# Bulk Locality and Quantum Error Correction in AdS/CFT

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- This is fine if you are interested in solving large $N$ strongly coupled QFT's, but in my view the most exciting interpretation of AdS/CFT is that it gives a *definition* of non-perturbative quantum gravity in asymptotically-AdS spacetime!
- Today I will explain how some surprising features of this definition can be naturally understood in the language of quantum error correction, a subject first developed as part of quantum computation theory. Almheiri/Dong/Harlow, Harlow/Pastawski/Preskill/Yoshida

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- The Hamiltonians are equivalent, as are the other generators of the AdS symmetries.
- For any bulk field $\phi(x)$, as we pull it to the boundary it becomes a CFT local operator:

$$\lim_{r\to\infty}\phi(t,r,\Omega)r^{\Delta}=\mathcal{O}(t,\Omega).$$

This is sometimes called the "extrapolate dictionary".

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The second point is quite correct, and will ultimately be important, but I will ignore it for now and see how far we can go before getting into trouble.

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The idea is to look for a CFT operator $\phi(x)$ that:

- Obeys the bulk equation of motion as an operator equation.
- Is consistent with the extrapolate dictionary.

These two conditions give us a PDE that we can hope to solve uniquely, at least order by order in $1/N$.

Banks/Douglas/Horowitz/Martinec, Hamilton/Kabat/Lifschytz/Low, Heemkerk/Marolf/Polchinski/Sully

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We all know that in the bulk we can write a free scalar as

$$\phi(x) = \sum_n f_n(x) a_n + f_n^*(x) a_n^\dagger,$$

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For concreteness, we can work in Poincare coordinates

$$ds^2 = \frac{dz^2 + d\vec{x}^2 - dt^2}{z^2},$$

and take

$$f_n(x) = \psi_{\vec{k}\omega}(z) e^{i(\vec{k}\cdot\vec{x} - \omega t)} \qquad \omega > 0,\ \omega^2 \geq \vec{k}^2,$$

with

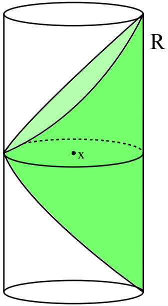
$$\psi_{\vec{k}\omega}(z) \to N_{\vec{k}\omega} z^\Delta \qquad (z \to 0).$$

Matching to the extrapolate dictionary gives

$$a_{\vec{k}\omega} = \frac{1}{N_{\vec{k}\omega}} \mathcal{O}_{\vec{k}\omega},$$

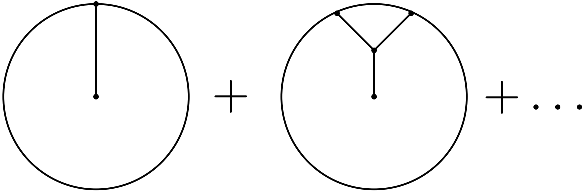
which we can substitute back into our expression for $\phi(x)$ to arrive at:

$$\phi(x) = \int_R dX \; K(x; X)\mathcal{O}(X)$$




This procedure is often called *global reconstruction*.

We can then include $1/N$ corrections by iterating the equations of motion on this expression, which has a nice diagrammatic representation:



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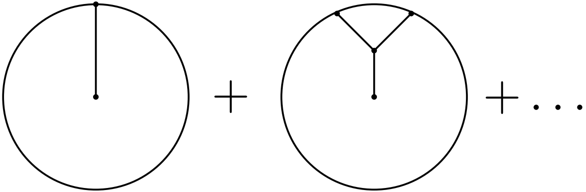

These corrections are important in understanding how this construction implements backreaction; for example if we consider a state with a planet in it then, as in electrodynamics, there will be an infinite subclass of diagrams that we should resum to correct the smearing function to be a solution in the new background.

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- So far this procedure works for *any* equations of motion, even the wrong ones! But clearly something should break if we don't use the right ones.
- One example of something that would break is the *algebra* of the operators; for example we want to have

$$\langle \Omega | \phi \ldots [\phi(x), \phi(y)] \ldots \phi | \Omega \rangle = 0 \qquad (x-y)^2 > 0,$$

but this usually won't be true in the CFT unless we use the right EOM. Kabat/Lifschytz/Lowe

# Bulk algebra in the CFT

- I want to emphasize that this does *not* establish the vanishing of this commutator as an operator statement; it has only been shown to vanish between states where we act on the vacuum with an $O(1)$ number of $\phi$'s.

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- It is consistent with the black hole argument I gave earlier, but is more rigorous.

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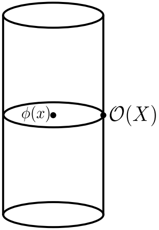

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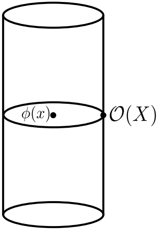


Here $\mathcal{O}(X)$ is some arbitrary local boundary operator. Do we have

$$[\phi(x), \mathcal{O}(X)] = 0?$$

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This would be inconsistent with a standard property of quantum field theory, which is called the "time-slice axiom" (or "primitive causality"):

- For any $\epsilon > 0$, any bounded operator that commutes with all local operators in a time slice of thickness $\epsilon$ about some Cauchy surface $\Sigma$ must be proportional to the identity operator. Streater/Wightman, Haag

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Intuitively, this is expressing the statement that the set of local operators on a time slice acts irreducibly on the Hilbert space:

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Thus we see that, unlike in boundary causality, bulk causality cannot be expressed as an operator equation in the CFT.

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- Finally I will make contact with the original holographic argument against bulk locality; we will see that the theory of quantum error correcting codes predicts the failure of bulk locality on the CFT side precisely at the point where the bulk argument demands it.

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- Finally I will make contact with the original holographic argument against bulk locality; we will see that the theory of quantum error correcting codes predicts the failure of bulk locality on the CFT side precisely at the point where the bulk argument demands it.
- Time permitting, I may also discuss recent work (this week!) introducing an exactly soluble model of AdS/CFT in which this proposal is realized explicitly. Harlow/Pastawski/Preskill/Yoshida.

# Subregion-subregion duality?

- In the last few years, there has been considerable interest in the following question: if we are given the quantum state of the CFT only on some subregion of the boundary, is there some subregion of the bulk that we can still describe? Bousso/Freivogel/Leichenauer/Rosenhaus/Zukowski, Czech/Karczmarek/Nogueira/Van Raamsdonk, Hubeny/Rangamani

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- There has also been a lot of discussion over which bulk region we might expect; eg the “causal wedge” or the “entanglement wedge”? Wall, Headrick/Hubeny/Lawrence/Rangamani, Freivogel/Jefferson/Kabir/Mosk/Yang

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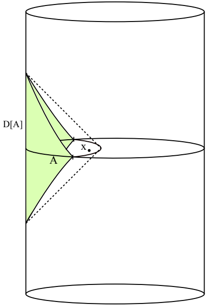
- These ideas were originally motivated by the Ryu Takayanagi formula, but by using the type of operator reconstruction I've already discussed one can also say something nontrivial about it. Hamilton/Kabat/Lifschytz/Lowe, Morrison

Indeed what I will be interested in is the so-called *AdS-Rindler reconstruction*:

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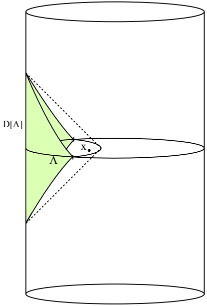
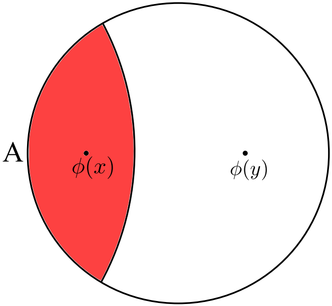



$$\phi(x)\Big|_{W[A]} = \int_{D[A]} dX\ \hat{K}(x;X)\mathcal{O}(X) + O(1/N).$$

In the bulk it is equivalent to the global reconstruction; they are related by a Bogoliubov transformation.

It is more intuitive from above:



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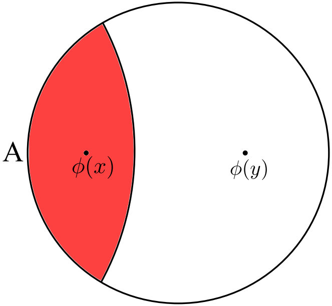


The operator $\phi(x)$ can be represented on $A$, but the operator $\phi(y)$ cannot.

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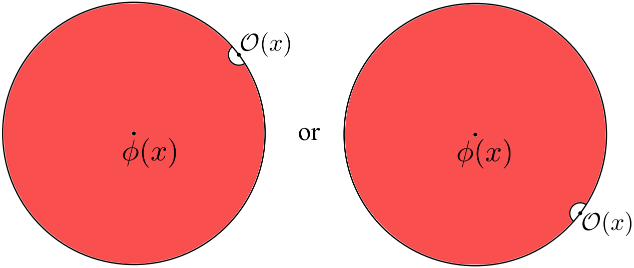

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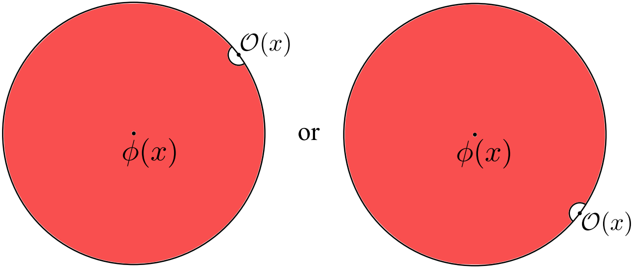


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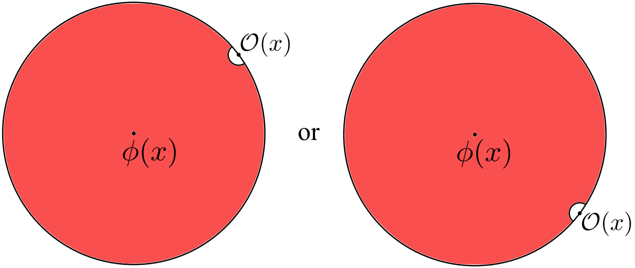
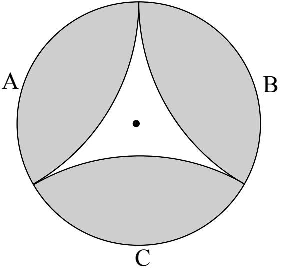



We can always find a wedge reconstruction of $\phi(x)$ such that $[\phi(x), \mathcal{O}(X)] = 0$.
This can only be consistent with the time-slice axiom if the different representations aren't actually equal as operators!

Another illustration:



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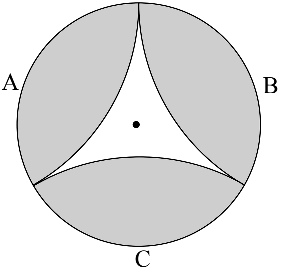


Now the operator in the center has no representation on $A$, $B$, or $C$, but it does have a representation either on $AB$, $AC$, or $BC$!

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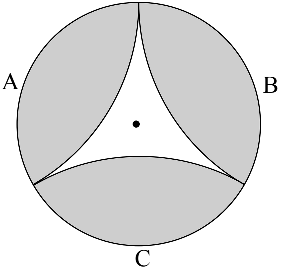


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Something interesting is going on here, but what is it?

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- Nonetheless, there is a way of a encoding the state which protects it against postal corruption - quantum error correction.
- QEC was first developed as a necessary part of building a quantum computer: decoherence of your memory is almost inevitable, so you need a way to fix it!

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The idea is to instead send you three qutrits in the state

$$|\widetilde{\psi}\rangle = \sum_{i=0}^{2} C_i |\widetilde{i}\rangle,$$

where $|\widetilde{i}\rangle$ is a basis for a special subspace of the full 27-dimensional Hilbert space, which is called the *code subspace*.

Explicitly, we take

$$\begin{aligned}
|\widetilde{0}\rangle &= \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right) \\
|\widetilde{1}\rangle &= \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right) \qquad (1) \\
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- This entanglement leads to the interesting property that in any state in the subspace, the density matrix on any one of the qutrits is maximally mixed, ie is given by $\frac{1}{3}\left(|0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|\right)$.

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- Note that this subspace is symmetric between the three qutrits, and each state is highly entangled.
- This entanglement leads to the interesting property that in any state in the subspace, the density matrix on any one of the qutrits is maximally mixed, ie is given by $\frac{1}{3}\left(|0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|\right)$.
- In other words, any single qutrit has *no information* about the encoded state $|\widetilde{\psi}\rangle$.

Explicitly, we take

$$\begin{aligned}
|\widetilde{0}\rangle &= \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right) \\
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- In other words, any single qutrit has *no information* about the encoded state $|\widetilde{\psi}\rangle$.
- This leads to the remarkable fact that we can *completely recover* the quantum state from any two of the qutrits!

To see this explicitly, we can define a two-qutrit unitary operation $U_{12}$ that acts as

$$\begin{array}{lll} |00\rangle \to |00\rangle & |11\rangle \to |01\rangle & |22\rangle \to |02\rangle \\ |01\rangle \to |12\rangle & |12\rangle \to |10\rangle & |20\rangle \to |11\rangle \\ |02\rangle \to |21\rangle & |10\rangle \to |22\rangle & |21\rangle \to |20\rangle \end{array} .$$

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$$U_{12}|\widetilde{i}\rangle = |i\rangle_1|\chi\rangle_{23},$$

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- By symmetry there must also exist $U_{13}$ and $U_{23}$.

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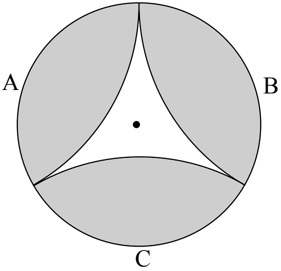
Generically this operator will have nontrival support on all three qutrits, but using our $U_{12}$ we can define

$$O_{12} \equiv U_{12}^{\dagger} O_1 U_{12},$$

which acts nontrivially only on the first two but still implements $O$ on the code subspace.

The point now is that we can interpret $O_{12}$, $O_{13}$, and $O_{23}$ as being analogous to the representations of $\phi(0)$ on $AB$, $AC$, and $BC$ in this example:



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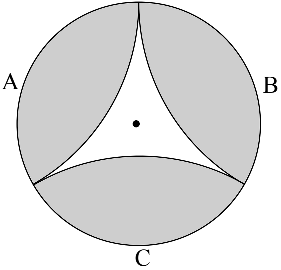


By using the entanglement of the code subspace, we can replicate the paradoxical properties of the AdS-Rindler reconstruction.

We can also make contact with the commutator puzzle: let's compute

$$\langle\widetilde{\psi}|[\widetilde{O}, X_3]|\widetilde{\phi}\rangle, \tag{2}$$

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This is the lesson to learn for AdS/CFT; the bulk algebra of operators *holds only on a subspace of states*!

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- Say that we have $n$ physical qubits, and we want to protect a $k$-qubit message from an erasure of $\ell$ or fewer of the physical qubits. Then we need

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and in fact for a typical $2^k$-dimensional code subspace this is *sufficient*.

This is quite intuitive; sending a bigger message that is better protected requires more qubits!

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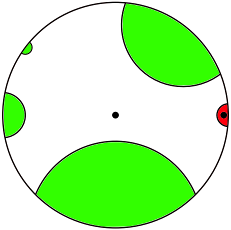

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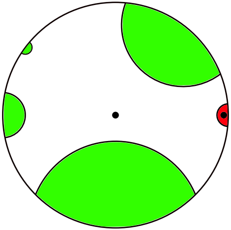

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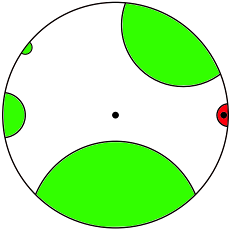
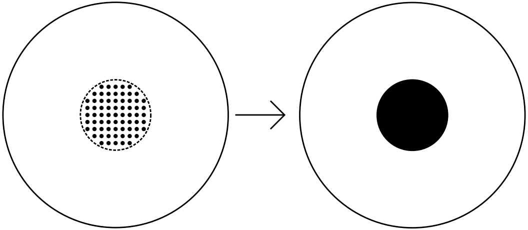


Indeed we need $\approx 1/2$ of the system to reconstruct the center.
Notice however that if we are NOT in the center we correct less well: this is a precise realization of the "radial direction $\leftrightarrow$ scale" correspondence.

We can now ramp up $k$:



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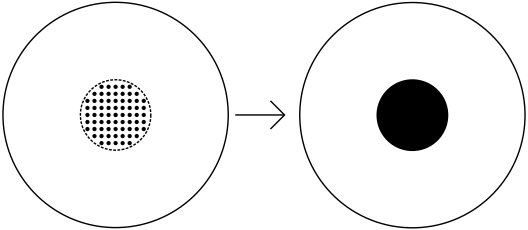

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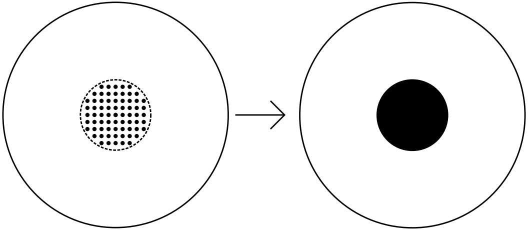

Clearly the answer will not change from $1/2$ until $k \sim N^2$, but on the bulk side this is just when we expect to create a huge black hole in the center! Thus we see that we are able to push our reconstruction of bulk operators just until the point where the old holographic arguments become relevant...

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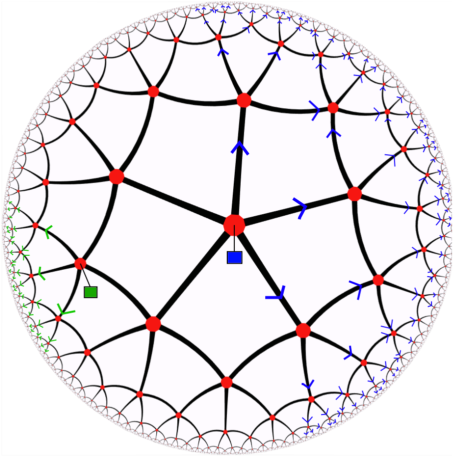
- The basic idea is to replace the CFT by a spin system and then just write down a set of states whose entanglement structure closely resembles that of the low energy states of a CFT.

Here is a picture:



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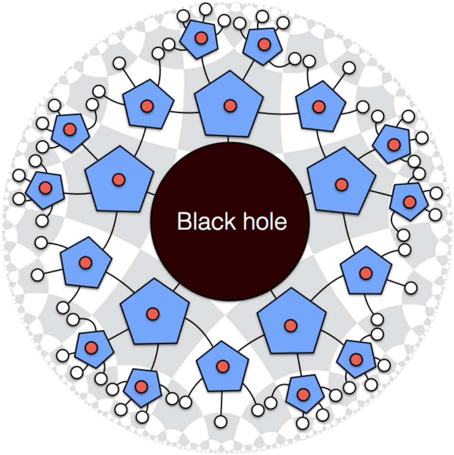

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Nonetheless I think they do illustrate the error-correcting properties of AdS/CFT quite clearly, and it would be interesting to see if they might be generalized to include these other features.

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- Thanks!