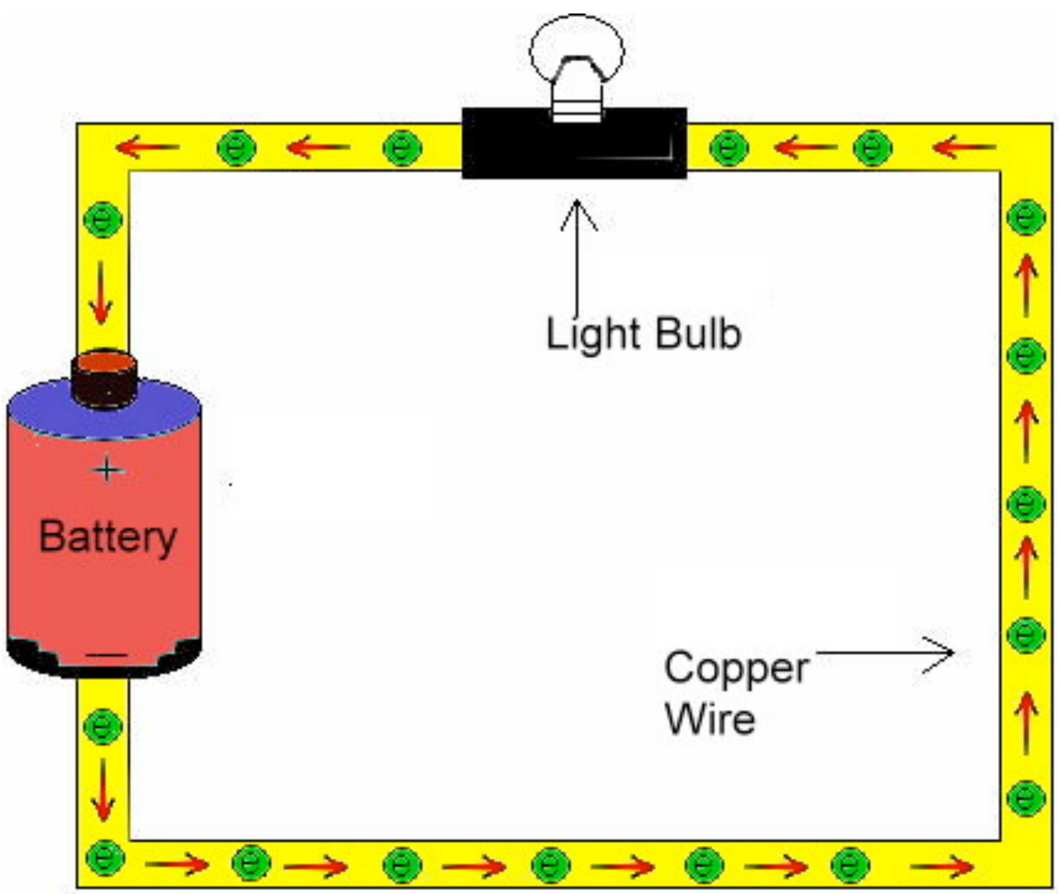
Physics without Momentum

Sean Hartnoll (Stanford)

Eurostrings, Cambridge - March 2015

Conventional metals

Simple equations



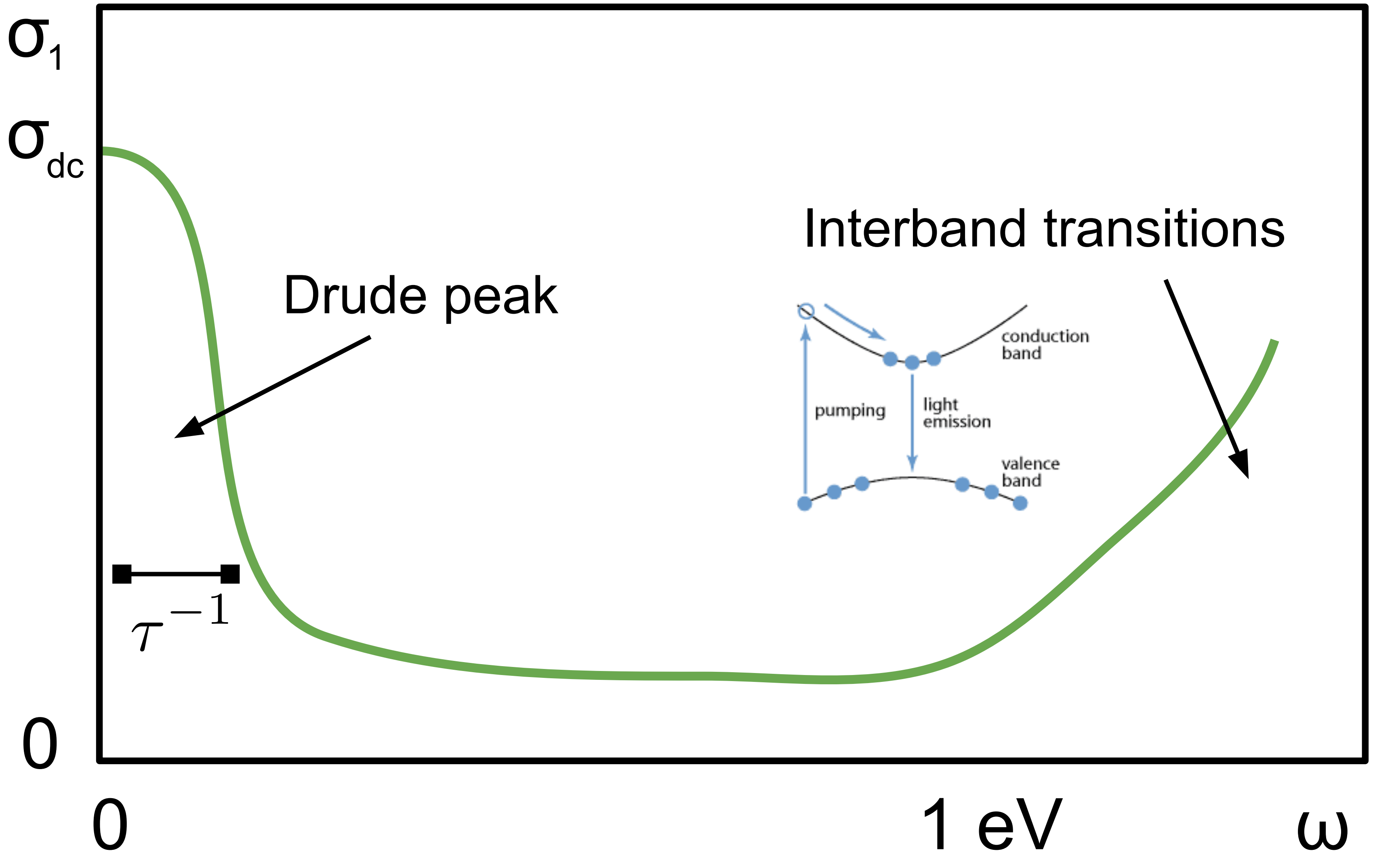
$$j(\omega) = \sigma(\omega) E(\omega)$$

$$\sigma(\omega) = \sigma_1(\omega) + i\,\sigma_2(\omega)$$

$$P = \sigma_1(\omega) E(\omega)^2$$

Joule Heating

Optical conductivity



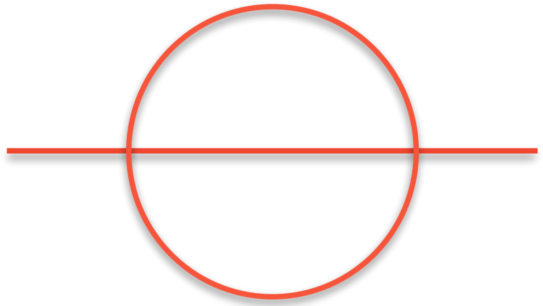
Essential facts

- **The quasiparticle lifetime τ is the longest timescale in the game.**
  ⇒sharp Drude peak.

- **The dc conductivity (Drude, 1900):**

$$\sigma_{\mathrm{dc}} = \frac{ne^2\tau}{m_\star}$$

- **Electron-electron scattering gives:**



$$\tau \sim \frac{\hbar}{k_B T}\frac{E_F}{k_B T} \gg \frac{\hbar}{k_B T}$$

(Landau Fermi Liquid)

Essential facts

- Computations are possible because the low energy effective field theory of a conventional metal has infinitely many almost conserved operators:

$$\delta n_k = c_k^\dagger c_k$$

- "Almost conserved"
  = conserved up to irrelevant operators.

- Correct theoretical framework: Boltzmann equation.

Challenges in unconventional metals

"Coherent" metals

[Lucas-Sachdev 1502.04704;
Hartnoll-Hofman 1201.3917;
Hartnoll-Kovtun-Muller-Sachdev 0706.3215]

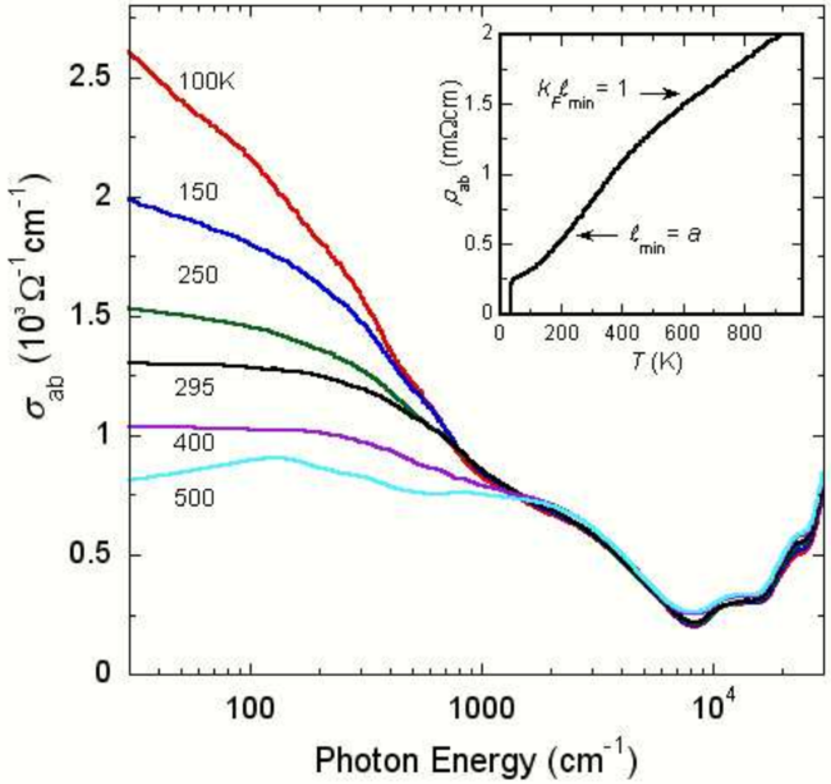
- What almost conserved quantities might a strongly interacting metal have?
- If $\exists$ a long wavelength continuum QFT description of the underlying lattice system, then there is an emergent almost conserved momentum.
- This is true even in the absence of quasiparticles.
- In these cases there is still a sharp 'Drude' peak with width: $\Gamma \ll T$

“Incoherent” metals

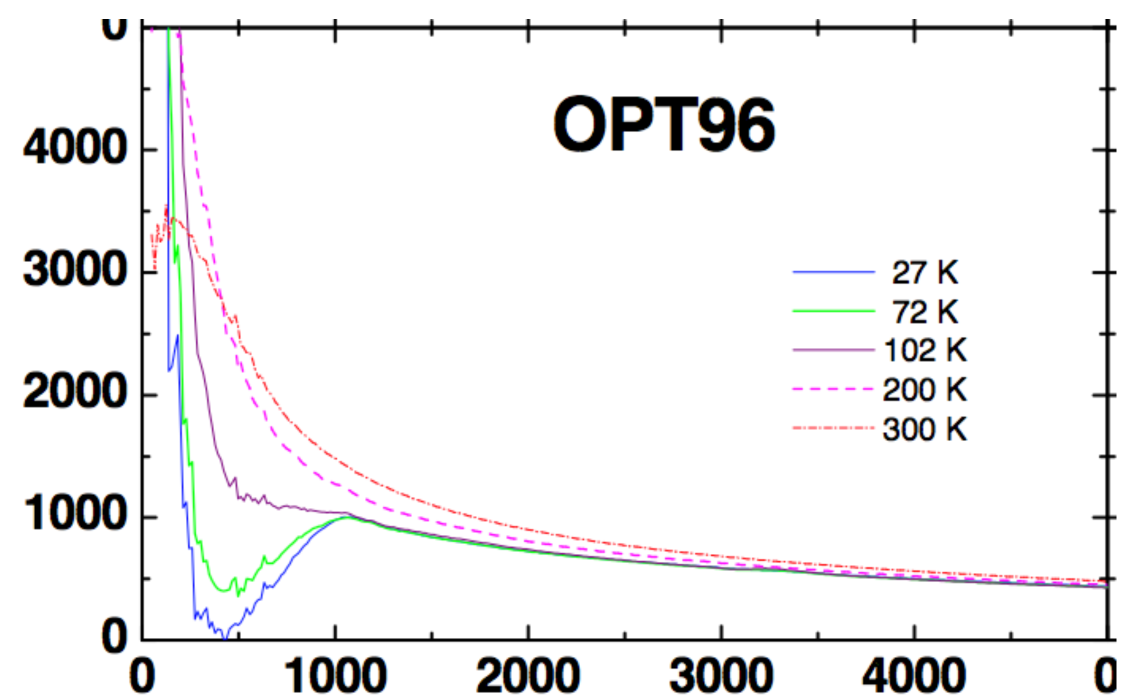
[Hartnoll 1405.3651]

- **But the most interesting cases don’t have a sharp Drude peak!**

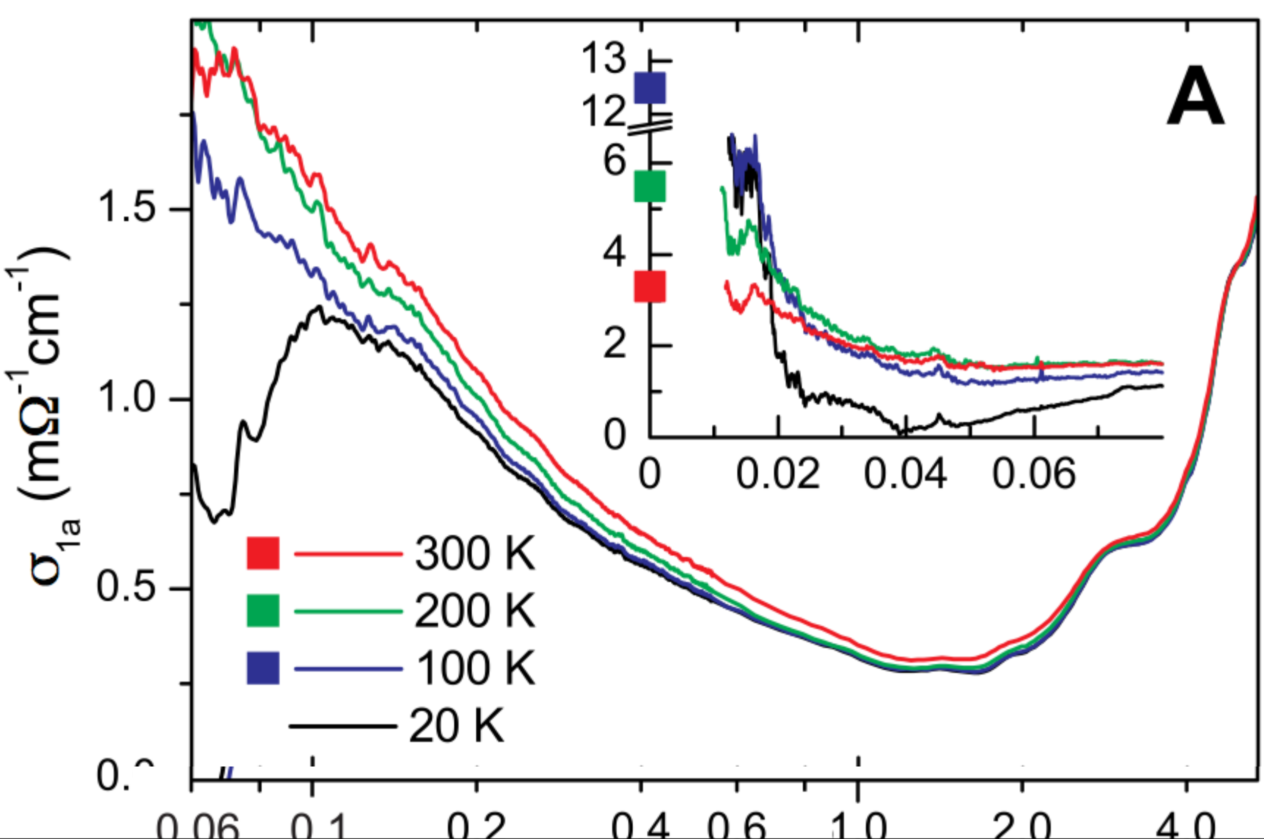
$$\frac{1}{\tau} \sim \frac{k_B T}{\hbar}$$



- **LSCO, Takenaka et al. ‘03**

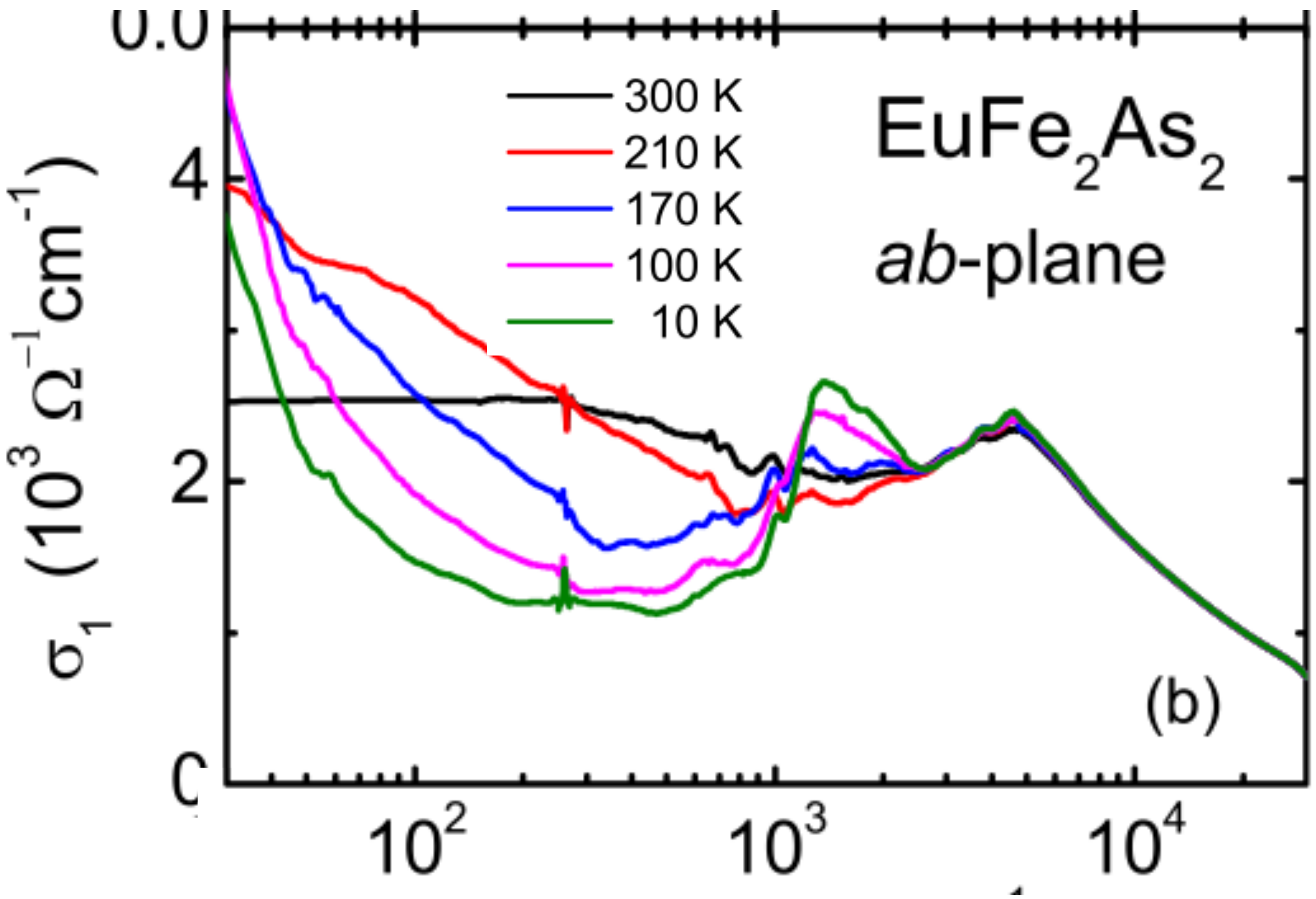


- **BSCCO, Hwang et al. ‘07**

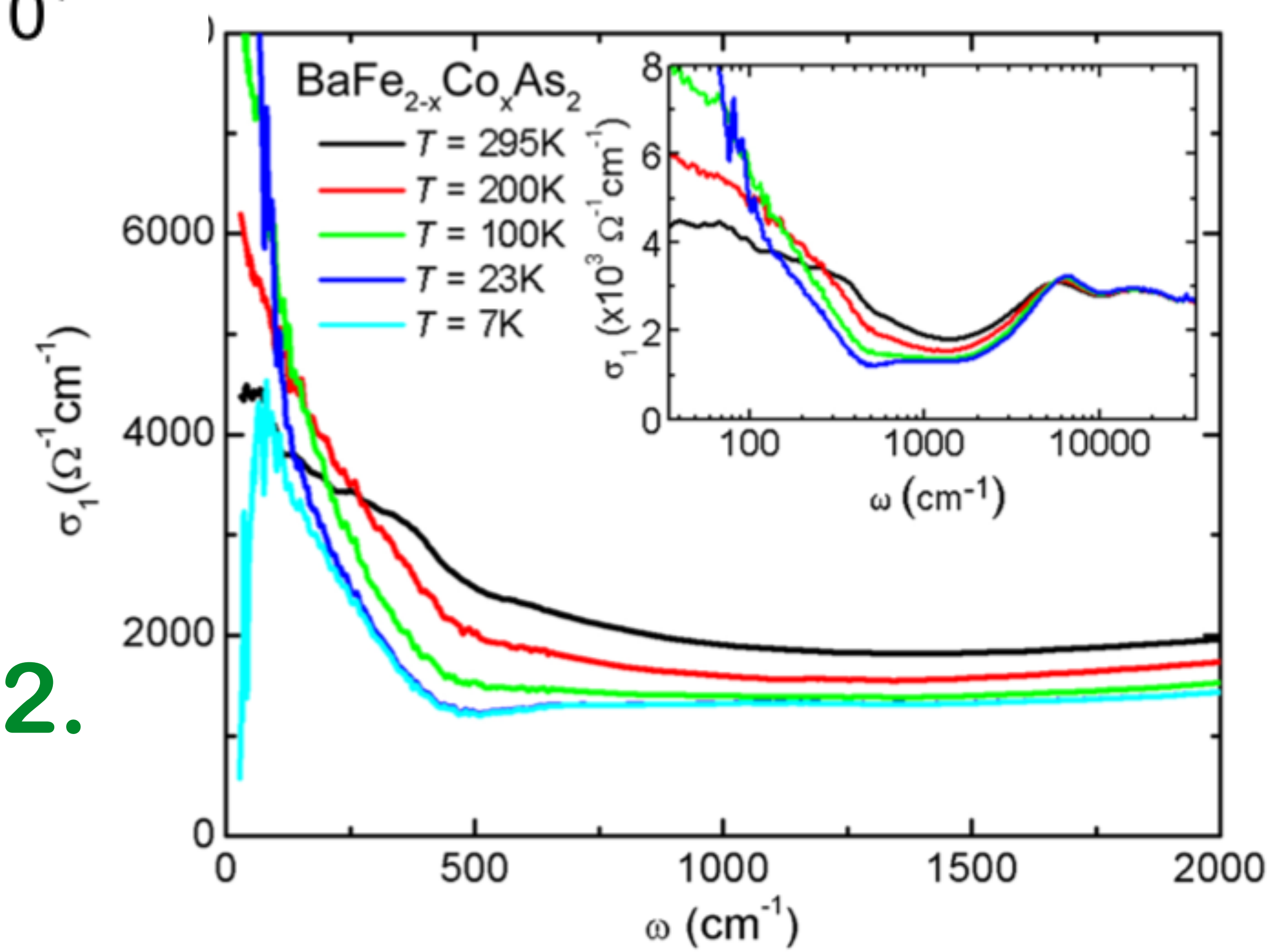


- **YBCO, Boris et al. ‘04**

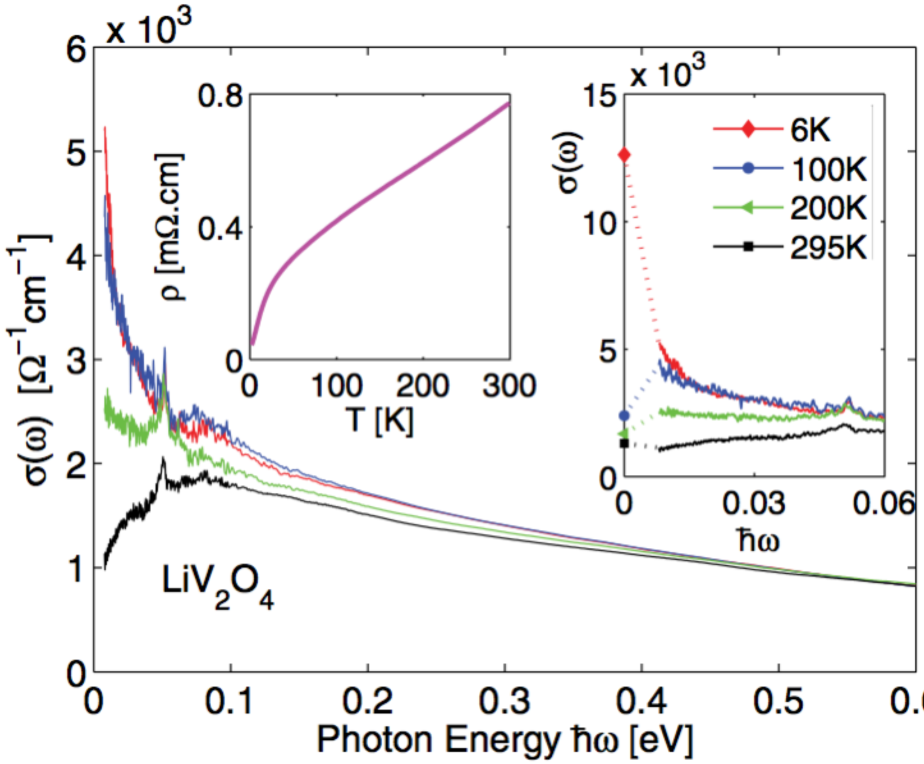
$$\frac{1}{\tau} \sim \frac{k_B T}{\hbar}$$



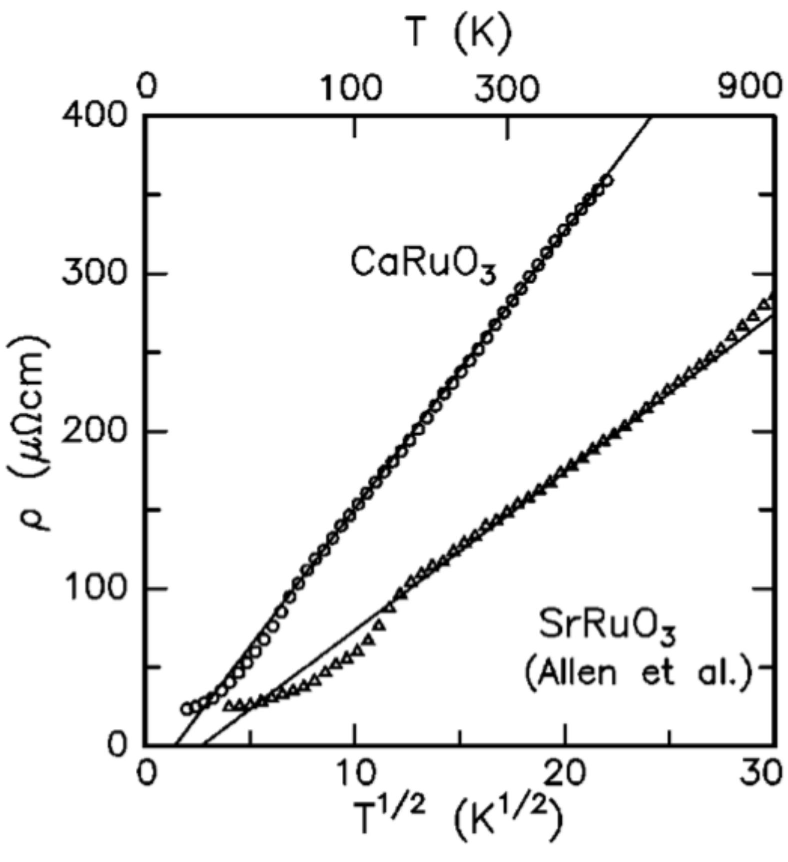
- **Wu et al. '09.**

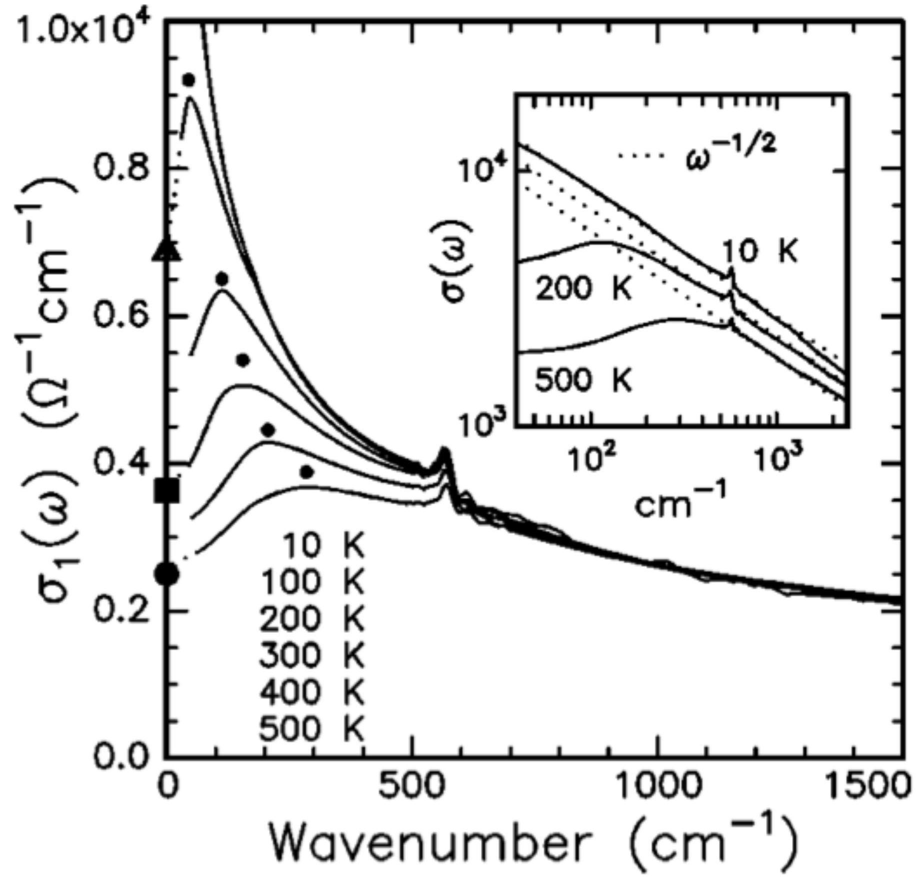


- **Schafgans et al. '12.**

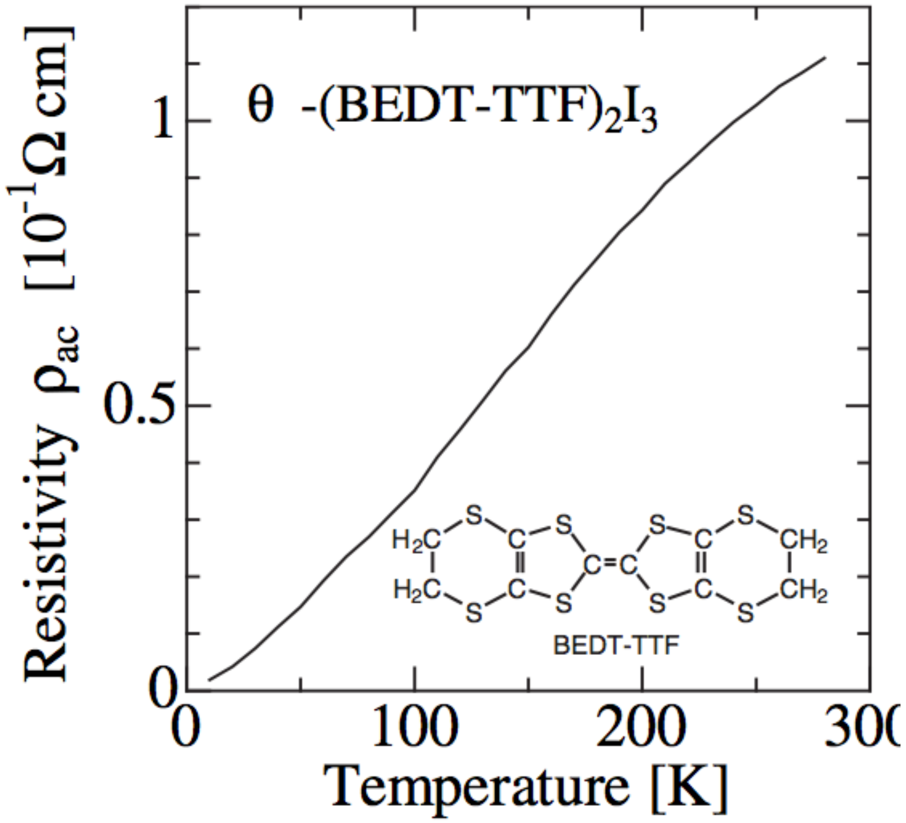


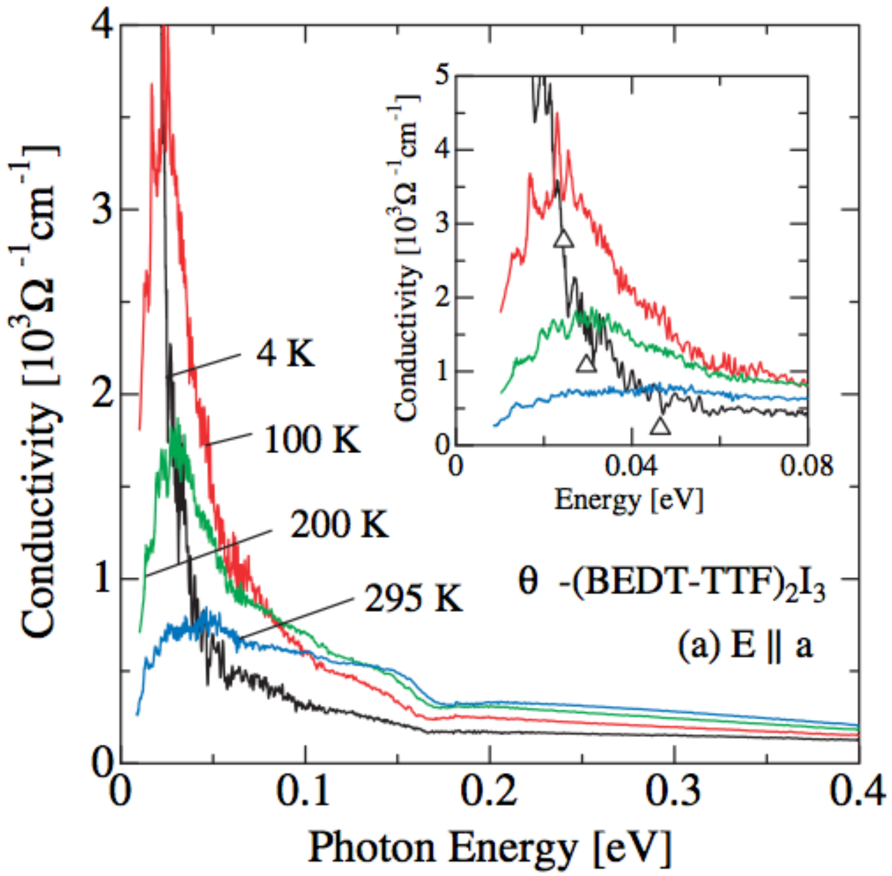
- **Jonsson et al. ’07.**



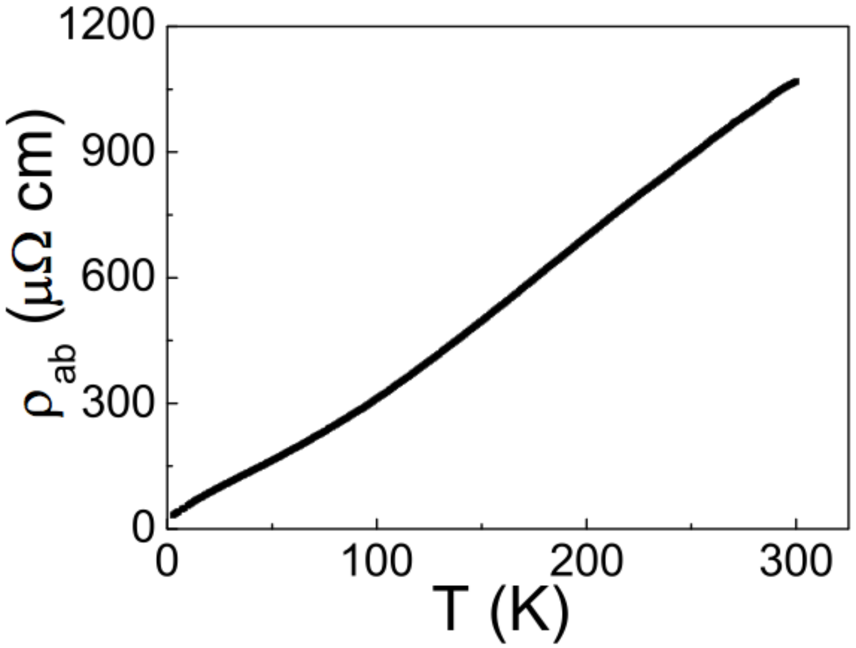


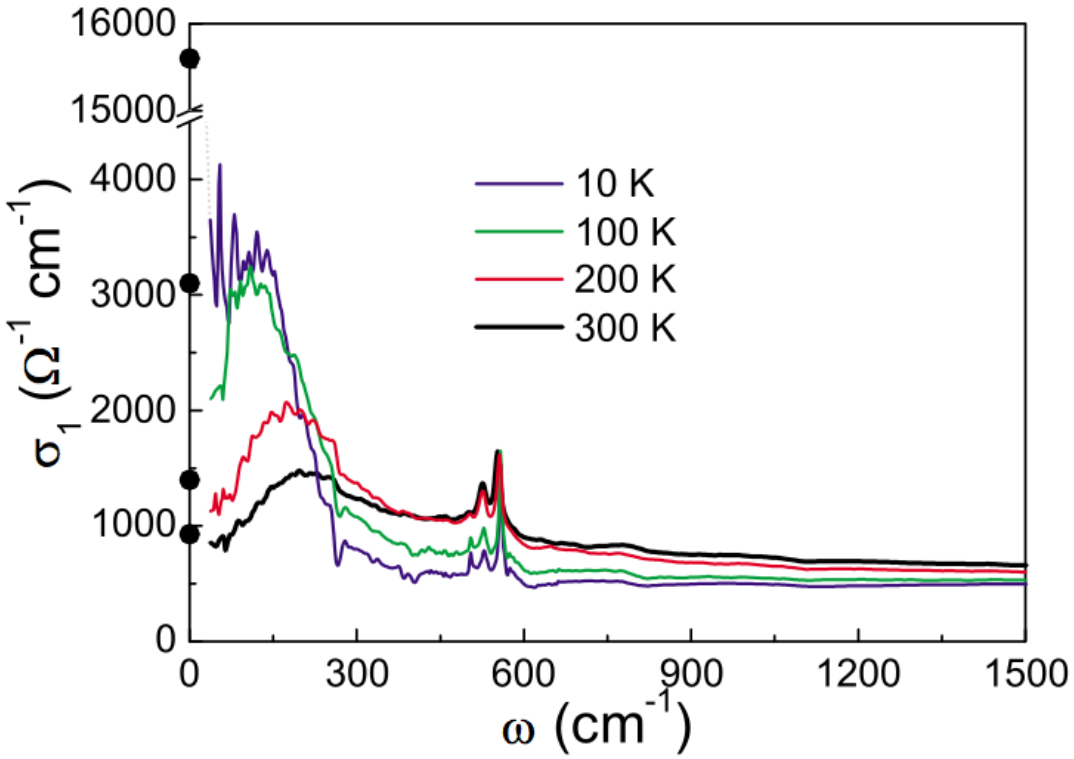
- **Lee et al. ’02.**





- Takenaka et al. '05.



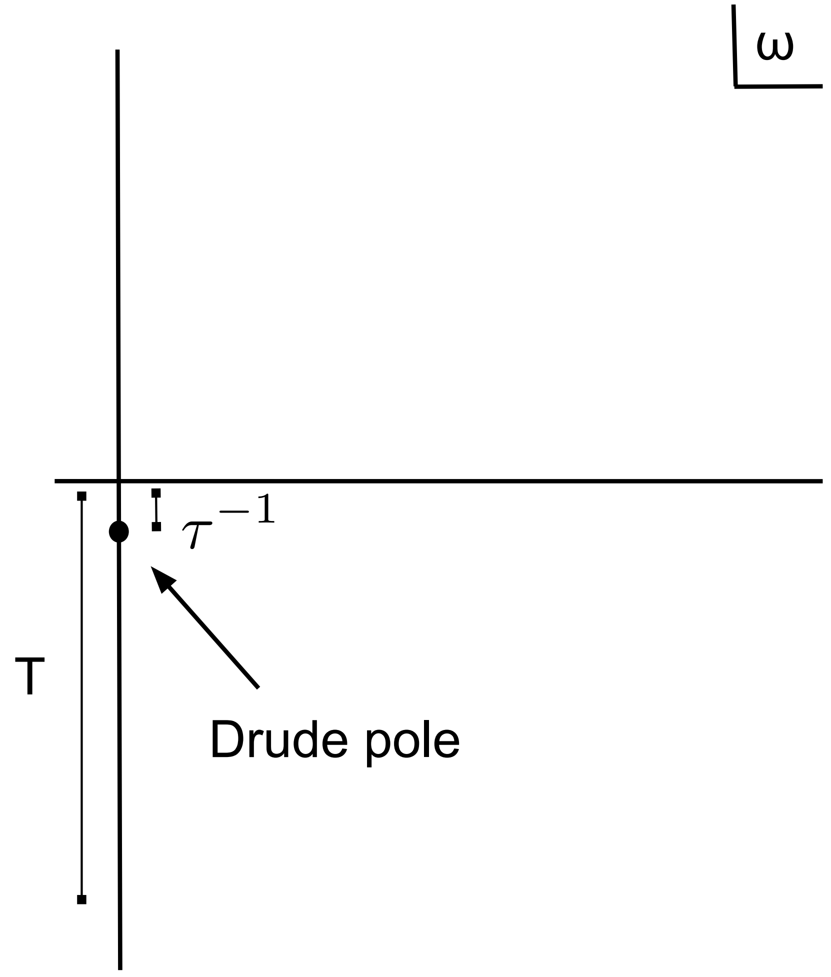


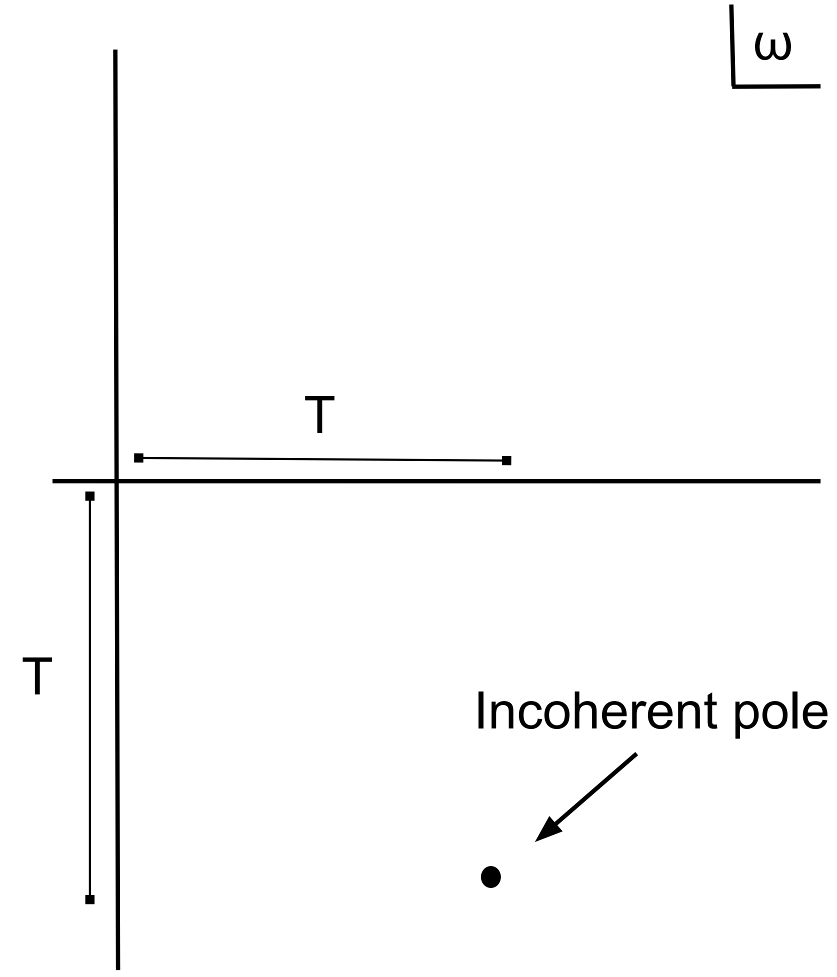
- Wang et al. '04.

$Na_{0.7}CoO_2$

No Drude peak

- **Conventional vs. Incoherent metal**





The theoretical challenge

[Hartnoll 1405.3651]

- How does one describe metals with no Drude peak? The effective description should not have a conserved momentum.
- What kind of physics might lead to this structure?
- An interesting class of tractable models involves clever symmetries [Gauntlett talk]. Good springboard for more realistic models [eg. Donos-Gauntlett].

Mott-like physics

[Sachdev, late 90s]

- One possibility is an emergent charge conjugation symmetry in the IR. Eg. CFT.
- In this case the IR momentum does not overlap with the current operator.
- This happens in special circumstances such as the Bose-Hubbard model at integer filling.
- In this case one can “have one’s cake and eat it”.

Disorder physics

- The simplest models of incoherent metals may come from disordered fixed points.
- Because disorder breaks translation invariance at all scales (unlike a lattice). It can easily have strong effects on the far IR.
- In QFT the way the continuum description and non-conservation of momentum are married is traditionally through the "replica trick".

Disorder physics

- However, attempts to find controlled interacting disordered fixed points in general dimension have not been successful.
- Eg. in attempts à la Wilson-Fisher, the second term in the beta function has the wrong sign:

$$\mu \frac{dV}{d\mu} = -\epsilon V - \# V^2 + \cdots$$

[e.g. Sachdev book]

Disorder physics

- Holography allows study of disorder physics without the replica trick.

Disordered fixed points in holography

[Hartnoll-Santos, 1402.0872]

- The relevance of a disordered coupling

$$\int dt d^d x \, h(x) \mathcal{O}(t, x)$$

- Is determined by the '**Harris criterion**':

$$\Delta > \frac{d+2}{2}$$

- If relevant, need to follow the flow to IR.

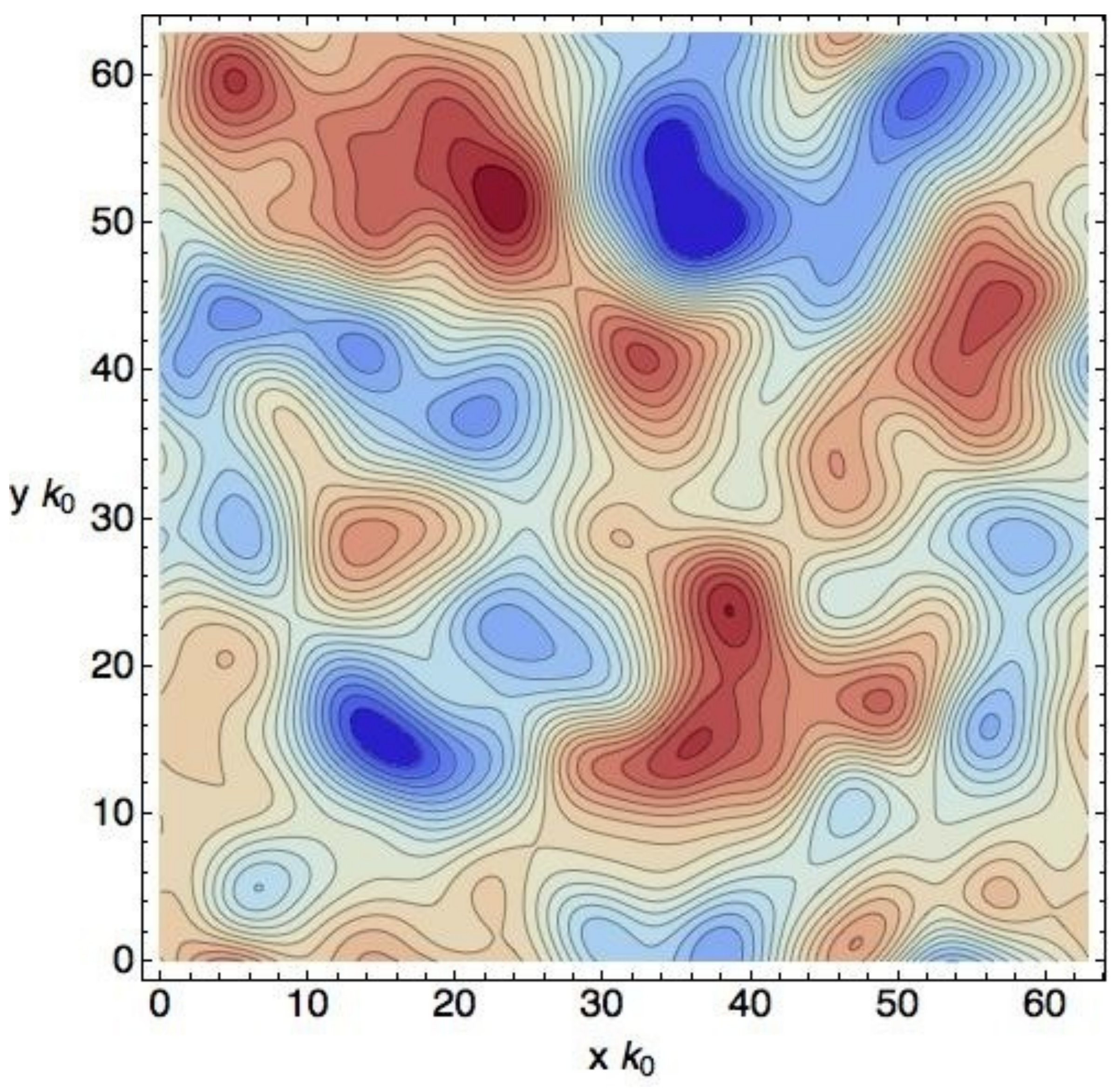
Disordered fixed points in holography

- A random coupling is generated by

$$h(x) = \bar{V} \sum_{n=1}^{N-1} 2\sqrt{\Delta k}\, \cos(n\Delta k\, x + \gamma_n)$$

- Solved Einstein-scalar bulk theory with a marginally relevant random source.
- Resummed the logarithmic growth to find stable disordered IR fixed points.
- Confirmed and extended results via numerical simulation of full disorder.

Disordered fixed points in holography



- **Zero temperature <u>averaged</u> IR geometry:**

$$\langle ds^2_{\mathrm{IR}} \rangle = -\frac{dt^2}{r^{2\bar{z}}} + \frac{dr^2 + dx^2 + dy^2}{r^2}$$

$$\bar{z} = 1 + \frac{\pi^{(d-1)/2}}{2}\Gamma\left(\frac{d+1}{2}\right)\bar{V}^2 + \cdots$$

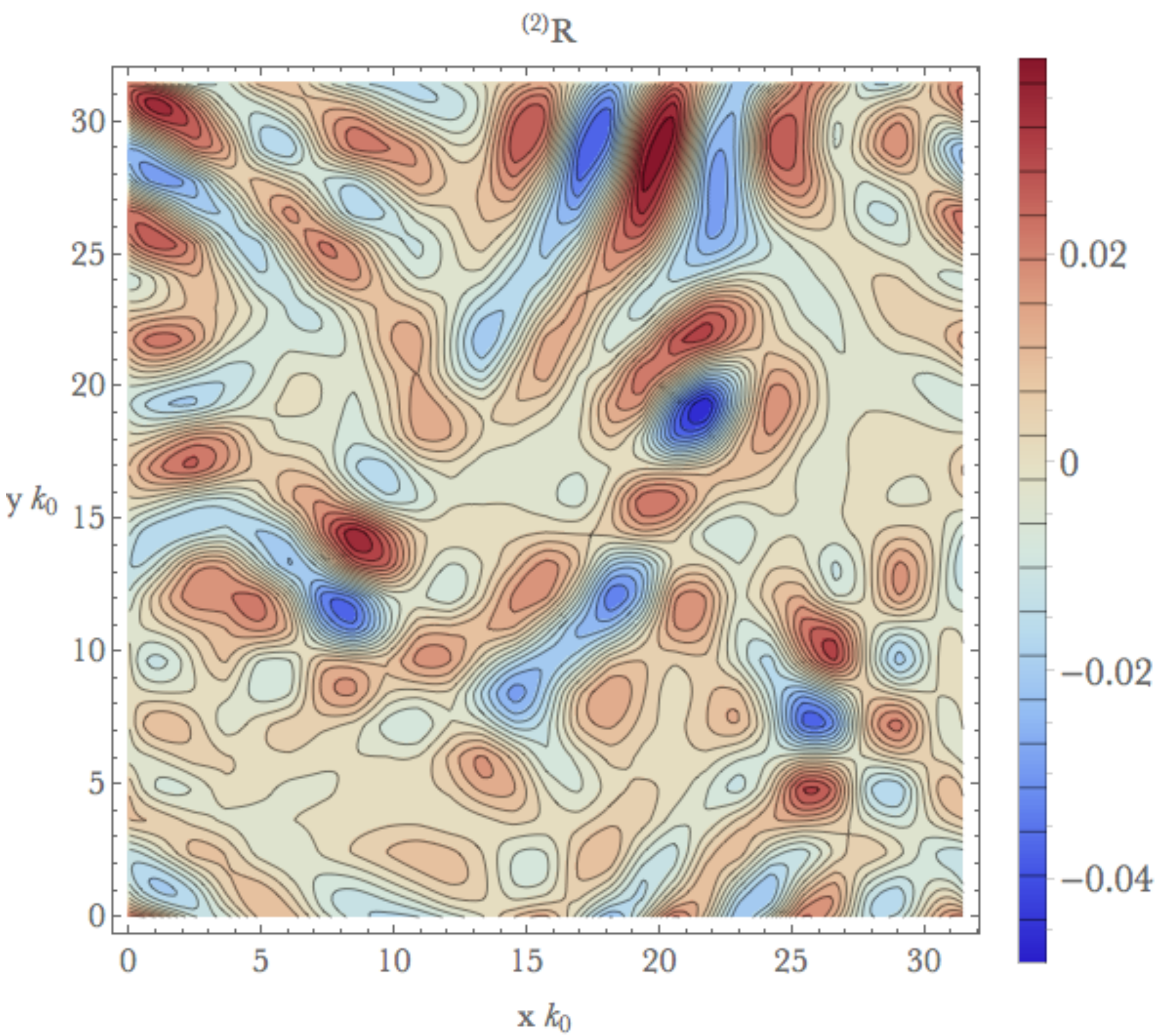
Disordered fixed points in holography

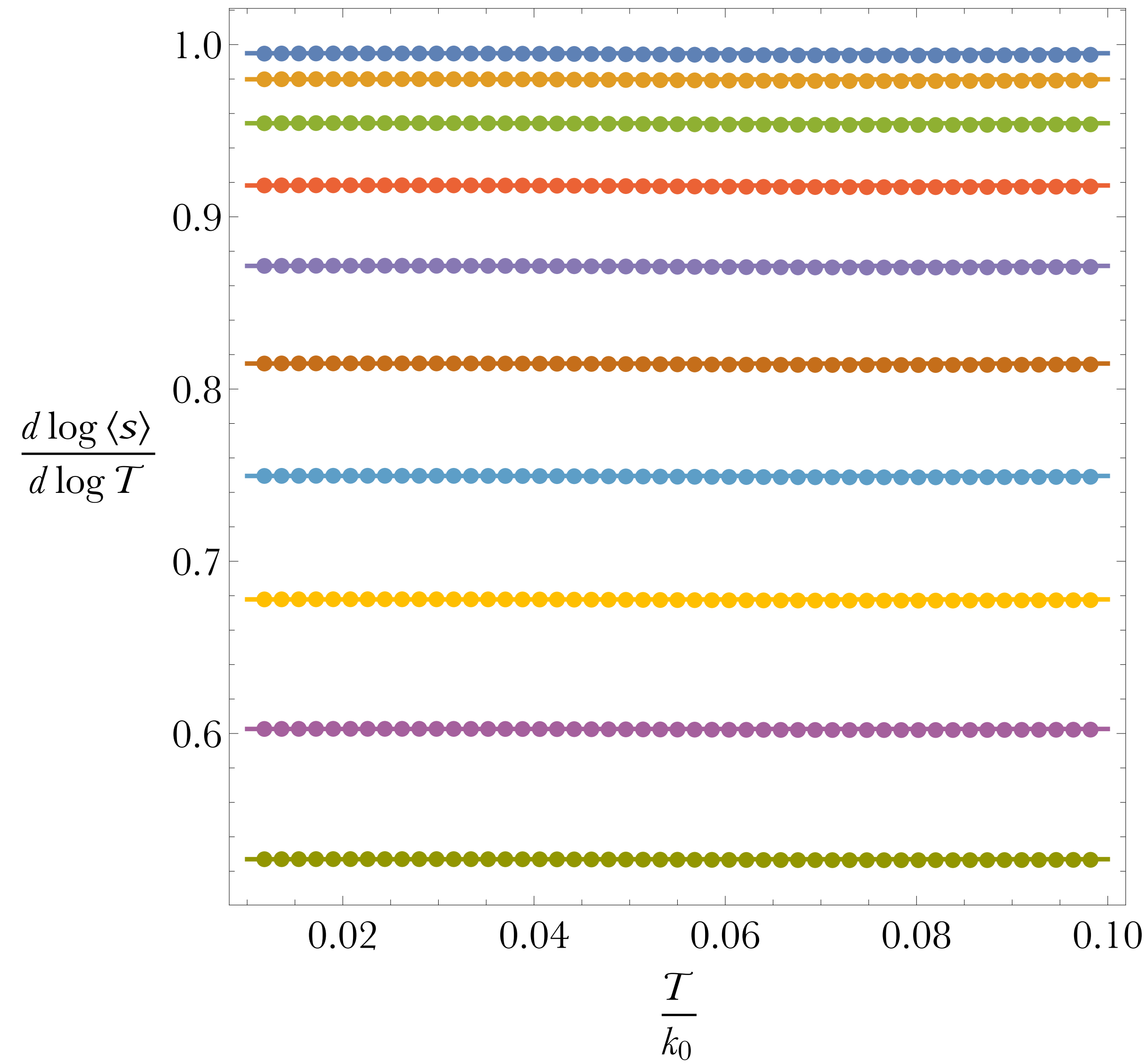
[Hartnoll-Ramirez-Santos, 1504.????]

- Scale invariance of the disorder averaged metric suggested an IR disordered fixed point.
- Not clear what quantities controlled by z.
- By constructing finite T solutions, have shown analytically and numerically:

$$s \sim T^{2/z}$$

Disordered fixed points in holography





Disordered fixed points in holography

- Compelling evidence for a disordered fixed point, of the type that is elusive in weakly interacting QFT.

- Transport calculations underway. Natural thing to look at is heat transport, model for incoherent 'metal'.

Big picture

- Much progress in holographic transport in past 3 years. Backreaction on conventional condensed matter theory and experiment.
- Current cutting edge — the remaining regime that lacks a distilled theory — is incoherent transport.
- Disordered fixed points have rich physics and seem to be accessible holographically.