Localization in Supergravity and Quantum AdS4/CFT3 Holography

João Gomes,
DAMTP

CFT

ABJM Theory

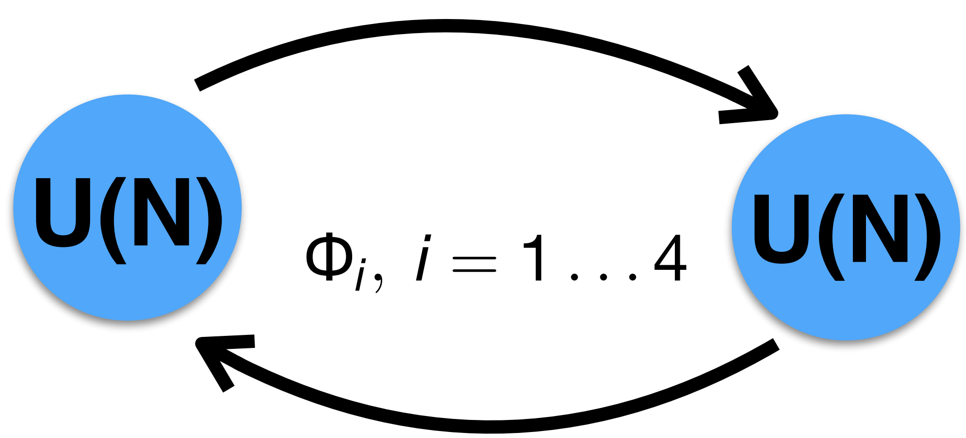
ABJM is a N=6 3d SCFT

Chern-Simons-matter-theory:

2 CS + 4 hypers bifundamental

[Aharony, Bergman, Jafferis, Maldacena]

$$U(N)_k \times U(N)_{-k}$$



't Hooft coupling

$$\lambda = \frac{N}{k}$$

It describes the low energy physics of

N $M2$-branes on $\mathbb{C}^4/\mathbb{Z}_k$

Dual to M-theory on AdS4

By AdS/CFT, conjecturally dual to

$$\text{M-theory on } AdS_4 \times S^7/\mathbb{Z}_k$$

$$\int_{S^7/\mathbb{Z}_k} \star F_4 = N$$

Large k, it is dual to

$$\text{II A on } AdS_4 \times \mathbb{CP}^3$$

$$S^7 \simeq \mathbb{CP}^3 \times S^1 \qquad g_s \propto \frac{1}{k}$$

ABJM on S^3 gives matrix model

We can use Localization on S^3 to compute exact partition function

[Kapustin, Yaakov, Willett]

$$Z_{S^3}(N,k) = e^{S_{ABJM}} \times Z_{1-loop} =$$

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$$\frac{1}{N!^2}\int \frac{d^N\mu}{(2\pi)^N}\frac{d^N\nu}{(2\pi)^N}\exp\left[\frac{ik}{4\pi}\sum_{i=1}^{N}(\mu_i^2-\nu_i^2)\right]\frac{\prod_{i<j}\left[2\sinh(\frac{\mu_i-\mu_j}{2})\right]^2\left[2\sinh(\frac{\nu_i-\nu_j}{2})\right]^2}{\prod_{i,j}\left[2\cosh(\frac{\mu_i-\nu_j}{2})\right]^2}$$

Vector multiplet scalar localizes in the Cartan directions

$$\sigma_{N\times N} = \mathrm{diag}(\mu^i)$$

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Perturbative partition function is Airy function

In a large N expansion we can systematically compute corrections to the free energy $F = \ln Z_{S^3}$

't Hooft expansion

$$1)\ N \gg 1,\ \lambda, \qquad F(g_s, \lambda) = \sum_{g \geq 0} g_s^{2g-2} F_g(\lambda) \qquad g_s = \frac{2\pi i}{k}$$

M-theory expansion

$$2)\ N \gg 1,\ k, \qquad F(N, k) = \sum_{g \geq 0} N^{2-2g} F_g(k)$$

1. **Matrix model is similar to pure CS on Lens space L(2,1) ~ ~topological string via large N duality [Marino, Drukker, Putrov; Marino, Putrov]**

2. **Fermi gas approach: one-dim ideal Fermi gas of N particles [Marino, Putrov]**

Perturbative partition function is Airy function

Free energy receives both ***perturbative and nonperturbative*** corrections

$$F_g(\lambda) = F_g^{pt}(\lambda) + F_g^{nonpt}(\lambda)$$

Nonperturbative corrections have interpretation of

IIA Worldsheet +
D2-brane instantons

$$F_g^{nonpt}(\lambda) \sim \mathcal{O}\left(e^{-2\pi\sqrt{\lambda}}, e^{-\pi k\sqrt{\lambda}}\right)$$

[Marino, Drukker, Putrov]

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Large N, λ

Perturbative partition function is Airy function

The perturbative part, on the other hand, can be exactly resummed to obtain an Airy function!

$$Z_{S^3} \simeq \exp F^{pert}(g_s, \lambda)$$

$$\propto Ai\left[\left(\frac{\pi^2 k}{2}\right)^{1/3}\left(N - \frac{k}{24} - \frac{1}{3k}\right)\right]$$

Also valid in M-theory limit: small k!!

[Fuji, Hirano, Moriyama; Marino, Putrov]

$$Ai(z) = \frac{1}{2\pi i}\int_{e^{-i\pi/3}\infty}^{e^{i\pi/3}\infty} dt \exp\left[\frac{t^3}{3} - zt\right]$$

ABJM dual to M-theory on AdS4

Since by AdS/CFT

$$ABJM\ (N,k) \sim \text{M-theory on } AdS_4 \times S^7/Z_k$$

We can use the ***master equation***

$$Z_{AdS_4} = Z_{CFT_3}$$

...to extract important information about

quantum corrections in M-theory/String theory

ABJM dual to M-theory on AdS4

For example, in a large N expansion

$$\ln Z_{S^3} = -\frac{\sqrt{2}\pi}{3} k^{1/2} N^{3/2} - \frac{1}{4} \ln N + \ldots$$

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Compute in M-theory by summing over field configurations

$$Z_{AdS_4} \simeq e^{S(EH+\Lambda)+1-loop+\ldots}$$

[Marino, Drukker, Putrov] [Marino, Sen, Bhattacharya, Grassi]

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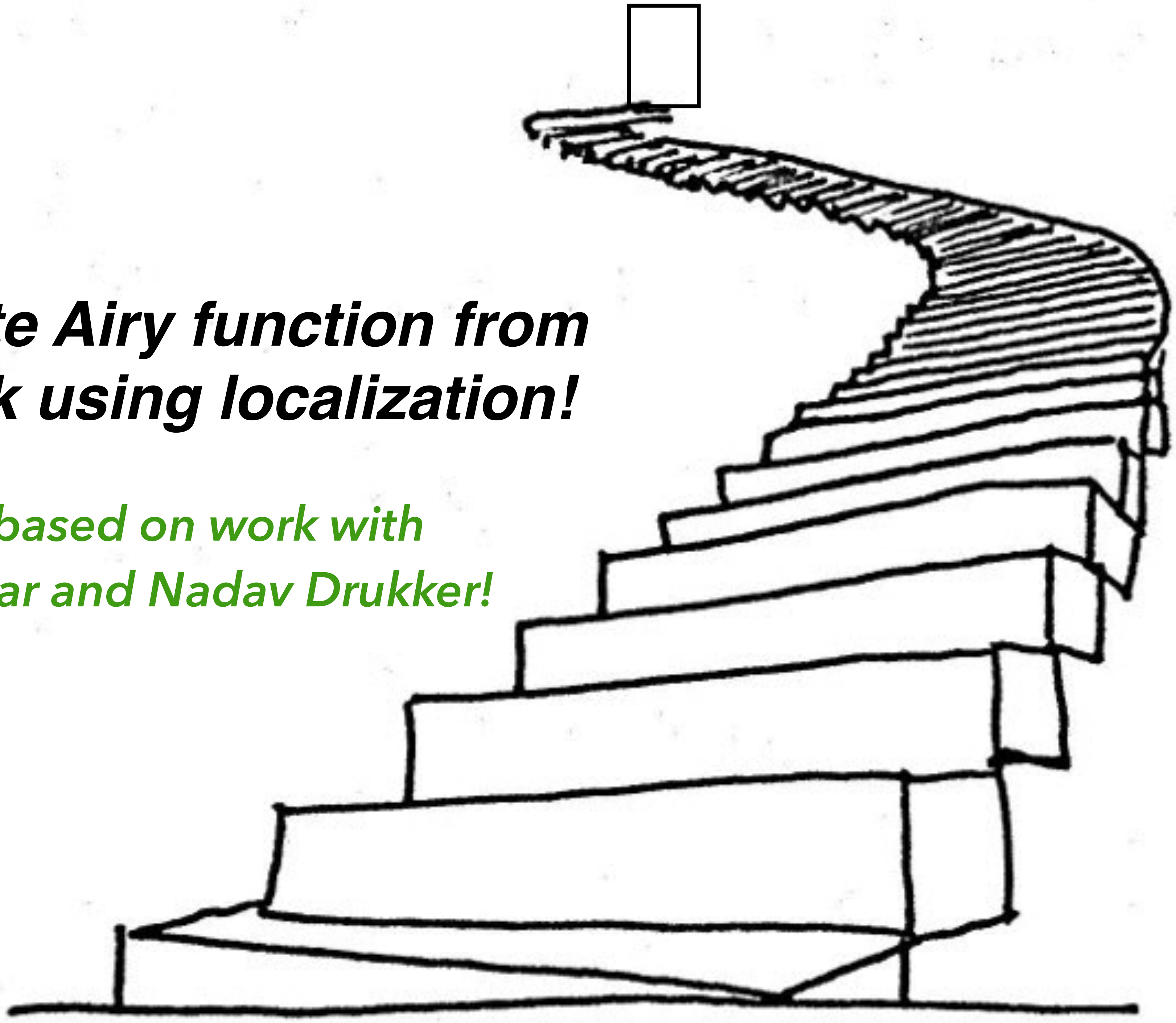
[Marino, Drukker, Putrov] [Marino, Sen, Bhattacharya, Grassi]

Goal here is to compute all 1/N corrections!!

[JG, Dabholkar, Drukker]

PLAN



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This talk is based on work with Atish Dabholkar and Nadav Drukker!

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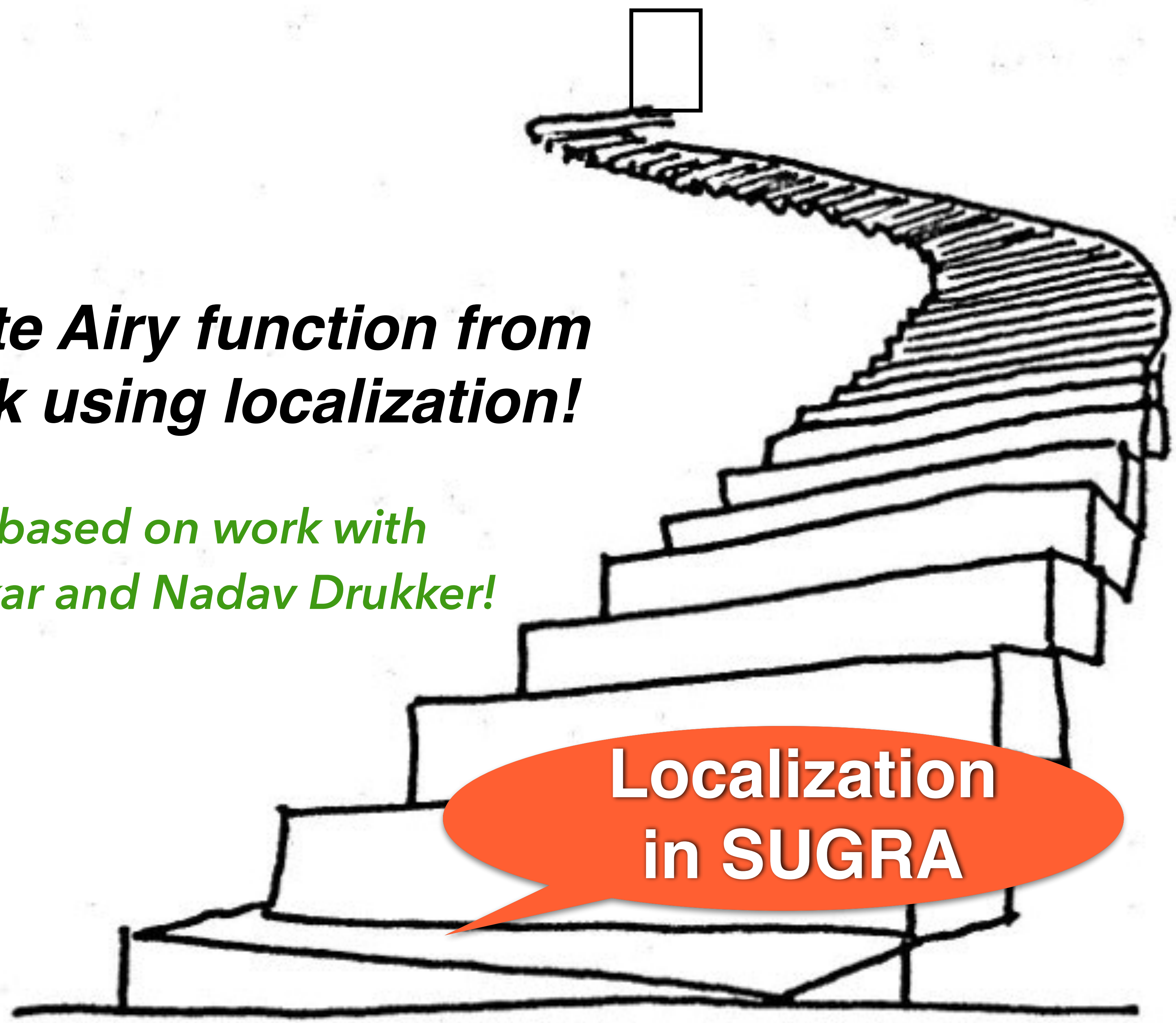

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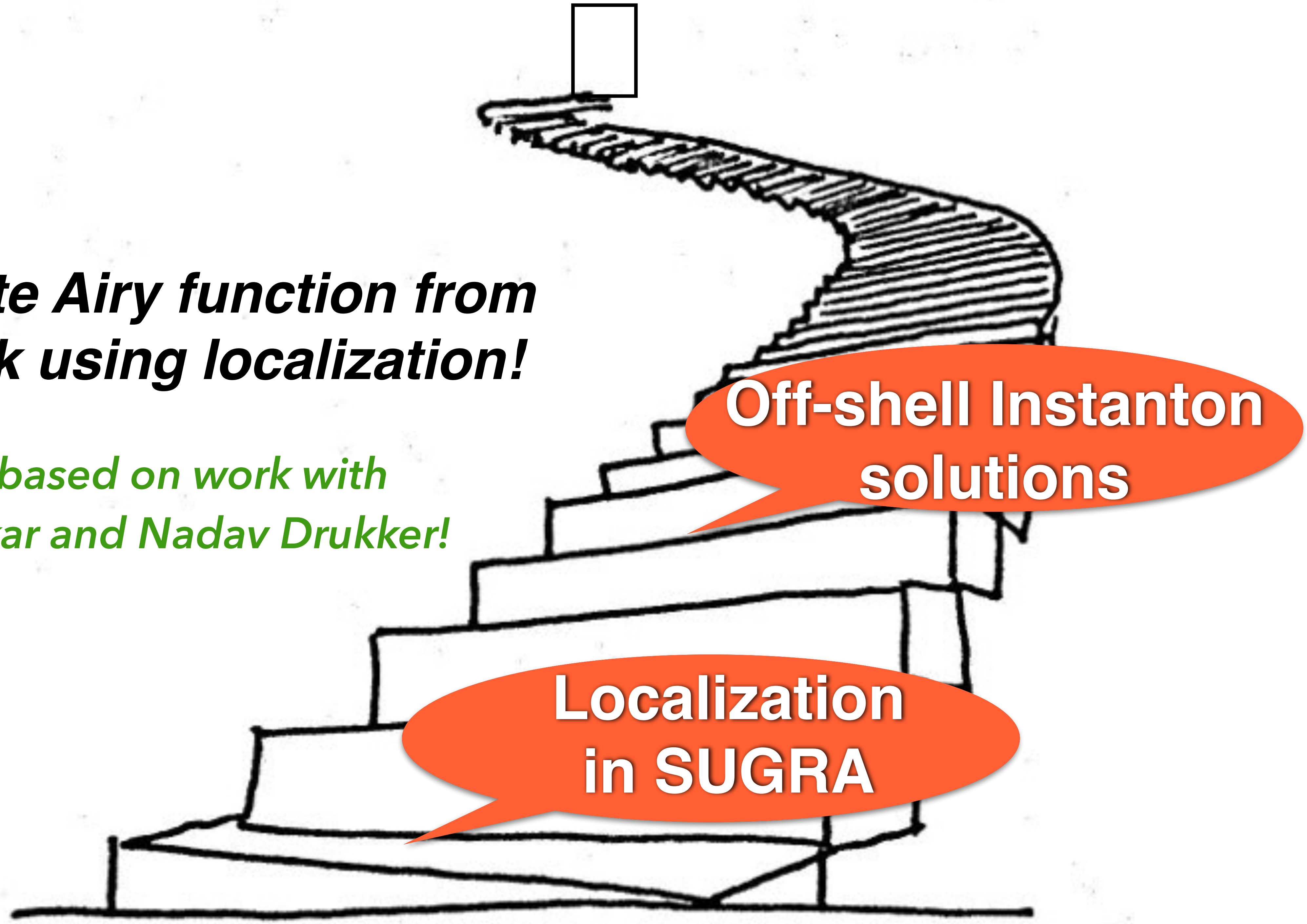

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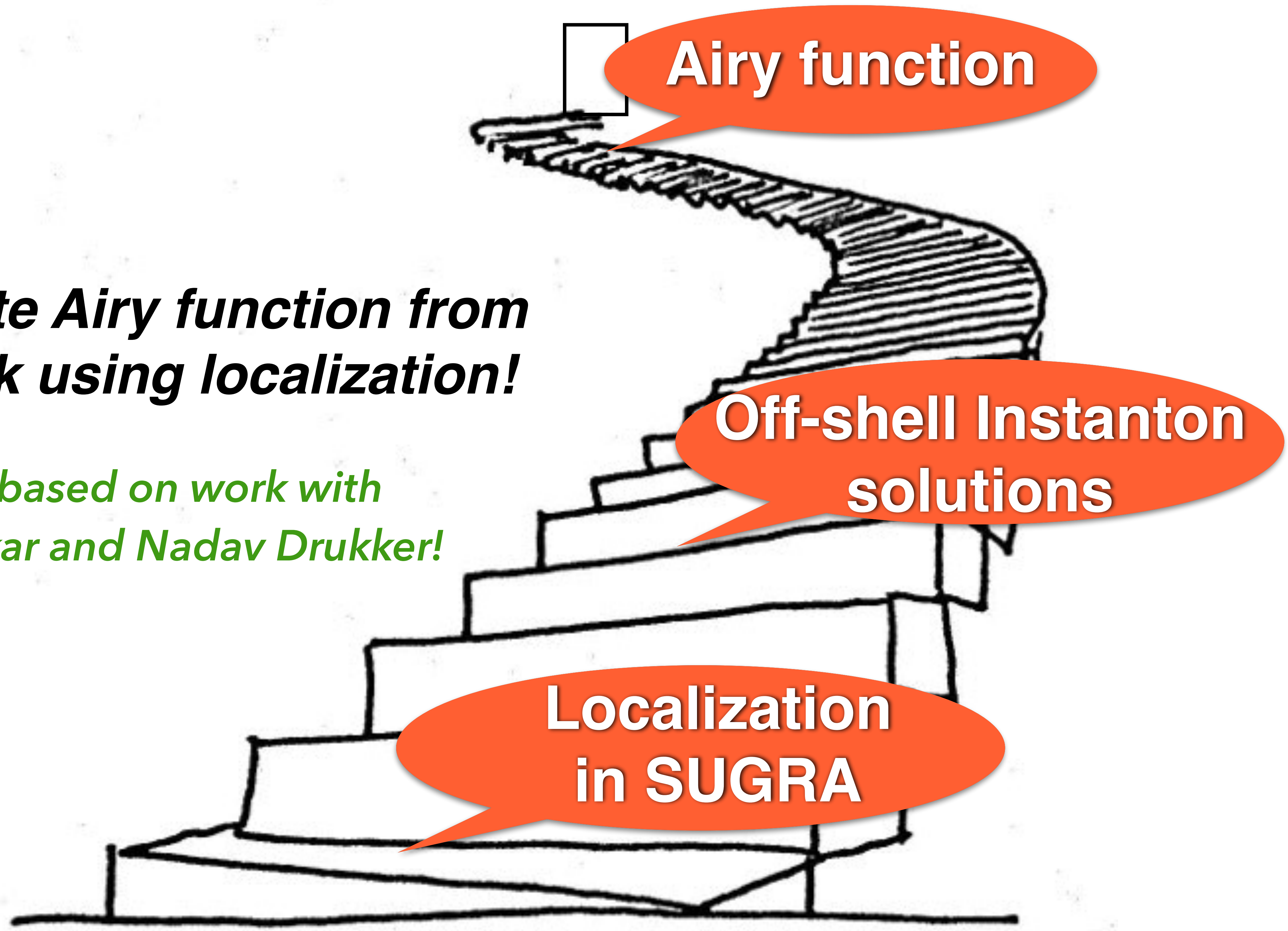

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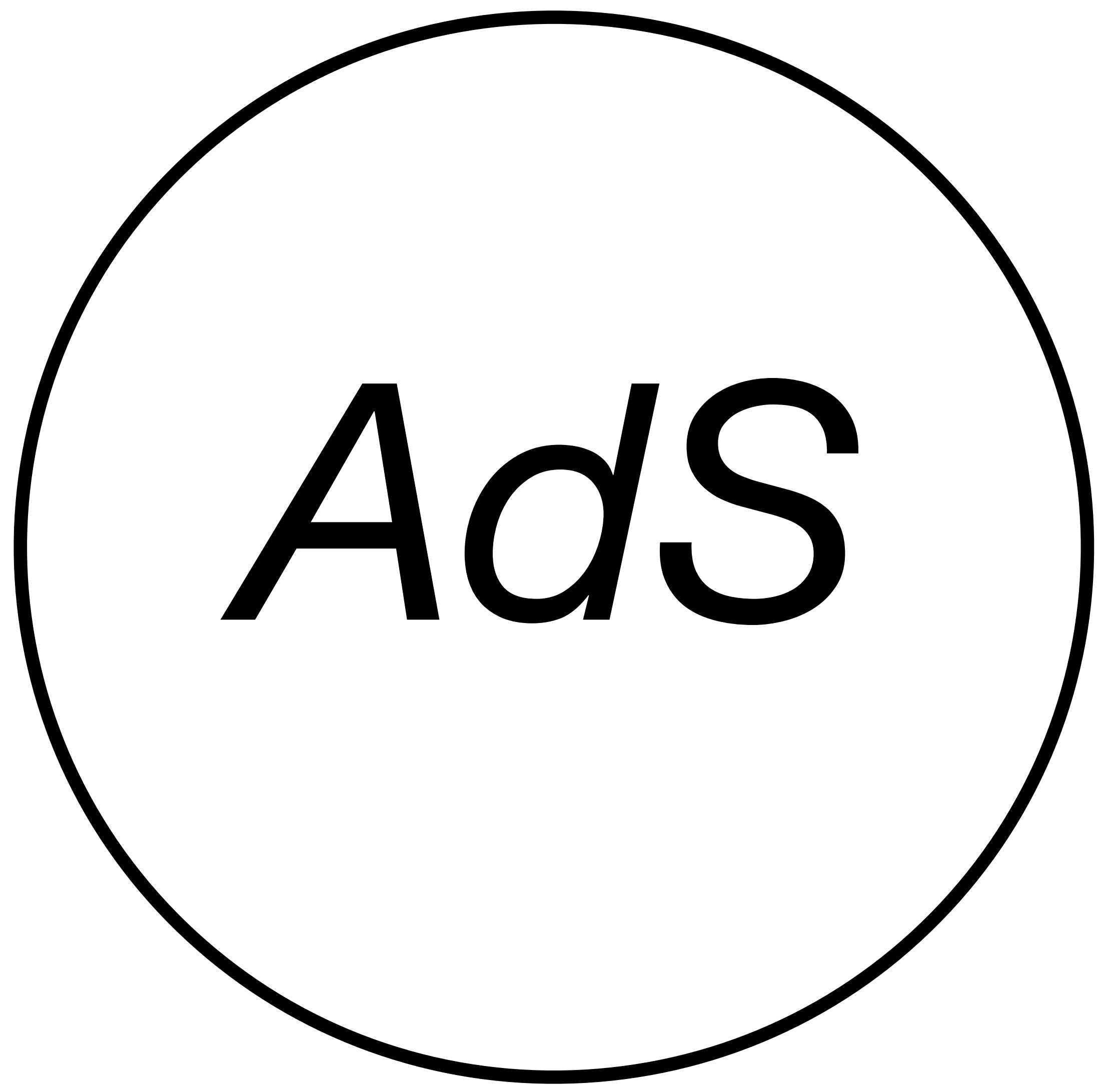
AdS
CFT


AdS

STEP 1

Localization in Supergravity

Localization & Black hole entropy

Recently we have assisted to spectacular progress in the computation of ***exact Black Hole entropy using Localisation techniques in Supergravity!***

$$d(q) = Z_{AdS_2}$$ [Sen]

Path integral in Supergravity ***localizes onto finite dimensional integral!*** In the case of ***N=8 SUGRA*** we can actually recover ***integers!!*** *(Cf Dabholkar's talk)*

$$Z_{AdS_2} = \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{\pi^2 q^2}{4t}} + \ldots \in \mathbb{N}$$

[JG, Dabholkar, Murthy]^3

[Murthy, Gupta; Murthy, Reys]

Localization gives exact computation

Consider supersymmetric path integral and deform it by a Q-exact term

$$\int e^{-S} \to Z(t) = \int e^{-S-tQV}$$

[Duistermaat, Heckman, Bott, Witten, Pestun…]

and chose supersymmetric deformation **Q^2V=0. Then it is easy to show that the deformation is exact**

$$\frac{\partial Z(t)}{\partial t} = 0 \Rightarrow Z(0) = Z(\infty)$$

In this limit path integral becomes ***1-loop exact!!***

$$Z(\infty) = \lim_{t\to\infty} \int e^{-S-tQV} = \sum_{\sigma\in\{QV=0\}} e^{-S(\sigma)} \times sdet(QV'')$$

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$$\sigma \neq EOM$$

M-theory truncation on $S^7/\mathbb{Z}_k$

Use a 4-dim consistent truncation down to AdS4 and apply localization at the level of the ***Off-shell theory.***

N=2 truncation consists of two vector multiplets coupled to a tensor multiplet. ***Prepotential is cubic*** *(CP^3 cohomology)*.

Dualise the theory to obtain ***square root prepotential plus a hypermultiplet charged under the gauge group.***

Same truncation for Sasaki-Einstein $SE_7 \simeq KE_6 \times S^1$, relevant for less supersymmetric quivers

[Gauntlett, Kim, Varela, Waldram]

4D superconformal gravity

Embed truncation in the **4d N=2 superconformal formalism of off-shell supergravity with electric gaugings**

[de Wit, Lauers, Proeyen,…]

Relevant Multiplets

- **Weyl** mult.: $(e_\mu^a, \psi_\mu^i, T_{ab}^-, \ldots)$
- **Vector** mult.: $(A_\mu, \Omega^i, X, Y_{ij})$
- **Hyper** mult.: $(A_i^\alpha, \xi^\alpha, F_i^\alpha)$

$i = 1, 2 \in SU(2)$ R-symmetry

$\alpha = 1, 2 \in Sp(2)$

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Auxiliary fields play very important role in Localization!

$i = 1, 2 \in SU(2)$ R-symmetry

$a = 1, 2 \in Sp(2)$

2-derivative Lagrangian

2-derivative bosonic Lagrangian with square root prepotential

$$\mathcal{N} = 2 \text{ prepotential} \qquad F(X^0, X^1) = \sqrt{X^0 (X^1)^3}$$

$$S = \int d^4x \sqrt{g} \Big[N_{IJ} \bar{X}^I X^J \left(\frac{R}{6} + D \right) + N_{IJ} \partial \bar{X}^I \partial X^J - \frac{1}{8} N_{IJ} Y^{ijI} Y^J_{ij} + N_{IJ} F^{I\mu\nu} F^J_{\mu\nu} + \ldots$$
$$\left(-\nabla A^i{}_\beta \nabla A_i{}^\alpha - \left(\frac{R}{6} - \frac{D}{2} \right) A^i{}_\beta A_i{}^\alpha + F^i{}_\beta F_i{}^\alpha + 4g^2 A^i{}_\beta \bar{X}^\alpha{}_\gamma X^\gamma{}_\delta A_i{}^\delta + g A^i{}_\beta Y^{jk\alpha}{}_\gamma A_k{}^\gamma \varepsilon_{ij} \right) d_\alpha{}^\beta \Big]$$

$$N_{IJ} = \frac{1}{2i} (F_{IJ} - \bar{F}_{IJ})$$

Couplings are derivatives of prepotential which give rise to intricate couplings.

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$$\left(-\nabla A^i{}_\beta\nabla A_i{}^\alpha-\left(\frac{R}{6}-\frac{D}{2}\right)A^i{}_\beta A_i{}^\alpha+F^i{}_\beta F_i{}^\alpha+4g^2A^i{}_\beta\bar{X}^\alpha_\gamma X^\gamma{}_\delta A_i{}^\delta+gA^i{}_\beta Y^{jk\alpha}_\gamma A_k{}^\gamma\varepsilon_{ij}\right)d_\alpha{}^\beta\Big]$$

Charged hypers lead to negative cosmological constant:

$$t_I A^\alpha_i=P_I(i\sigma^3)^\alpha{}_\beta A^\beta_i,\ I=0,1$$

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On-shell solution

AdS4 Metric: $$ds^2 = L^2\left(\frac{dr^2}{r^2-1} + (r^2-1)d\Omega_3^2\right)$$

$$\int_{S^7/\mathbb{Z}_k} \star F_4 = (2\pi)^6 N \Rightarrow N = \frac{6L^6 \mathrm{vol}(\hat{S}^7)}{k(2\pi)^6}$$

For symmetry reasons: $$F_{\mu\nu} = T_{\mu\nu} = 0$$

Attractor solution: all scalars are functions of N,k

Vec: $$X^I = iJ^I,\ J = \mathrm{const}$$ $$Y^I = -2i\frac{J^I}{L},$$

Hyper: $$A_i^a \propto \delta_i^a$$ $$F_i^a = 0$$

Gauging leads to hyperbolic space: $$2gJ^I P_I = -\frac{1}{L}$$

Renormalized On-shell action

Let's compute on-shell action $\int R - 2\Lambda|_{on-shell}$

$$S = \int d^4x\sqrt{g}\mathcal{L} = \int d\Omega_3 \int_1^{\infty} dr(r^2 - 1)\mathcal{L} = \rightarrow \infty$$

Renormalized On-shell action

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$$S = \int d\Omega_3 \int_1^{r_0} dr(r^2 - 1)\mathcal{L} = \left(\frac{r_0^3}{3} - r_0 + \frac{2}{3}\right) C$$

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$$S \to S + \int_{\partial AdS_4} \mathcal{L}_b$$

Add boundary couterterms to remove cutoff dependent terms

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$$\mathrm{Ren(S)} = \frac{2}{3} C$$

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Add boundary couterterms to remove cutoff dependent terms

$$\mathrm{Ren}(S) = -\frac{\pi\sqrt{2}}{3} k^{1/2} N^{3/2}$$

Exact matching with ABJM!

[Marino, Drukker, Putrov]

STEP 2

Off-shell instanton solutions

Localization equations & Solutions

Deform action by introducing Q-exact term

$$S \to S - t\sum Q\left[(Q\Psi)^\dagger \Psi\right] \quad Q^2 = \partial_{Hopf}$$

Bosonic part of deformation is definite positive!

$$QV|_{bosonic} = (Q\Psi)^\dagger Q\Psi \geq 0$$

Theory localizes very off-shell

$$\lim_{t\to\infty} e^{-S-tQV} \overset{\text{Localizes}}{\Longrightarrow} \sum_{\sigma\in\{Q\Psi=0\}} e^{-S(\sigma)}$$

Localization equations & Solutions

Assume there's no background fluctuations!
Solve off-shell BPS equations for vectors & hypers on AdS4

$$Q\Psi_{vec} = 0$$

$$X^I = \frac{C^I}{r} + iJ^I, \quad Y^I = -2i\frac{J^I}{L} + 2\frac{C^I}{r^2}$$

C^I : parametrize 2-dim space of off-shell solutions

All the other fields remain in their attractor background!

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Similar solutions found on AdS2xS^2 in the context of black hole entropy

[JG, Dabholkar, Murthy]

Cf Dabholkar's talk

Localization equations & Solutions

Assume there's no background fluctuations!
Solve off-shell BPS equations for vectors & hypers on AdS4

$$Q\Psi_{hyper} = 0$$

$$A_i^a \propto \delta_i^a, \quad F_i^a = 2g\frac{C^I P_I}{r}(\sigma^3)_i^a$$

Remark: *the hyper solution still solves all BPS equations!*
On the contrary, for vectors, only half of them are satisfied!

STEP 3

The Airy function

Off-shell renormalised action

Localization leads to huge simplification! It reduces a very complicated path integral to a finite dimensional integral!!

$$Z_{AdS4} = \int e^{-S_{SUGRA}} \rightarrow \sum_{\sigma \in Q\Psi = 0} e^{-S_{SUGRA}(\sigma)} \times Z_{det}$$

$$\sigma \sim C^{I}$$

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Here we focus only on the "classical" part!

Need to compute complicated integral on AdS4 and renormalise!

Off-shell renormalised action

$$Z_{AdS_4} = \int dC^0 dC^1 \exp[-\text{Ren}\, S(C^0, C^1)]$$

$$\text{Ren}(S) = S\left[X(r), Y(r), F_i^a(r)\right] + \text{bnd ct}$$

Given the square root prepotential, the action on the localisation locus is a non-trivial problem! But ***action happens to be integrable!!***

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$$Z_{AdS_4} = \int d\phi^0 d\phi^1 \exp\left[\frac{2\sqrt{2}}{\pi\sqrt{3}}\sqrt{\phi^0(\phi^1)^3} - N\phi^1 - k\phi^0\right]$$

[JG, Dabholkar, Drukker]

$$\phi \propto (J + iC) \sim X(r = 1)$$

Airy function

Massaging a bit the renormalised action…

$$Z_{AdS_4} = \int d\phi^0 d\phi^1 \exp\left[-k\left(\sqrt{\phi^0} - \frac{\sqrt{2}}{\pi k\sqrt{3}}(\phi^1)^{3/2}\right)^2 + \frac{2}{3\pi^2 k}(\phi^1)^3 - N\phi^1\right]$$

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$$\boxed{\propto Ai\left(\left(\frac{\pi^2}{2}k\right)^{1/3} N\right)}$$

[JG, Dabholkar, Drukker]

Comparison with exact answer

We obtain

$$\propto Ai\left((\frac{\pi^2}{2}k)^{1/3} N \right)$$

but exact answer contains **renormalizations of N**

$$\propto Ai\left((\frac{\pi^2}{2}k)^{1/3} \left(N - \frac{k}{24} - \frac{1}{3k} \right) \right)$$

[Bergman, Hirano]

These are easy to implement once we know the **renormalized value of N** already at the **"classical level"**

$$Z_{AdS_4} = \int \exp\left[\frac{2\sqrt{2}}{\pi\sqrt{3}} \sqrt{\phi^0 (\phi^1)^3} - (N + \ldots)\phi^1 - k\phi^0 \right]$$

Conclusions

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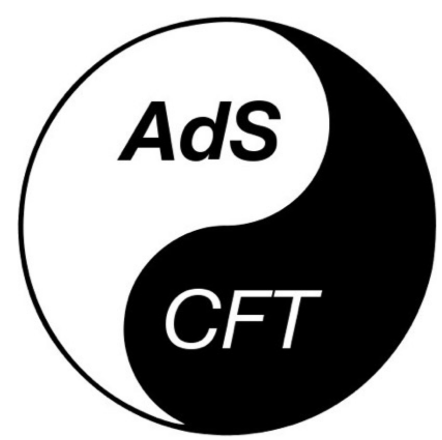
ABJM partition function on S^3 via Localization



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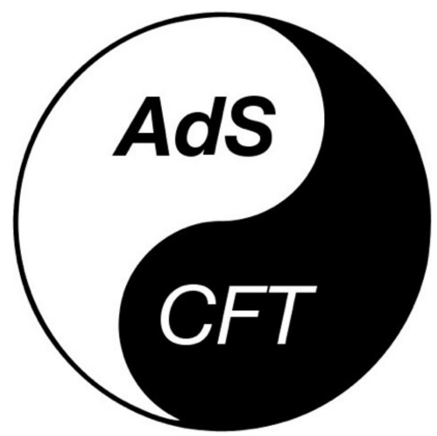

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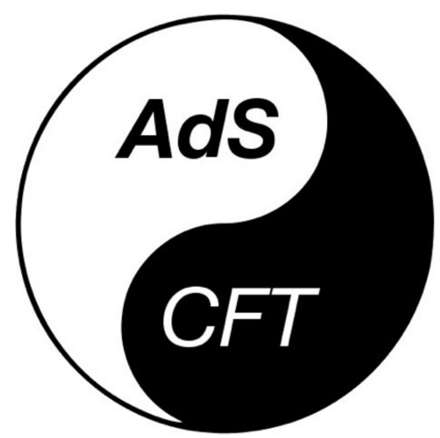

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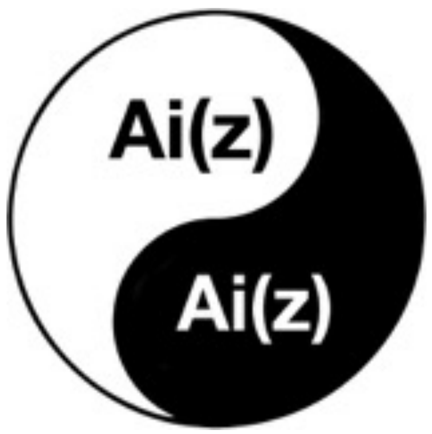

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Comments & Remarks

- More ***general quiver diagrams*** which preserve at least N=3 SUSY (12 Q's) also show ***Airy function behaviour***. This is consistent with our computation which only relies on N=2 SUGRA (8 Q's)!

- ***ABJ matrix model (U(N)xU(N+M))*** can also be solved exactly; the perturbative partition function is given by same ***Airy function*** but with ***additional renormalizations to N***.

- On the M-theory side we can turn on ***torsion*** on $S^7/\mathbb{Z}_k$. This corresponds to ***fractional holonomy*** $\int C_3 \propto \frac{M}{k}$; The same computation will follow and ***Airy function*** is found.

- The bulk computation was done in a frame where all ***gaugings are electric***. However, the truncation corresponds to mixed ***magnetic-electric gaugings***. This might explain measure (**work in progress…**).

- It should be possible to do the computation from the world-sheet and argue using localisation that the partition function is a topological string partition function (**work in progress…**).