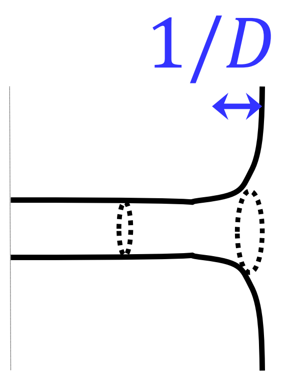
Limitations

1/D expansion breaks down when $\partial_z \sim D$

- Highly non-uniform black strings



- AdS black funnels



Time evolution

Long wavelength, slow evolution $\partial_{t,z} \sim D^0$

can lead to

large gradients, fast evolution $\partial_{t,z} \sim D$

if so, breakdown of expansion

Conclusions

1/D expansion of GR is very efficient at capturing dynamics of horizons

Reformulation of a sector of GR: bhs in terms of surfaces (membrane-like)

decoupled from bulk (grav waves)

1/D: it works

(not obvious beforehand!)

Spherical reduction of Einstein-Hilbert

$$ds_{nh}^2 = \frac{4r_0^2}{D^2}\left(-g_{\mu\nu}^{(2)} dx^\mu dx^\nu\right) + r_0^2 e^{-\frac{4\Phi(x)}{D-2}} d\Omega_{D-2}^2$$

$$g_{\mu\nu}^{(2)}(x)\,,\ \Phi(x)$$

$$I = \int d^2x\sqrt{-g}e^{-2\Phi}\left(R + 4\frac{D-3}{D-2}(\nabla\Phi)^2 + \frac{(D-3)(D-2)}{r_0^2}e^{4\Phi/(D-2)}\right)$$

$\Longrightarrow$ 2d dilaton gravity

Spherical reduction of Einstein-Hilbert

$$ds_{nh}^2 = \frac{4r_0^2}{D^2}\left(-g_{\mu\nu}^{(2)}dx^\mu dx^\nu\right) + r_0^2 e^{-\frac{4\Phi(x)}{D-2}} d\Omega_{D-2}^2$$

$$D \to \infty$$

Soda, Grumiller et al

$$I \to \int d^2x\sqrt{-g}e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 + \frac{D^2}{r_0^2}\right)$$

$\Longrightarrow$ 2d **string** gravity

$$\ell_{string} \sim \frac{2r_0}{D}$$

BH perturbations: How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

BH perturbations: How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

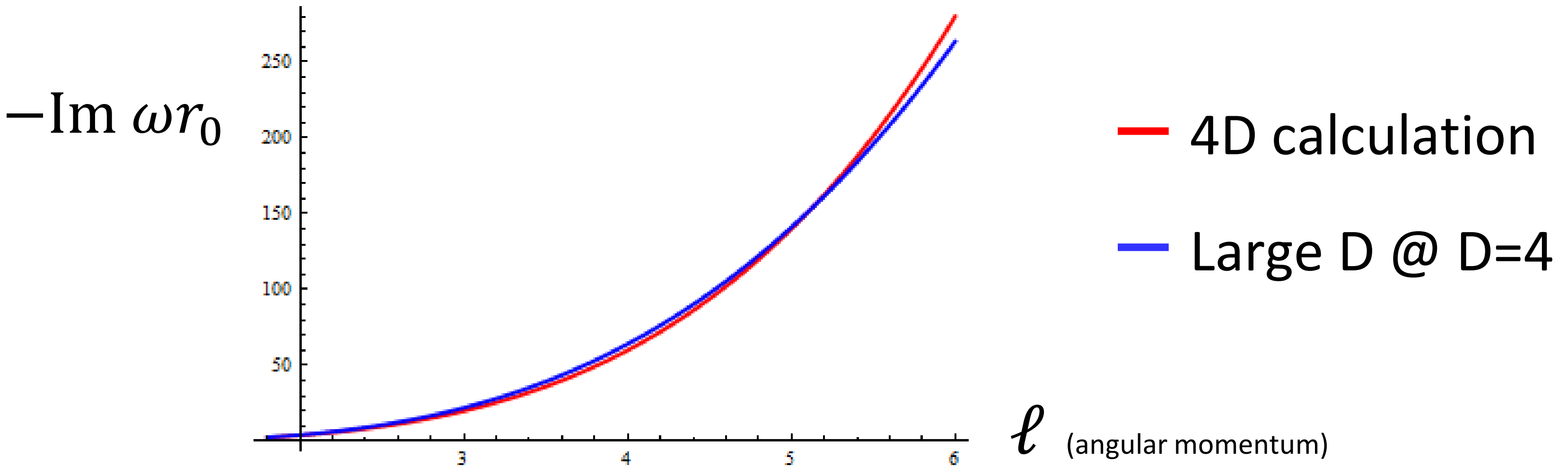
But it seems to be $\frac{1}{\mathbf{2}(D-3)}$

not *so* bad in $D = 4$, if we can compute higher orders

(in AdS: $\frac{1}{2(D-1)}$)

Quite accurate

Quasinormal frequency in $D = 4$ (vector-type)



Calculation up to $\frac{1}{D^3}$ yields 6% accuracy in $D = 4$

$$6\% = \frac{1}{\left(2(D-3)\right)^4}\Big|_{D=4}$$

Quantum effects?

Dimensionful scale:

$$L_{Planck} = (G\hbar)^{\frac{1}{D-2}}$$

Quantum effects governed by $\frac{r_0}{L_{Planck}}$

If $\dfrac{r_0}{L_{Planck}} \sim D^0$ the bh is fully quantum:

Entropy $\rightarrow 0$

Temperature $\rightarrow \infty$

Evaporation lifetime $\rightarrow 0$

But other scalings are possible

Scaling $\frac{r_0}{L_{Planck}}$ with D:

how large are the black holes,

which quantum effects are finite at large D

Finite entropy: $r_0/L_{Planck} \sim D^{1/2}$

Finite temperature: $r_0/L_{Planck} \sim D$

Finite energy of Hawking radn: $r_0/L_{Planck} \sim D^2$

Black hole perturbations

Given the general master equation, it's a straightforward perturbative analysis

Leading order is simple and universal
(solving in 2D string bh): static modes $\omega \sim \frac{1}{D}\left(\frac{D}{r_0}\right) \to 0$

Higher order perturbations are not universal, but organized by simple leading order solution