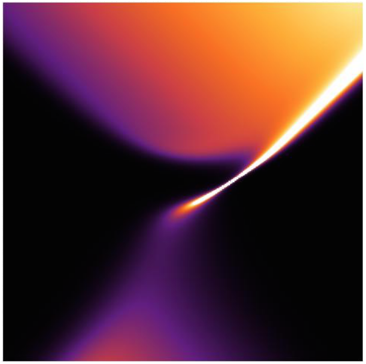
Holographic Fermi surfaces from top-down constructions

Steve Gubser



Based on 1112.3036, 1207.3352, 1312.7347 with O. DeWolfe and C. Rosen
and 1411.5384 with C. Cosnier-Horeau

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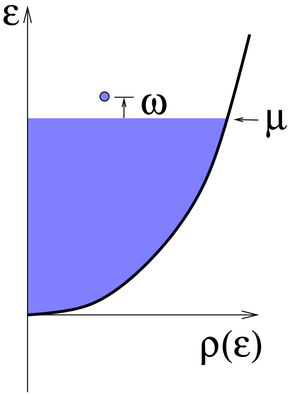
1. Introduction

Fermi surfaces at finite chemical potential μ are straightforward at weak coupling:

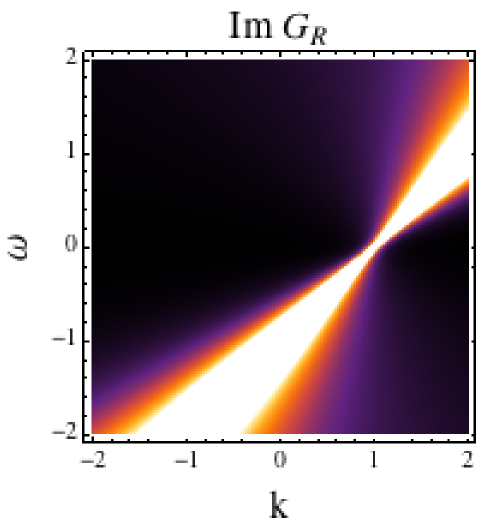


- System's response to adding a fermion is measured by Green's function:

$$G_R(t,x) = i\theta(t)\langle\{\psi^\dagger(t,x),\psi(0,0)\}\rangle_{\text{Fermi sea}}$$

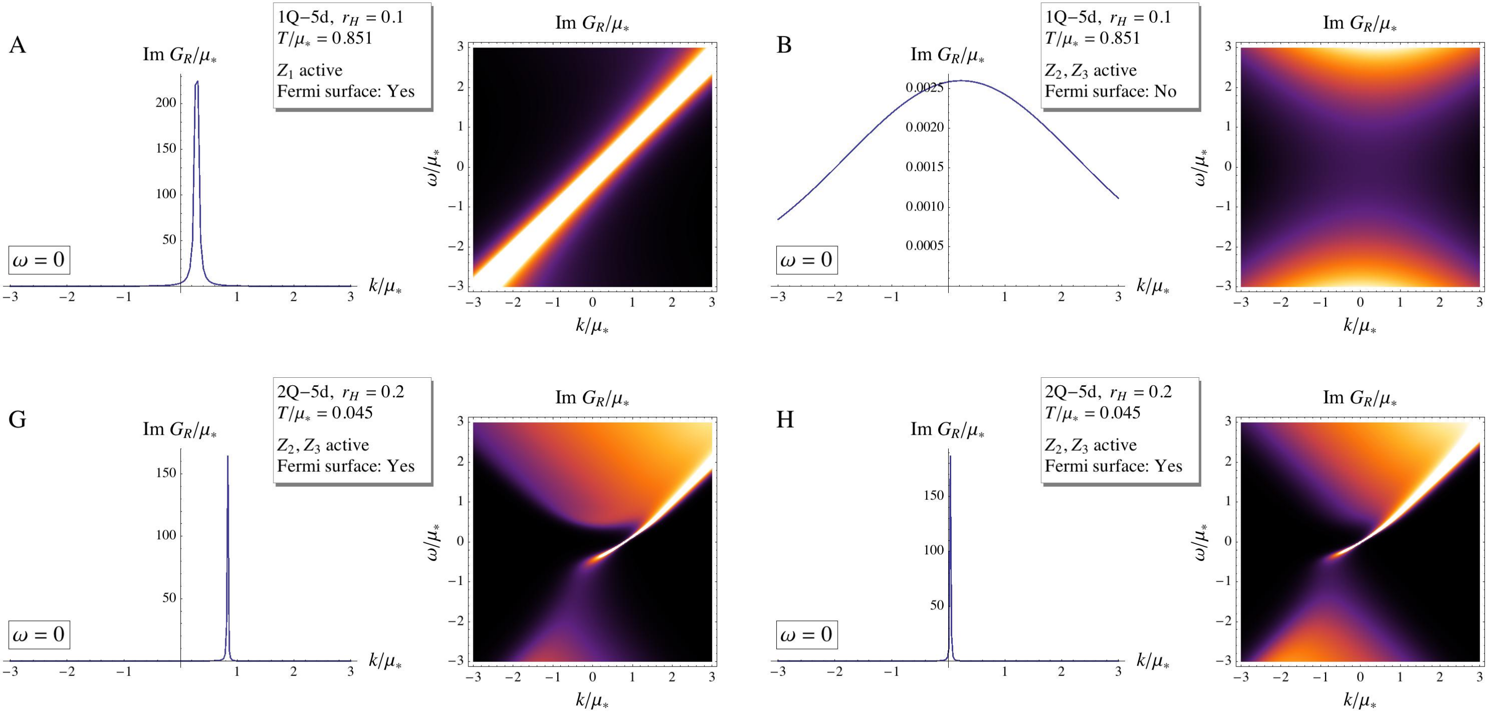
- Find a pole in Fourier space:

$$G_R(\omega,k) = \frac{Z}{\epsilon(k)-\mu-\omega-i\Gamma} + \ldots$$
$$\approx \frac{Z}{v_F(k-k_F)-\omega-i\Gamma}$$



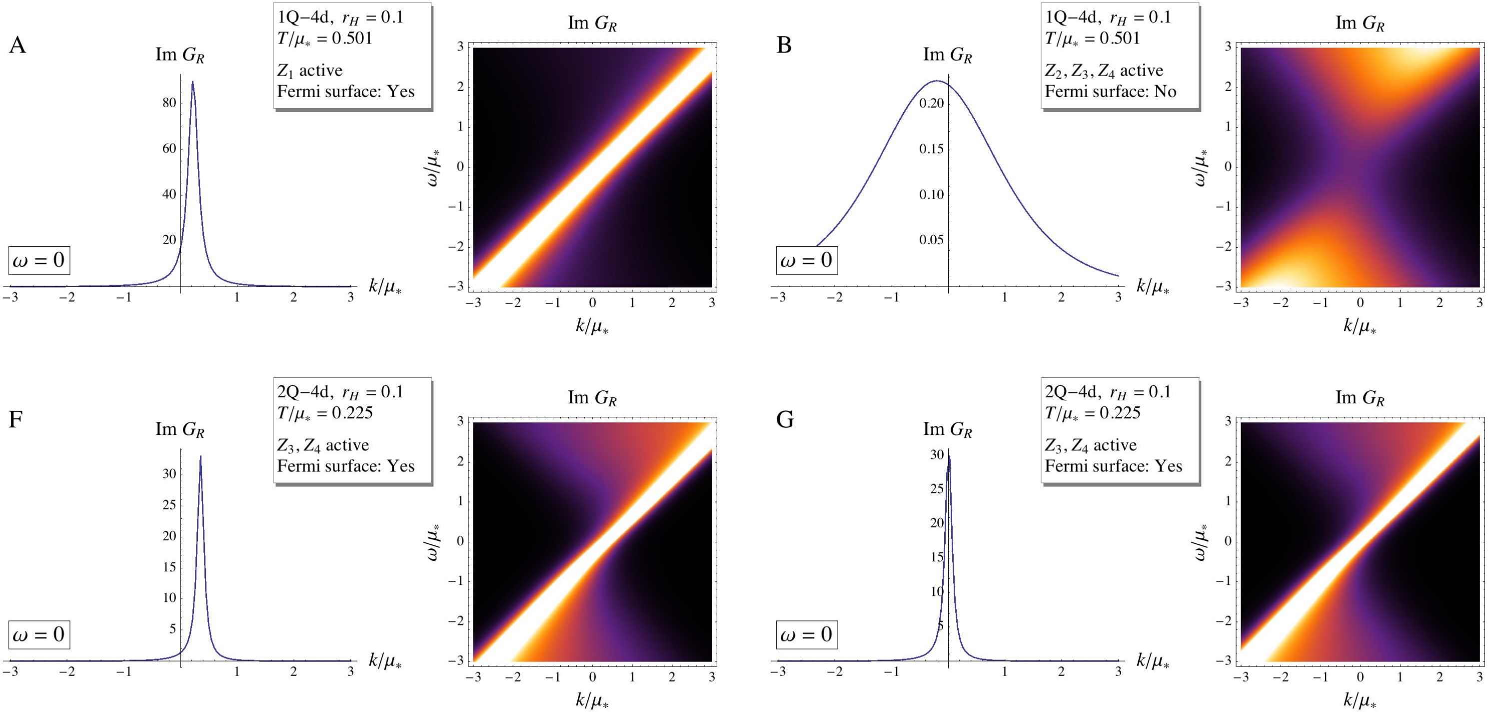
- $\Gamma \sim \omega^2 +$ (finite temperature) in Landau theory: long-lived quasi-particles.
- I plotted $\mathrm{Im}\,\frac{1}{(k-1)-\omega-\frac{i}{10}(\omega^2+0.02)}$ to convey the idea.

Fermi surfaces arise in $\mathcal{N} = 4$ super-Yang-Mills and ABJM theory, exhibiting several different behaviors. Here are typical examples from $AdS_5 \times S^5$:



A significant caveat: states showing Fermi surfaces often have bosonic instabilities.

Here are typical examples from $AdS_4 \times S^7$.



The key question: Knowing the dual operator in $\mathcal{N} = 4$ SYM or ABJM theory, how can we *anticipate* whether there will be a Fermi surface, and/or estimate k_F?

1.1. An overview of holographic fermions

- Study finite density theories in $\mathbf{R}^{2,1}$ at finite density by analyzing a dual black hole in AdS_4 with electric charge behind the horizon.

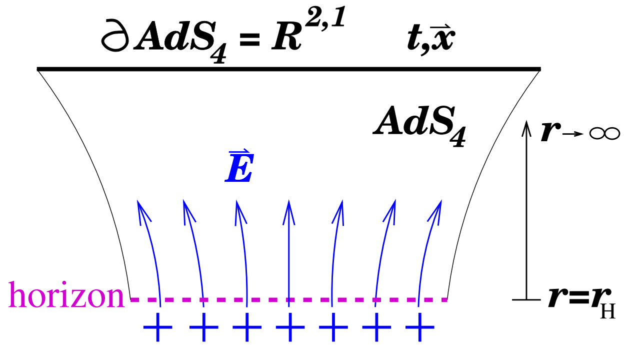
$$S = \int d^4x \left[R - \frac{1}{4} F_{\mu\nu}^2 + \frac{6}{L^2} \right]$$

  A_μ gauges in the bulk a $U(1)$ symmetry of the boundary theory.

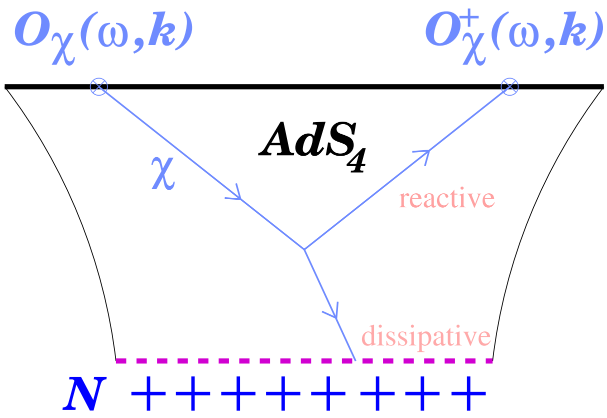
- Dual field theory (from coincident M2-branes) has fermionic operators $\mathcal{O}_\chi \sim \mathrm{tr}\, \lambda X$ where λ is a fermion and X a boson. Trace is over color.

- Use AdS/CFT to compute $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$.

Electric field comes from charge behind horizon.

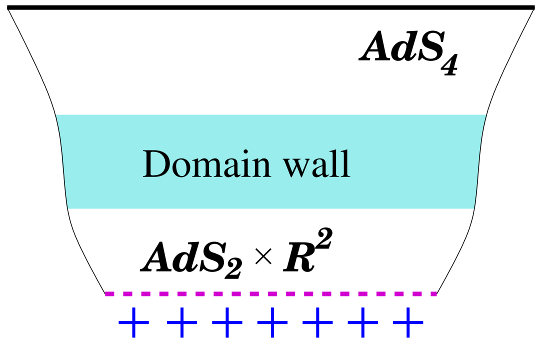


AdS/CFT prescription is to insert a supergravity fermion from the boundary with $\mathcal{O}_\chi$ and see how much of it falls into the black hole.

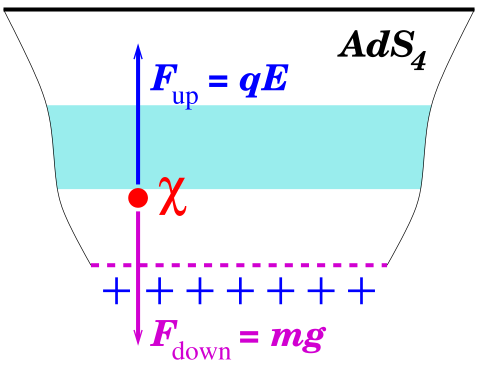


- Boundary theory is a strongly interacting CFT involving both fermions and bosons, so it's not *a priori* clear there should be fermi surfaces at all.
- But supergravity picture makes it fairly clear that there should be *if fermion charge is large enough*.

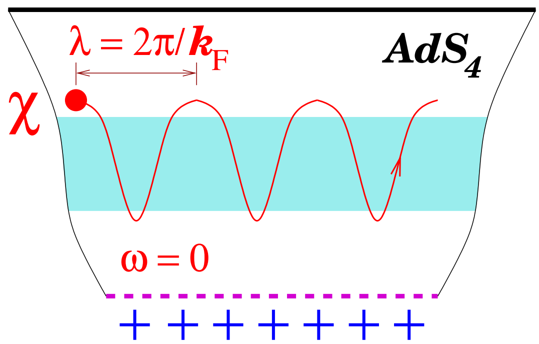
Back-reacted geometry is a domain wall from AdS_4 to $AdS_2 \times \mathbf{R}^2$.



Gravitational attraction and electrostatic repulsion compete to determine behavior of test particles.



Normalizable fermions at $\omega = 0$ and $k = k_F \neq 0$ stay above the horizon and below the boundary.



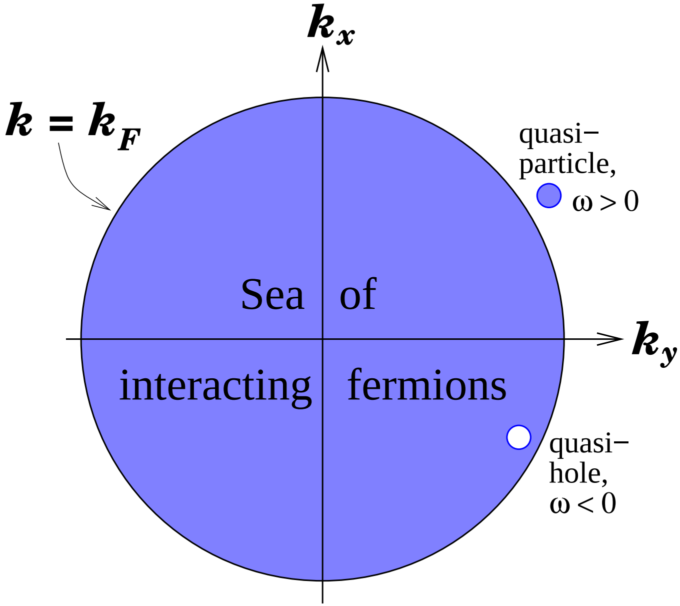
1.2. A generic answer from AdS/CFT for the fermionic Green's function

[Faulkner-Liu-McGreevy-Vegh 0907.2694], cf [S.S. Lee 0809.3402, Cubrovic-Zaanen-Schalm 0904.1993]

$$G(\omega,k) = \left\langle \mathcal{O}_\chi(\omega,\vec{k})\mathcal{O}_\chi^\dagger(-\omega,-\vec{k})\right\rangle \approx \frac{h_1}{(k-k_F) - \frac{1}{v_F}\omega - h_2 e^{i\gamma}\omega^{2\nu_F}}$$

when $k \approx k_F$ and $\omega \approx 0$.

- A singularity in $G(\omega,k)$ at $\omega=0$ and finite $k=k_F$ *defines* the presence of a Fermi surface.
- v_F is Fermi velocity.
- Assuming $\nu_F > 1/2$, low-energy dispersion relation is $\omega \approx v_F(k-k_F)$.
- If $\nu_F > 1/2$ or if $e^{i\gamma}$ is nearly real, quasi-particles' width is much smaller than their energy.
- Can easily obtain $\nu_F \leq 1/2$, i.e. far from perturbative Landau regime.



1.3. The simplest supergravity calculation

[Faulkner et al '09; Hartman-Hartnoll 1003.1918; DeWolfe-Gubser-Rosen 1112.3036]

- Simplest charged black hole background is extremal RNAdS_4:

$$ds^2 = \frac{r^2}{L^2}(f dt^2 - d\vec{x}^2) - \frac{L^2}{r^2}\frac{dr^2}{f} \qquad A_\mu dx^\mu = \mu\left(1 - \frac{r_H}{r}\right)$$
$$f = 1 - 4\left(\frac{r_H}{r}\right)^3 + 3\left(\frac{r_H}{r}\right)^4 \tag{1}$$

- Simplest fermion to consider obeys massless charged Dirac equation:

$$\gamma^\mu(\nabla_\mu - iqA_\mu)\chi = 0\,. \tag{2}$$

Fermi surfaces in boundary theory correspond to fermion normal modes in the bulk.

- Supergravity gives relations $q = \frac{1}{\sqrt{2}L}$ and $\mu = \frac{\sqrt{6}r_H}{L}$. Generally we'll choose $L = 1$. If also $r_H = 1$, then one finds a normal mode at

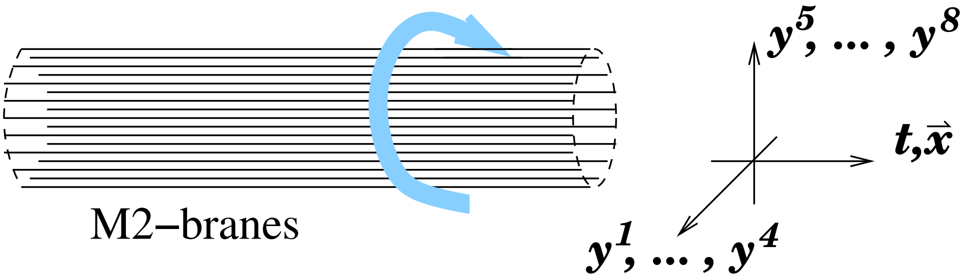
$$\omega = 0 \qquad k = k_F \equiv 0.9185\,. \tag{3}$$

1.4. Problems with holographic fermions and their solutions

- Previous calculations mainly focus on ad hoc lagrangians, e.g. [Liu-McGreevy-Vegh '09, Cubrovic-Zaanen-Schalm '09].
  - Instead, let's work with fermions of maximal gauged supergravity in $D = 4$ and $D = 5$: reductions / truncations of M-theory on S^7 and type IIB on S^5.
- AdS-Reissner-Nordstrom black holes have non-zero entropy at $T = 0$, which is hard to understand in field theory.
  - Work with classical variants of RNAdS_4 which can be embedded in M-theory or type IIB and have no zero-point entropy.
- Field theory interpretation, e.g. in ABJM theory or $\mathcal{N} = 4$ super-Yang-Mills theory, has been obscure.
  - Formulate "boson rule" and "fermion rule" which capture results of many supergravity calculations in terms of field theory quantities.
- Supergravity calculations are hard work!
  - Find some strong collaborators.

2. Supergravity backgrounds and spinning branes

Charged black holes in AdS_5 come from spinning D3-branes.
Charged black holes in AdS_4 come from spinning M2-branes.



$D = 4$, $\mathcal{N} = 8$ supergravity [de Wit and Nicolai, 1982] has $SO(8)$ gauge symmetry associated with the S^7 directions coming from $y^1 \ldots y^8$.

- For a semi-pedagogical refresher, see [de Wit, hep-th/0212245].
- Field content is: graviton $g_{\mu\nu}$, 8 gravitini ψ^i_μ, 28 gauge fields A^{ij}_μ, 56 Majorana spinors χ^{ijk}, and 70 real scalars ϕ^{ijkl}.
- Eight-valued indices $i, j, \ldots$ characterize either the internal symmetry group $SU(8)$ or the gauge group $SO(8)$ (in a spinorial rep wrt S^7).

It's useful to pass to an $SO(8)$ triality frame more simply related to S^7:

$$\begin{pmatrix} A_\mu^{12} \\ A_\mu^{34} \\ A_\mu^{56} \\ A_\mu^{78} \end{pmatrix} = \frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} A_\mu^a \\ A_\mu^b \\ A_\mu^c \\ A_\mu^d \end{pmatrix} . \tag{4}$$

Now A_μ^a encodes spin in the y^1-y^2 plane, A_μ^b encodes spin in the y^3-y^4 plane, etc.

With $A_\mu^a \neq A_\mu^b \neq A_\mu^c \neq A_\mu^d$, one must turn on three of the 70 scalars to find consistent solutions. Relevant part of $\mathcal{D} = 4$, $\mathcal{N} = 8$ action is

$$\mathcal{L} = R - \frac{1}{2}(\partial\vec{\phi})^2 + \frac{2}{L^2}(\cosh\phi_1 + \cosh\phi_2 + \cosh\phi_3) - \frac{1}{4}\sum_{i=a,b,c,d} e^{-\lambda_i}(F_{\mu\nu}^i)^2 \tag{5}$$

where

$$\begin{pmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{6}$$

These scalars parametrize oblateness / prolateness of the S^7.

The general charged 4-d black brane solution we want to consider is

$$ds_4^2 = e^{2A(r)}\left[-h(r)dt^2 + d\vec{x}^2\right] + \frac{e^{2B(r)}}{h(r)}dr^2 \qquad A^i = \Phi_i(r)dt \qquad \phi_A = \phi_A(r)$$

where

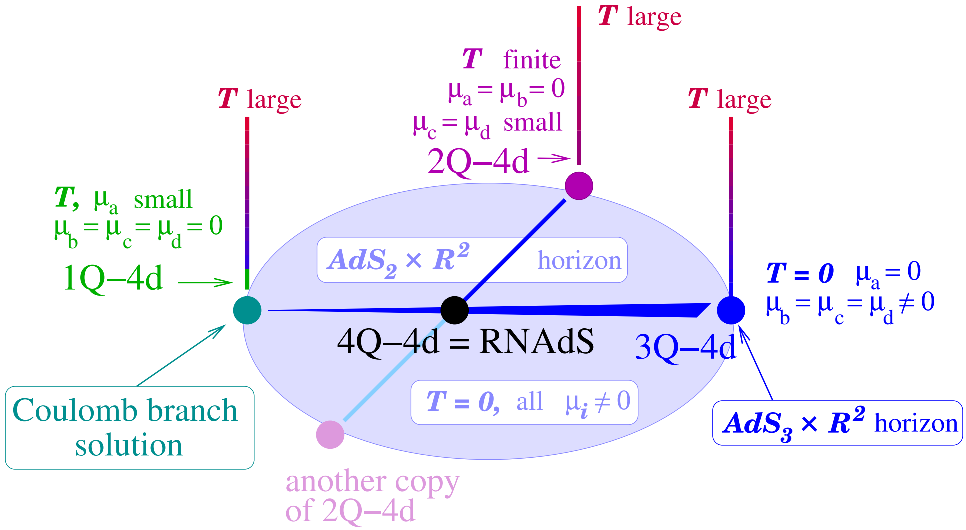
$$\begin{aligned}
A = -B &= \log\frac{r}{L} + \frac{1}{4}\sum_i \log H_i \\
h &= 1 - \frac{r}{r_H}\prod_i \frac{r_H + Q_a}{r + Q_a} \\
\lambda_i &= -2\log H_i + \frac{1}{2}\sum_j \log H_j \\
\Phi_i &= \frac{1}{L}\sqrt{\frac{Q_i}{r_H}}\frac{\sqrt{\prod_j(r_H + Q_j)}}{r_H + Q_i}\left(1 - \frac{r_H + Q_i}{r + Q_i}\right) ,
\end{aligned} \tag{7}$$

and one can show

$$s = \frac{1}{4GL^2}\sqrt{\prod_j (r_H + Q_j)} \tag{8}$$

with $s \to 0$ as $r_H \to 0$ provided at least one of the $Q_j = 0$.

There are several qualitatively different behaviors for these charged black branes, and we aim to explore all of them, *especially the cases with* $s \to 0$.



- 1Q-4d, 2Q-4d, 3Q-4d are the main cases we'll consider; 4Q-4d was the simplest RNAdS_4 case, already discussed.
- $r_H \to 0$ limit is singular for 1Q-4d, 2Q-4d, 3Q-4d.
- To make sure that supergravity is applicable, we'll turn on small non-zero r_H.
- Order of limits gets subtle: For example, 2Q-4d is a $r_H \to 0$ limit with $\mu_a = \mu_b = 0$, not the same as a $\mu_a = \mu_b \to 0$ limit with $T = 0$.

3. Fermion equations of motion

$D = 4, \mathcal{N} = 8$ supergravity lagrangian is schematically

$$\mathcal{L} = \mathcal{L}_b + \frac{1}{2}\bar{\chi} D_\chi \chi + \bar{\psi}_\mu O_{\text{mix}} \chi + \frac{1}{2}\bar{\psi}_\mu D_{\text{Rarita-Schwinger}} \psi_\mu + \mathcal{O}(\text{fermion}^4) \quad (9)$$

Our main task is to decouple the quadratic fermion action and solve resulting linear equations to get two-point functions $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$.

- Some of the **56** fermions χ_{ijk} can mix with the **8** gravitini ψ_μ^i, giving them a mass (super-Higgs). We don't want these.

- Because bosonic background has no charged fields under $U(1)^4$, we know that χ_{ijk} can't couple with ψ_μ^i if it has an $SO(8)$ weight not in the **8**. There are 32 such χ_{ijk}, and dual operators are schematically $\operatorname{tr} \lambda Z$.

- Of the 24 remaining χ_{ijk}, there are 16 which don't couple to the ψ_μ^i, and 8 that do, but we haven't worked out which are which. So ignore them all and focus on the special 32.

- Similar results are available from [Gubser-DeWolfe-Rosen '13] in the case of $D = 5$, $\mathcal{N} = 8$ supergravity; fields of interest are dual to operators $\operatorname{tr} \lambda Z$.

In 4-dim: The fermion equations of motion we want to study take the form

$$\left[i\gamma^\mu \nabla_\mu + \gamma^\mu A_\mu^j \mathbf{Q}_j + \sigma^{\mu\nu} F_{\mu\nu}^j \mathbf{P}_j + \mathbf{M}\right] \vec{\chi} = 0\,. \qquad (10)$$

$\vec{\chi}$ is a 32-component vector, and the matrices $\mathbf{Q}_j$, $\mathbf{P}_j$, and $\mathbf{M}$ all commute (!).

Simultaneous eigenvectors satisfy

$$\left[i\gamma^\mu \nabla_\mu + \frac{1}{4}\sum_j \left(q_j \gamma^\mu A_\mu^j + \frac{i}{2} p_j e^{-\lambda_j/2} \sigma^{\mu\nu} F_{\mu\nu}^j + m_j e^{\lambda_j/2}\right)\right] \chi = 0\,. \qquad (11)$$

∇_μ includes spin connection but not gauge connections

Gauge couplings and Pauli couplings.

Spatially variable mass term, $m \to 0$ at ∂AdS_4.

and we can tabulate the parameters (q_j, p_j, m_j).

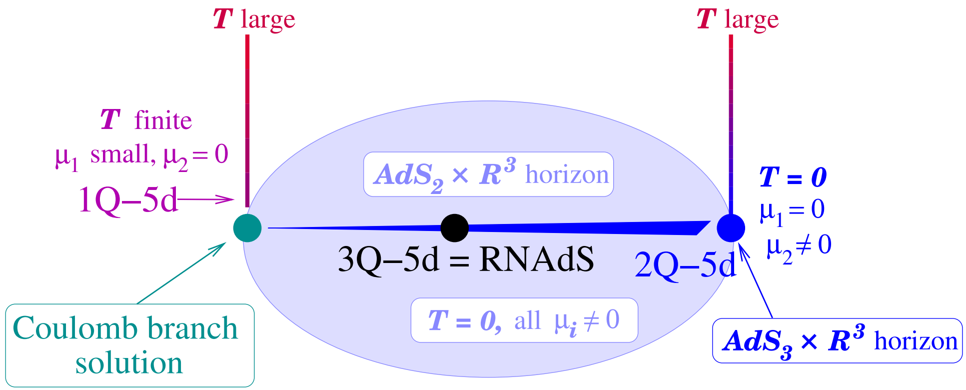
Dual operators follow from values of q_j: E.g. $q_j = (3, 1, 1, -1)$ corresponds to $\mathrm{tr}\, \lambda Z$ where

$$[\lambda]_{SO(8)} = (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}) \qquad [Z]_{SO(8)} = (1, 0, 0, 0) \quad \text{i.e.} \quad Z = X_1 + iX_2 \qquad (12)$$

We'll denote $Z_j = X_{2j-1} + iX_{2j}$ for $j = 1, 2, 3, 4$.

In 5-dim: Gauge group is $SO(6) \supset U(1)^3$, but we restricted to the case

$$A_\mu^a \equiv A_\mu^{12} = a_\mu \qquad A_\mu^b \equiv A_\mu^{34} = A_\mu^c \equiv A_\mu^{56} = A_\mu$$
$$\mu_a = \mu_1 \qquad \mu_b = \mu_c = \mu_2/\sqrt{2} \tag{13}$$



Only one scalar in supergravity is active, ϕ in the $\mathbf{20}'$ of $SO(6)$; it is dual to $\mathcal{O}_\phi = \mathrm{tr}(2|Z_1|^2 - |Z_2|^2 - |Z_3|^2)$, where $Z_j = X_{2j-1} + iX_{2j}$.

24 of the $\mathbf{48}$ fermions χ_{abc} are dual to $\mathrm{tr}\,\lambda Z$ and obey equations of the form

$$\Big[i\gamma^\mu \nabla_\mu + 2q_1\gamma^\mu a_\mu + 2q_2\gamma^\mu A_\mu + ip_1 e^{-2\phi/\sqrt{6}}\gamma^{\mu\nu} f_{\mu\nu} + ip_2 e^{\phi/\sqrt{6}}\gamma^{\mu\nu} F_{\mu\nu} \\ - 2(m_1 e^{-\phi/\sqrt{6}} + m_2 e^{2\phi/\sqrt{6}}) \Big] \chi = 0 \tag{14}$$

4. Green's functions and Fermi surfaces

Setting $\chi(t,\vec{x},r) = \frac{1}{\sqrt[4]{-\det g_{mn}}} e^{-i\omega t + ikx^1}\psi(r)$ where $m,n = t,1,2$, we find:

- Infalling solution at the horizon is $\psi \propto (r - r_H)^{-\frac{i\omega}{4\pi T}}$.
- Asymptotic forms at large r are related to retarded Green's function:

$$\psi_{\alpha+} = A_\alpha r^{m-\frac{d}{2}} + B_\alpha r^{-m-1-\frac{d}{2}} \qquad \psi_{\alpha-} = C_\alpha r^{m-1-\frac{d}{2}} + D_\alpha r^{-m-\frac{d}{2}}$$
$$G_R(\omega,\vec{k}) = -i\int \frac{d^3k}{(2\pi)^3} e^{i\omega t - i\vec{k}\cdot\vec{x}}\theta(t)\langle[\mathcal{O}_\chi(t,x),\mathcal{O}_\chi^\dagger(0,0)]\rangle = \frac{D_\alpha}{A_\alpha}\,. \tag{15}$$

- $\alpha = 1,2$ refers to spinor index of boundary operator. $G_R = G_{\alpha\beta}$ is *diagonal*, so we can consider one value of α at a time.
- $A_\alpha = 0$ makes fermion wave-function normalizable at boundary.
- Dissipationless modes are possible at $\omega = 0$: Fermion normal mode if also $A_\alpha = 0$. Thus a Fermi surface ($G_R = \infty$) corresponds to a normal mode.
- As far as we can tell, no analytic results are available; all results for G_R were obtained by numerically solving (a close equivalent of) the Dirac equation.

4.1. Examples

Thanks to a relation $G_{11}(\omega, k) = G_{22}(\omega, -k)$, we can get all information from G_{22}. Cases examined in 5-d were the following:

#	Dual operator	m_1	m_2	q_1	q_2	p_1	p_2	1Q-5d	2Q-5d
1	$\lambda_1 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	1	$-\frac{1}{4}$	$\frac{1}{2}$	Y[A]	N
2	$\lambda_2 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	-1	$-\frac{1}{4}$	$-\frac{1}{2}$	Y[A]	N
3	$\bar\lambda_3 Z_1$, $\bar\lambda_4 Z_1$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$	0	$-\frac{1}{4}$	0	Y[A]	N
4	$\lambda_1 Z_2$, $\lambda_1 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{1}{4}$	0	N[B]	Y[G]
5	$\bar\lambda_2 Z_2$, $\bar\lambda_2 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	2	$-\frac{1}{4}$	0	N	Y[G]
6	$\lambda_3 Z_2$, $\lambda_4 Z_3$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	1	$-\frac{1}{4}$	$-\frac{1}{2}$	N	Y[H]
7	$\bar\lambda_3 Z_3$, $\bar\lambda_4 Z_2$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{1}{2}$	N[B]	Y[H]

"Boson Rule:" You get a Fermi surface for $\mathrm{tr}\,\lambda Z$ iff Z has an expectation value.

- 1Q-5d has $\langle \mathrm{tr}(2|Z_1|^2 - |Z_2|^2 - |Z_3|^2)\rangle > 0$, so $\langle \mathrm{tr}\,|Z_1|^2\rangle > 0$.
- 2Q-5d has $\langle \mathrm{tr}(2|Z_1|^2 - |Z_2|^2 - |Z_3|^2)\rangle < 0$, so $\langle \mathrm{tr}\,|Z_2|^2\rangle > 0$, $\langle \mathrm{tr}\,|Z_3|^2\rangle > 0$.

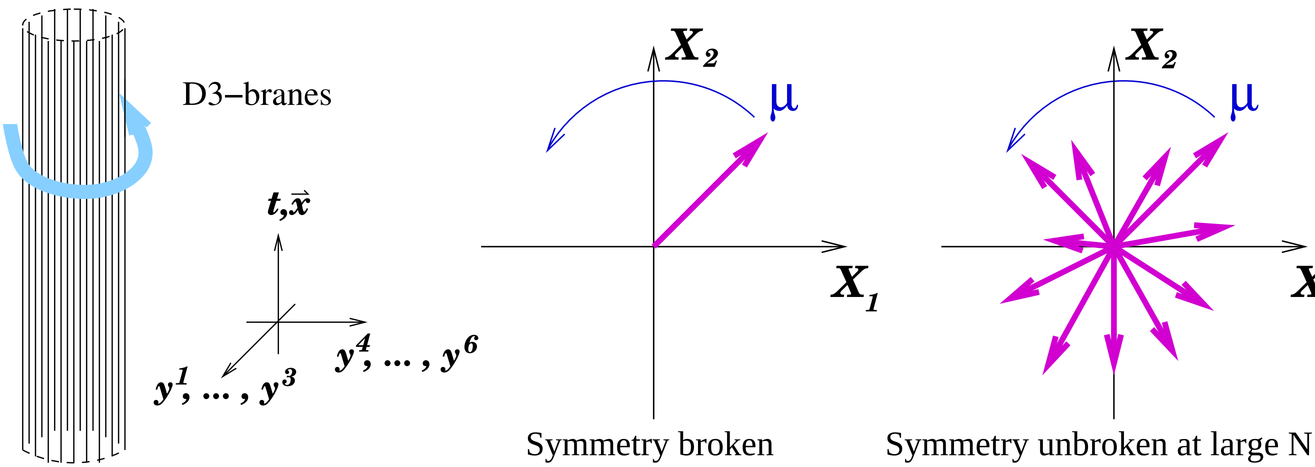
4-d cases are a bit more intricate:

#	Active boson	q_a	q_b	q_c	q_d	m_a	m_b	m_c	m_d	1Q-4d	2Q-4d	3Q-4d
1	Z_1	3	−1	1	1	−3	1	1	1	Y^{A}	N	N
2	Z_1	3	1	−1	1	−3	1	1	1	Y^{A}	N	N
3	Z_1	3	1	1	−1	−3	1	1	1	Y^{A}	N	N
4	Z_2	−1	3	1	1	1	−3	1	1	N^{B}	N	Y
5	Z_2	1	3	−1	1	1	−3	1	1	N	N	Y
6	Z_2	1	3	1	−1	1	−3	1	1	N	N	Y
7	Z_3	−1	1	3	1	1	1	−3	1	N^{B}	Y^{F}	Y
8	Z_3	1	−1	3	1	1	1	−3	1	N	Y^{F}	Y
9	Z_3	1	1	3	−1	1	1	−3	1	N	Y^{G}	Y
10	Z_4	−1	1	1	3	1	1	1	−3	N^{B}	Y^{F}	Y
11	Z_4	1	−1	1	3	1	1	1	−3	N	Y^{F}	Y
12	Z_4	1	1	−1	3	1	1	1	−3	N	Y^{G}	Y
13	Z_1	3	−1	−1	−1	−3	1	1	1	Y^{A}	N	N
14	Z_2	−1	3	−1	−1	1	−3	1	1	N^{B}	N	Y
15	Z_3	−1	−1	3	−1	1	1	−3	1	N^{B}	Y^{G}	Y
16	Z_4	−1	−1	−1	3	1	1	1	−3	N^{B}	Y^{G}	Y

But boson rule works in every case: non-zero bosons are Z_1 for 1Q-4d; Z_3 and Z_4 for 2Q-4d; and Z_2, Z_3, Z_4 for 3Q-4d.

Suggested interpretation: The singularity in $\langle \mathcal{O}_\chi \mathcal{O}_\chi^\dagger \rangle$ is due to a Fermi surface of a colored fermion, co-existing with a scalar condensate which (at large N) leaves the $U(1)$ symmetry unbroken.

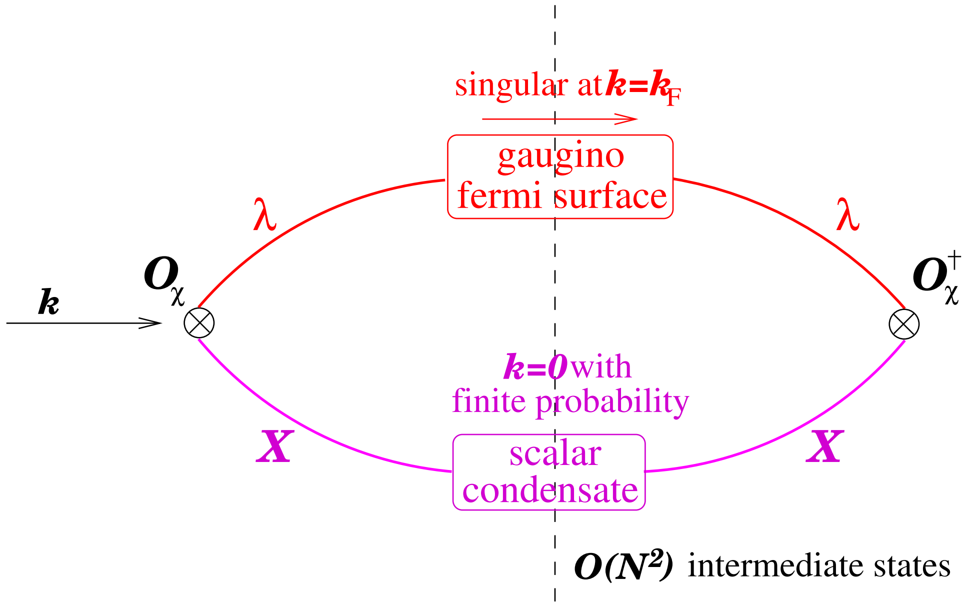
Easiest for me to think about the case of $\mathcal{N} = 4$ SYM in $d = 4$. Large N allows $U(1)$ to remain unbroken even with non-zero scalar condensate:



A common worry is that scalar condensate can run away along flat directions. But perhaps this is not relevant at large N. Here's why:

- Only a subleading fraction of directions satisfy $[X^I, X^J] = 0$.
- Cases considered are finitely far from SUSY limit, so it's probably more representative to think of non-commuting directions.
- In non-commuting directions, condensate is limited by $V \sim g^2 \operatorname{tr}[X^I, X^J]^2$.

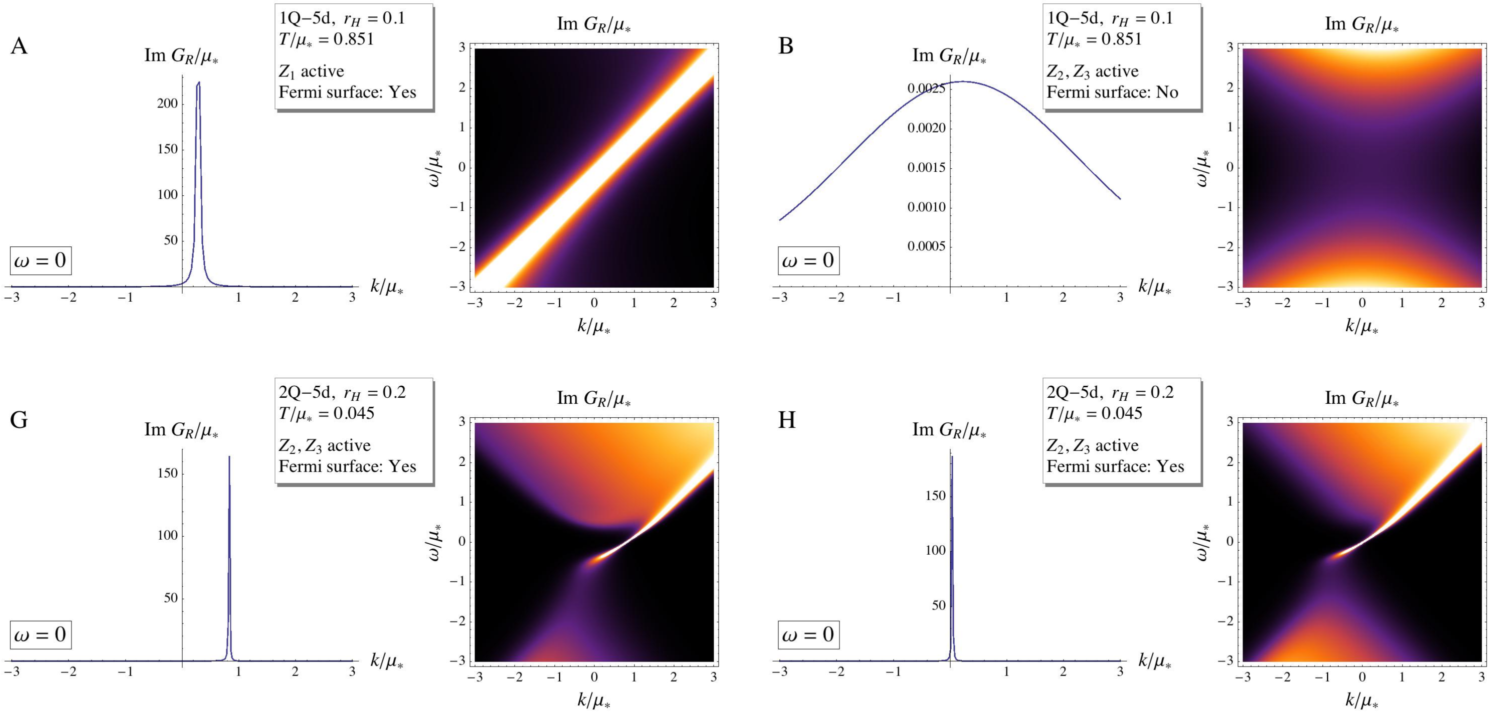
So—plausibly—the singularity at $k = k_F$, with residue $\sim N^2$ in AdS_5 calculations, owes to diagrams in $\mathcal{N} = 4$ SYM roughly like this:



This account contrasts strongly with the proposal the Fermi surfaces are best understood in terms of color singlet fermions *in the gauge theory* [Huijse-Sachdev '11], and if colored fermions have Fermi surfaces, they are hidden from supergravity calculations.

A closer look at examples shows that k_F is often significantly smaller than the natural scale

$$\mu_* = \sqrt{T^2 + \mu_1^2 + \mu_2^2} \quad \text{(5-d)} \qquad \mu_* = \sqrt{T^2 + \textstyle\sum_j \mu_j^2} \quad \text{(4-d)}\,. \tag{16}$$



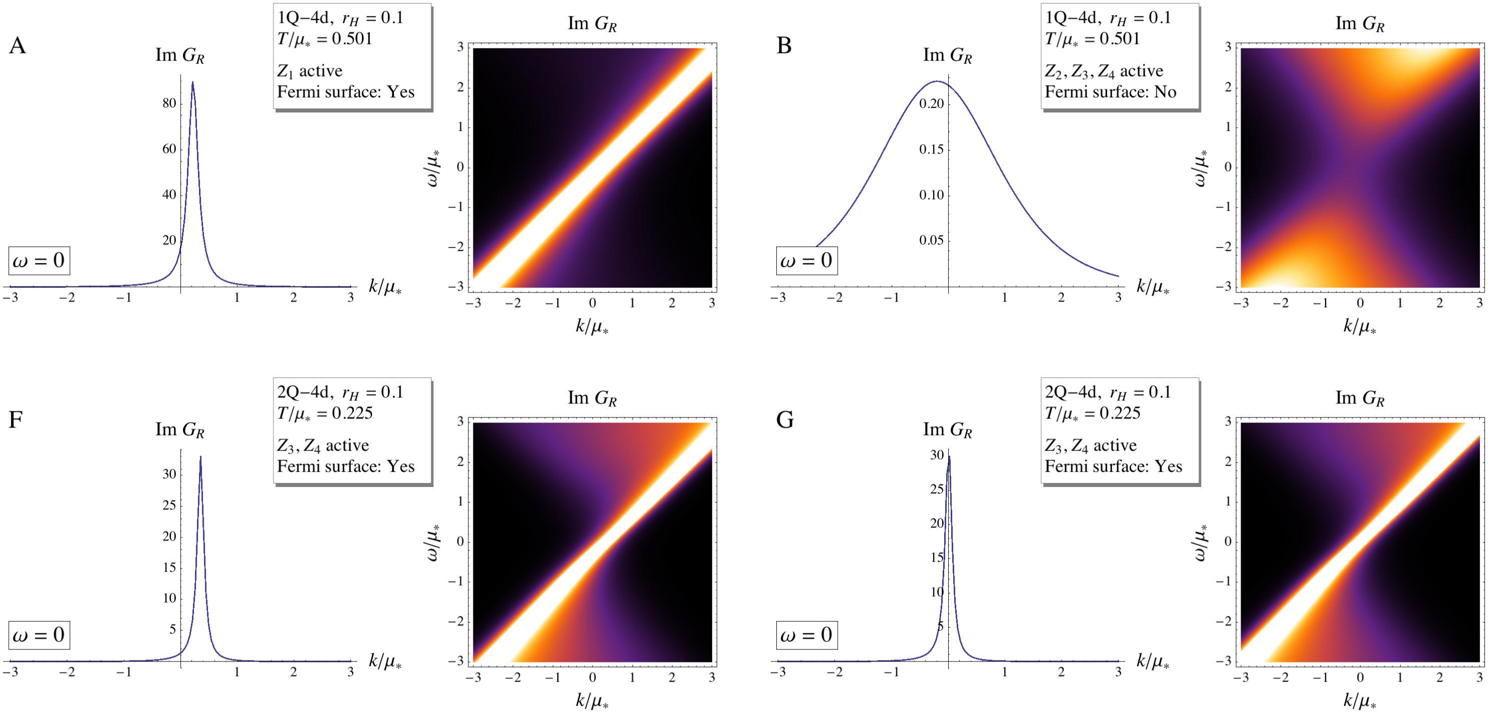
There are two unrelated reasons for this:

1. $\mu_1 \ll \mu_*$ for the 1Q-5d (Case A), so we naturally have small Fermi surfaces.
2. Case G involves the gaugino $\lambda_1^{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)}$, which carries charge under $U(1)$ of the 2Q-BH background, whereas Case H involves the gaugino $\lambda_3^{\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)}$, which is *neutral* under this $U(1)$.

Viewing #1 as trivial, we suggest the following

> "Fermion Rule:" The value of k_F is suppressed, though it may not vanish, when λ is neutral under the $U(1)$ charge of the black hole.

A detailed look at 4-d cases provide supports the boson rule and gives some additional evidence in favor of the fermion rule.



- Chemical potential μ_a is small for case A.
- k_F is rather larger for case F (charged λ) than for case G (neutral λ).

5. A tentative Luttinger count

Consider just the 2Q-5d case: An AdS_5 black hole with two equal charges, $A_\mu^{34} = A_\mu^{56}$ with $A_\mu^{12} = 0$.

- A lovely sharp Fermi surface arises for λ_1^{+++} and $\bar{\lambda}_2^{-++}$ (more precisely for $\mathrm{tr}\,\lambda Z$ operators involving these fermions): $k_F/\mu_* \approx 0.812$.

- A count of the total charge carried by the fermions is given, according to Luttinger, by

$$\rho_{\text{fermions}} = N_{\text{fermions}} q_{\text{fermions}} \int_{k<k_F} \frac{d^3k}{(2\pi)^3} . \tag{17}$$

  We know $q_{\text{fermions}} = 1$ (from $SO(6)$ group theory), and it's reasonable to suppose $N_{\text{fermions}} = N^2$.

- On the other hand, we know the *total* $\rho_{\text{total}} = \dfrac{Q_2}{\pi r_H} s$.

- After a bit of calculation, find $\dfrac{\rho_{\text{fermion}}}{\rho_{\text{total}}} \approx 0.972$.

  OK since it's not decisively greater than one—but a bit strange since I'd have thought more charge would be carried by bosons.

6. Summary

- Field theory understanding of holographic Fermi surfaces is probably easier without extremal entropy complicating the story.

- Holographic Fermi surfaces appear or don't appear in correlators of $\mathcal{O}_\chi = \mathrm{tr}\, \lambda Z$ precisely if Z has an expectation value.

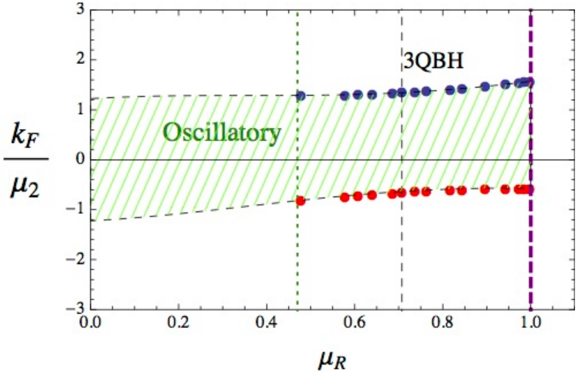
- Probably what's going on is that we're seeing a Fermi surface of the color-charged fermions λ, not some composite color-singlet created by $\mathcal{O}_\chi$.

- Neutral fermions (wrt black hole's chemical potential) have smaller Fermi surfaces, though their k_F may not be exactly 0.

- Better understanding of field theory is very desirable. Also, we need some examples without boson instabilities.

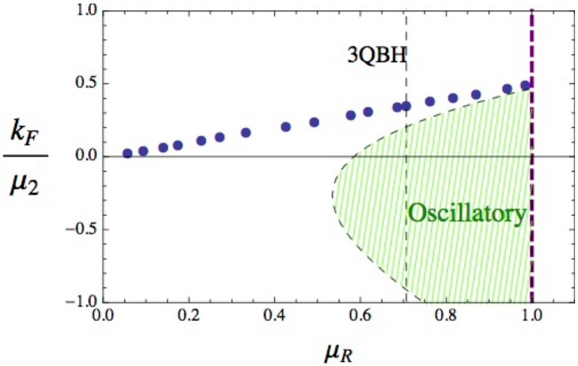
Revisiting cases with non-zero entropy

What about 5d cases with all three charges non-zero? Hard because s remains finite as $T \to 0$. A few examples are helpful:



Fermion is $\operatorname{tr} \lambda_1 Z_2 = \operatorname{tr} \lambda^{+++} Z^{010}$.
There "should" be a Fermi surface everywhere.

Oscillatory region is where BF bound is violated in AdS_2 region.

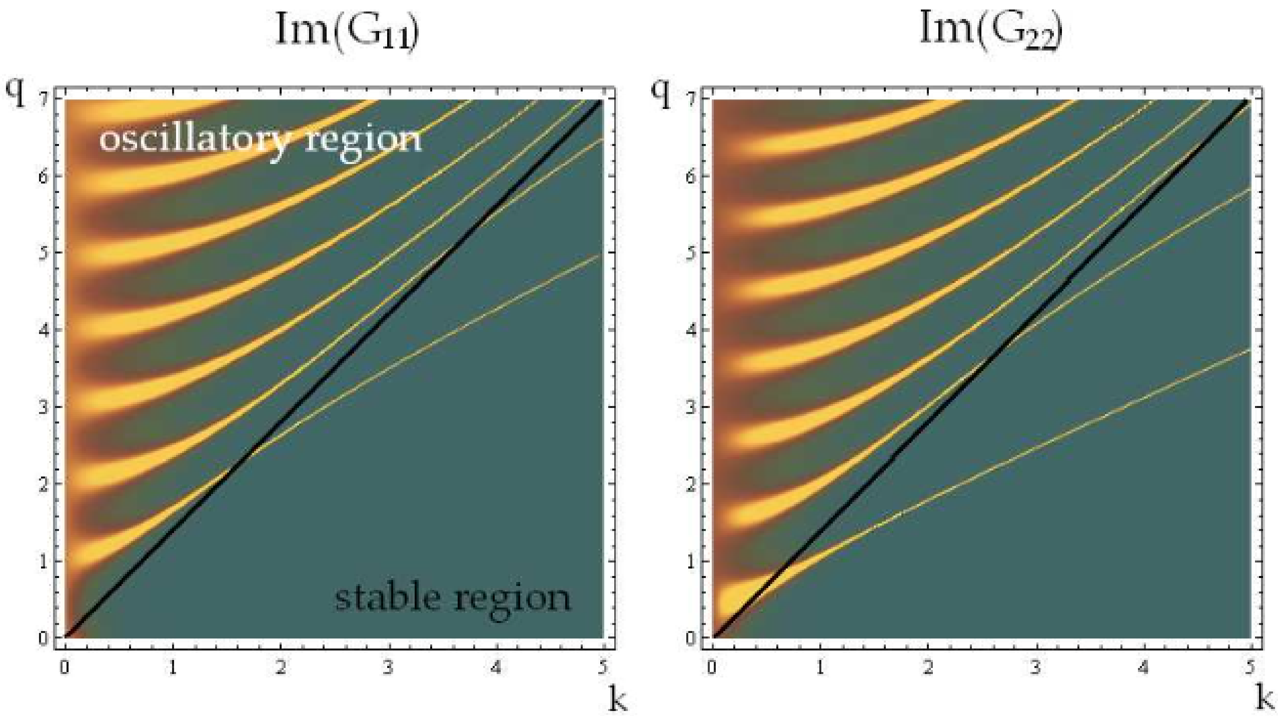


Fermion is $\operatorname{tr} \bar{\lambda}_3 Z_1 = \operatorname{tr} \lambda^{+-+} Z^{100}$.
There "should" be a Fermi surface everywhere.

$\mu_R = \frac{\mu_a}{\sqrt{2}\mu_b}$, with $\mu_b = \mu_c$.

$\mu_R = \frac{1}{\sqrt{2}}$ is the equal-charge black hole.

A Fermi surface vanishing into an oscillatory region probably indicates only finite but small width developing at zero temperature; c.f. [Liu-McGreevy-Vegh '09]



Contrast with fermions such as $\operatorname{tr}\lambda_4 Z_1 = \operatorname{tr}\lambda^{--+}Z^{100}$, where there are no Fermi surfaces. Makes sense because overall charge of λ_4 is negative—so no Fermi sea wants to form.