ENTANGLEMENT & GRAVITY

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Based on: M. Headrick, VH, A. Lawrence, & M. Rangamani: 1408.6300;
J. Bhattacharya, VH, M. Rangamani, T. Takayanagi: 1412.5472;
& previous works w/ {H. Maxfield, M. Rangamani, & E. Tonni}: 1306.4004, 1306.4324, & 1312.6887

Motivation from AdS/CFT

String theory ($\ni$ gravity) $\Longleftrightarrow$ gauge theory (CFT)

"in bulk" asymp. AdS $\times$ K *"on boundary"*

Applied AdS/CFT:

- study specific system via its dual
- e.g. AdS/QCD, AdS/CMT, ...

Fundamentals of AdS/CFT:

- why/how does the duality work
- map between the 2 sides

Holographic Entanglement Entropy

Quantum Gravity

Entanglement

- Most non-classical manifestation of quantum mechanics
  - "Best possible knowledge of a whole does not include best possible knowledge of its parts — and this is what keeps coming back to haunt us" [Schrodinger '35]
- New quantum resource for tasks which cannot be performed using classical resources [Bennet '98]
- Plays a central role in wide-ranging fields
  - quantum information (e.g. cryptography, teleportation, …)
  - quantum many body systems
  - quantum field theory
- Hints at profound connections to geometry…

Entanglement Entropy (EE)

Suppose we only have access to a subsystem A of the full system = A + B. The amount of entanglement is characterized by Entanglement Entropy S_A :

- reduced density matrix $\rho_A = \mathrm{Tr}_B \, |\psi\rangle\langle\psi|$

  (more generally, for a mixed total state, $\rho_A = \mathrm{Tr}_B \rho$)

- EE = von Neumann entropy $S_A = -\mathrm{Tr}\, \rho_A \, \log \rho_A$

Defined if we can divide a quantum system into a subsystem A and its complement B, such that the Hilbert space decomposes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Entanglement Entropy (EE)

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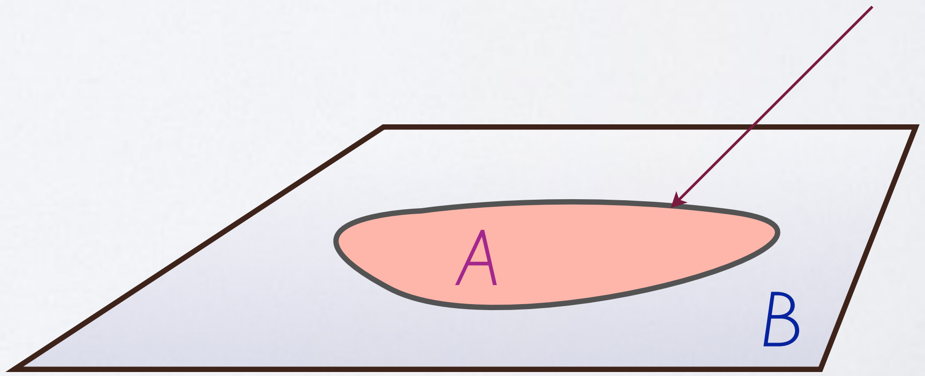
- e.g. in local QFT:

  A and B can be spatial regions, separated by a smooth entangling surface



The good news & the bad news

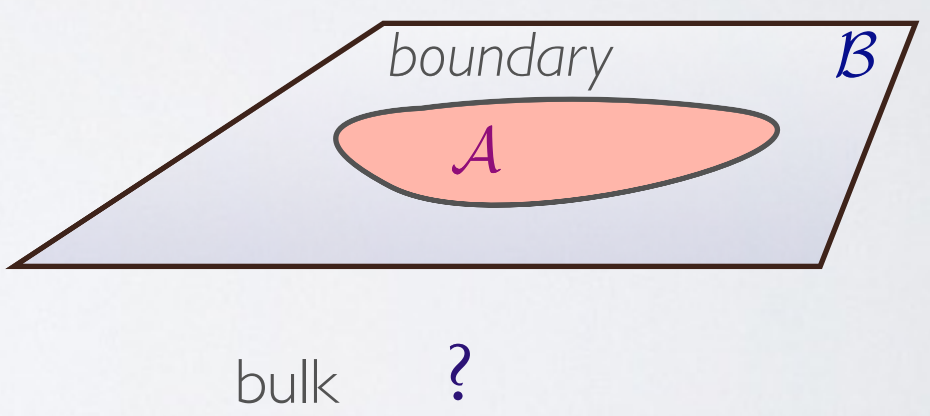
- But EE is hard to deal with…
  - non-local quantity, intricate & sensitive to environment
  - difficult to measure
  - difficult to calculate

… especially in strongly-coupled quantum systems

- AdS/CFT to the rescue?

~ Is there a natural bulk dual of EE?

(= "Holographic EE")

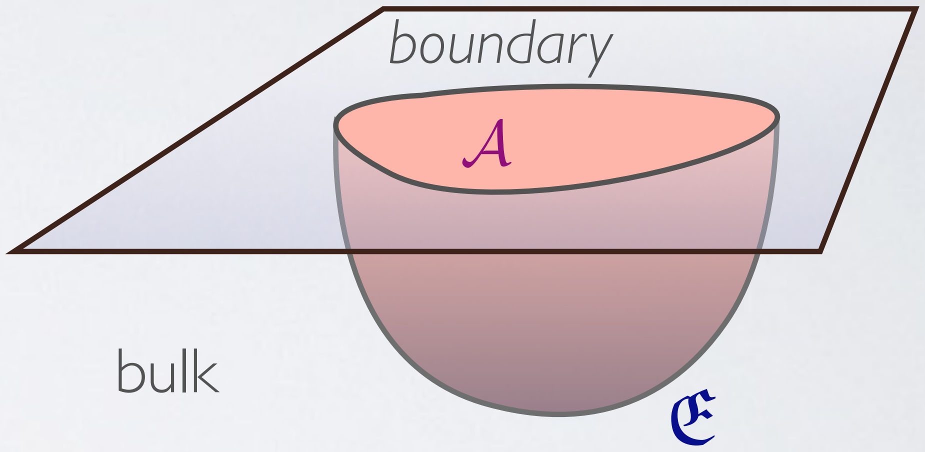


Yes! - described geometrically…

Holographic Entanglement Entropy

Proposal [Ryu & Takayanagi, '06] for *static* configurations:

In the bulk, EE $S_{\mathcal{A}}$ is captured by the area of minimal co-dimension 2 bulk surface $\mathfrak{E}$ *(at constant t)* anchored on $\partial\mathcal{A}$.



$$S_{\mathcal{A}} = \min_{\partial\mathfrak{E}=\partial\mathcal{A}} \frac{\text{Area}(\mathfrak{E})}{4\,G_N}$$

Remarks:

- cf. black hole entropy…
- Large body of evidence, culminating in [CHM, …, Lewkowycz, Maldacena]
- Beautifully geometrizes profound & important relations,
  e.g. $S_{\mathcal{A}} = S_{\mathcal{A}^c}$ for pure states, $|\, S_{\mathcal{A}} - S_{\mathcal{A}^c} \,| \leq S_{tot} \leq S_{\mathcal{A}} + S_{\mathcal{A}^c}$

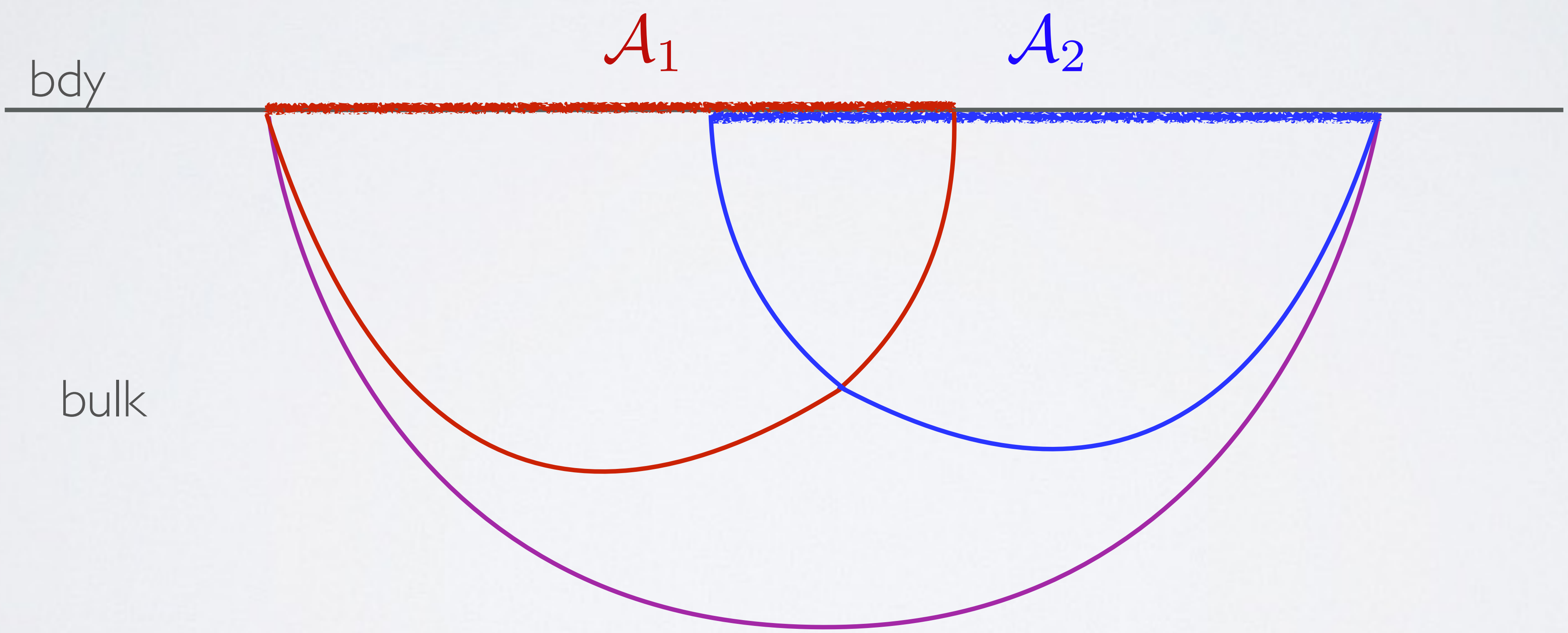
  ↙ Araki-Lieb ↘ subadditivity

Subadditivity

- Subadditivity:

$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \geq S_{\mathcal{A}_1 \cup \mathcal{A}_2}$$

- Manifest in the gravity dual



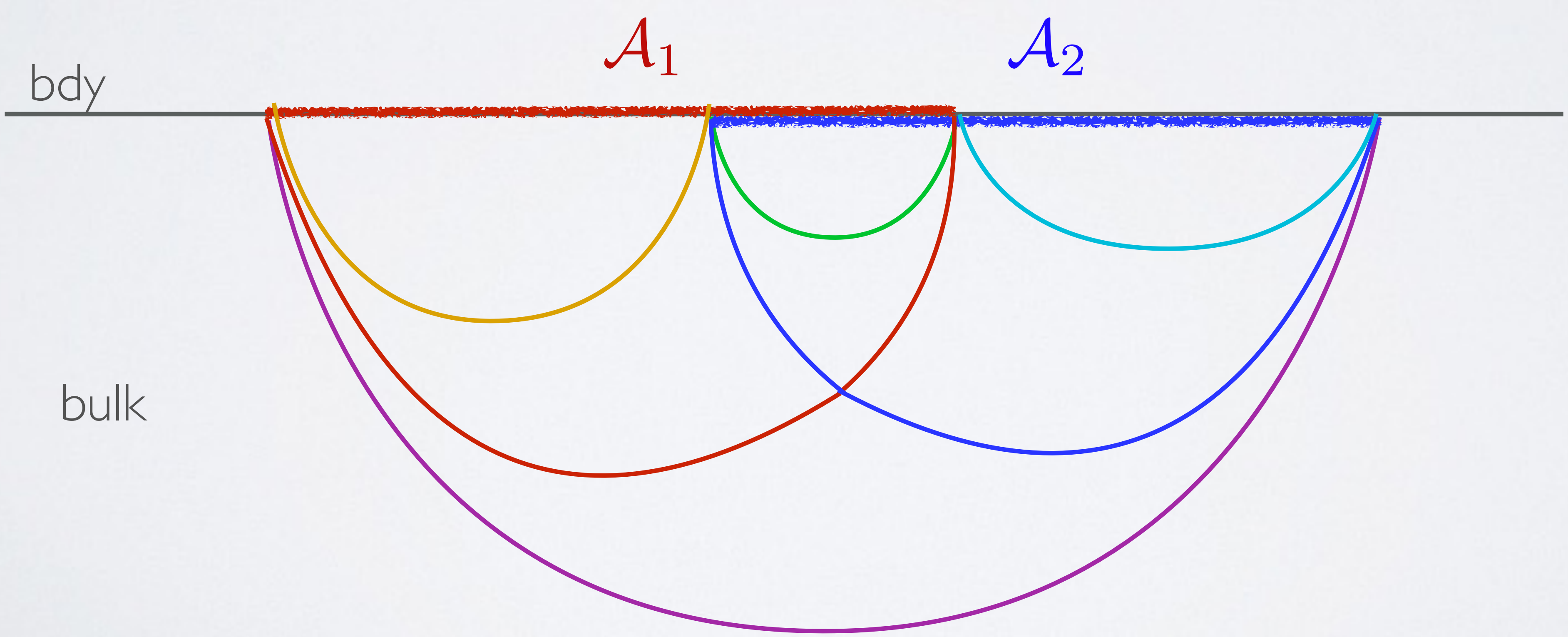
- Implies positivity of mutual information: $I(\mathcal{A}_1, \mathcal{A}_2) = S_{\mathcal{A}_1} + S_{\mathcal{A}_2} - S_{\mathcal{A}_1 \cup \mathcal{A}_2}$

Strong Subadditivity

- strong subadditivity:

$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \;\geq\; S_{\mathcal{A}_1 \cup \mathcal{A}_2} + S_{\mathcal{A}_1 \cap \mathcal{A}_2}$$

$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \;\geq\; S_{\mathcal{A}_1 \backslash \mathcal{A}_2} + S_{\mathcal{A}_2 \backslash \mathcal{A}_1}$$

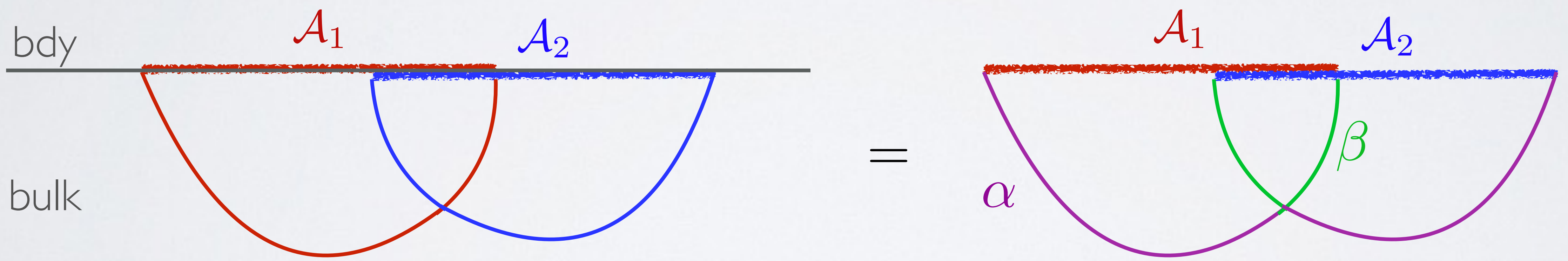


Proof of Strong Subadditivity

- strong subadditivity:

$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \geq S_{\mathcal{A}_1 \cup \mathcal{A}_2} + S_{\mathcal{A}_1 \cap \mathcal{A}_2}$$

- proof in static configurations [Headrick&Takayanagi]



$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} = \alpha + \beta$$

Proof of Strong Subadditivity

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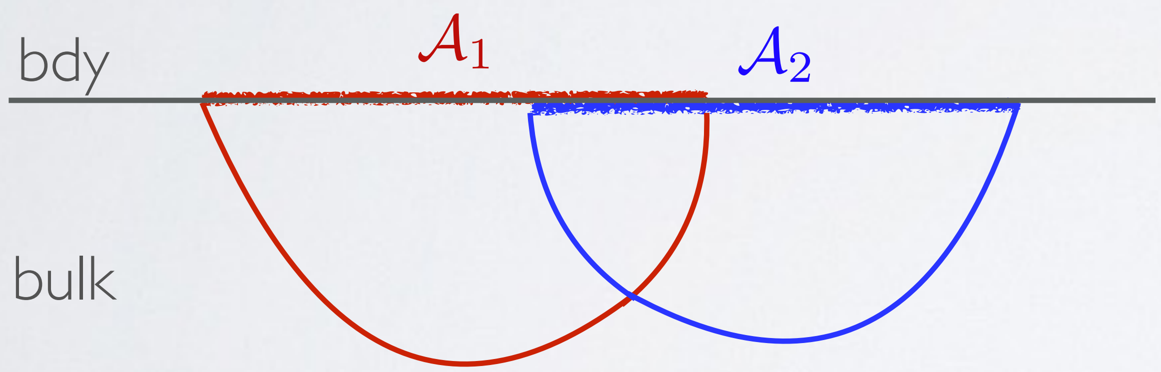
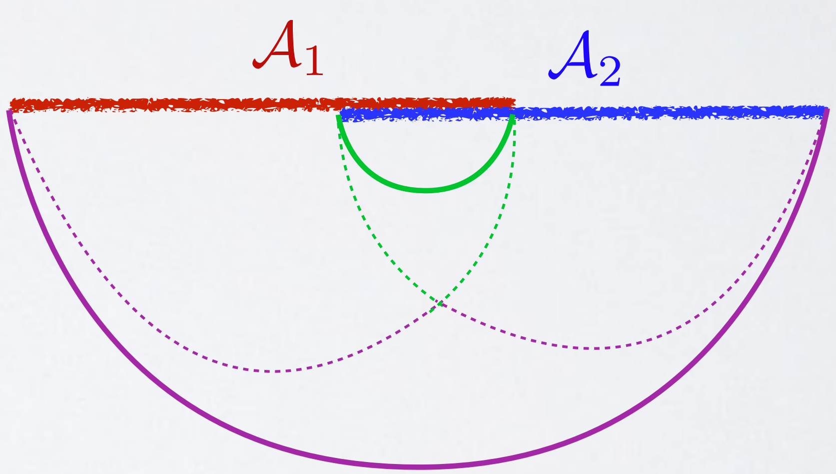


$\geq$

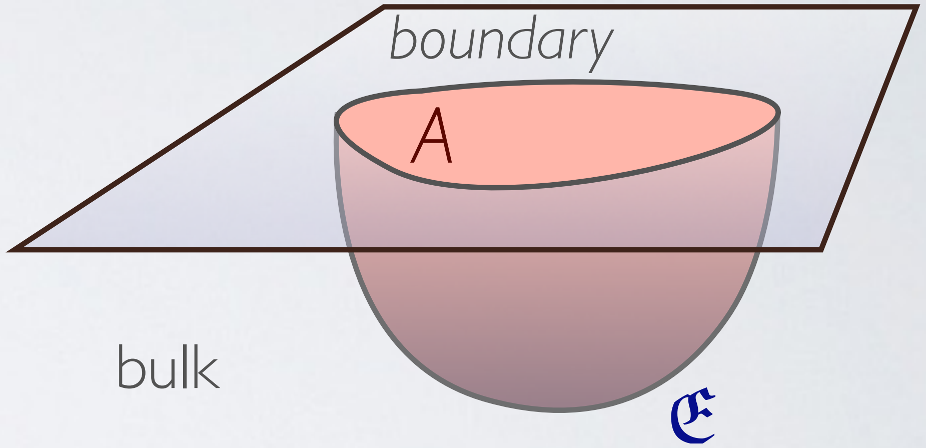


$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} = \alpha + \beta \geq S_{\mathcal{A}_1 \cup \mathcal{A}_2} + S_{\mathcal{A}_1 \cap \mathcal{A}_2}$$

Covariant Holographic EE

But the RT prescription is not well-defined outside the context of static configurations:

- In Lorentzian geometry, we can decrease the area arbitrarily by timelike deformations
- In time-dependent context, no natural notion of "const. t" slice…



In *time-dependent* situations, RT prescription must be covariantized:

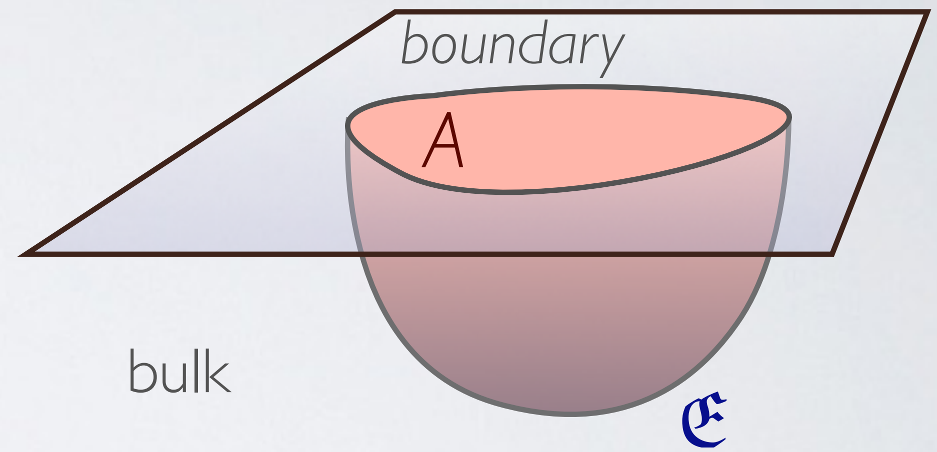
[VH, Rangamani, Takayanagi '07]

- ✴ minimal surface → extremal surface
- ✴ equivalently, $\mathfrak{E}$ is the surface with zero null expansions; (cf. light sheet construction [Bousso])
- ✴ equivalently, maximin construction: maximize over minimal-area surface on a spacelike slice [Wall]

Covariant Holographic EE

HRT Prescription: [VH, Rangamani, Takayanagi '07]

In the bulk EE $S_{\mathcal{A}}$ is captured by the area of extremal co-dimension 2 bulk surface $\mathfrak{E}$ anchored on $\partial\mathcal{A}$ & homologous to $\mathcal{A}$



$$S_{\mathcal{A}} = \min_{\partial\mathfrak{E}=\partial\mathcal{A}} \frac{\text{Area}(\mathfrak{E})}{4\,G_N}$$

This gives a well-defined quantity in any (arbitrarily time-dependent asymptotically AdS) spacetime ⇒ equally robust as in CFT

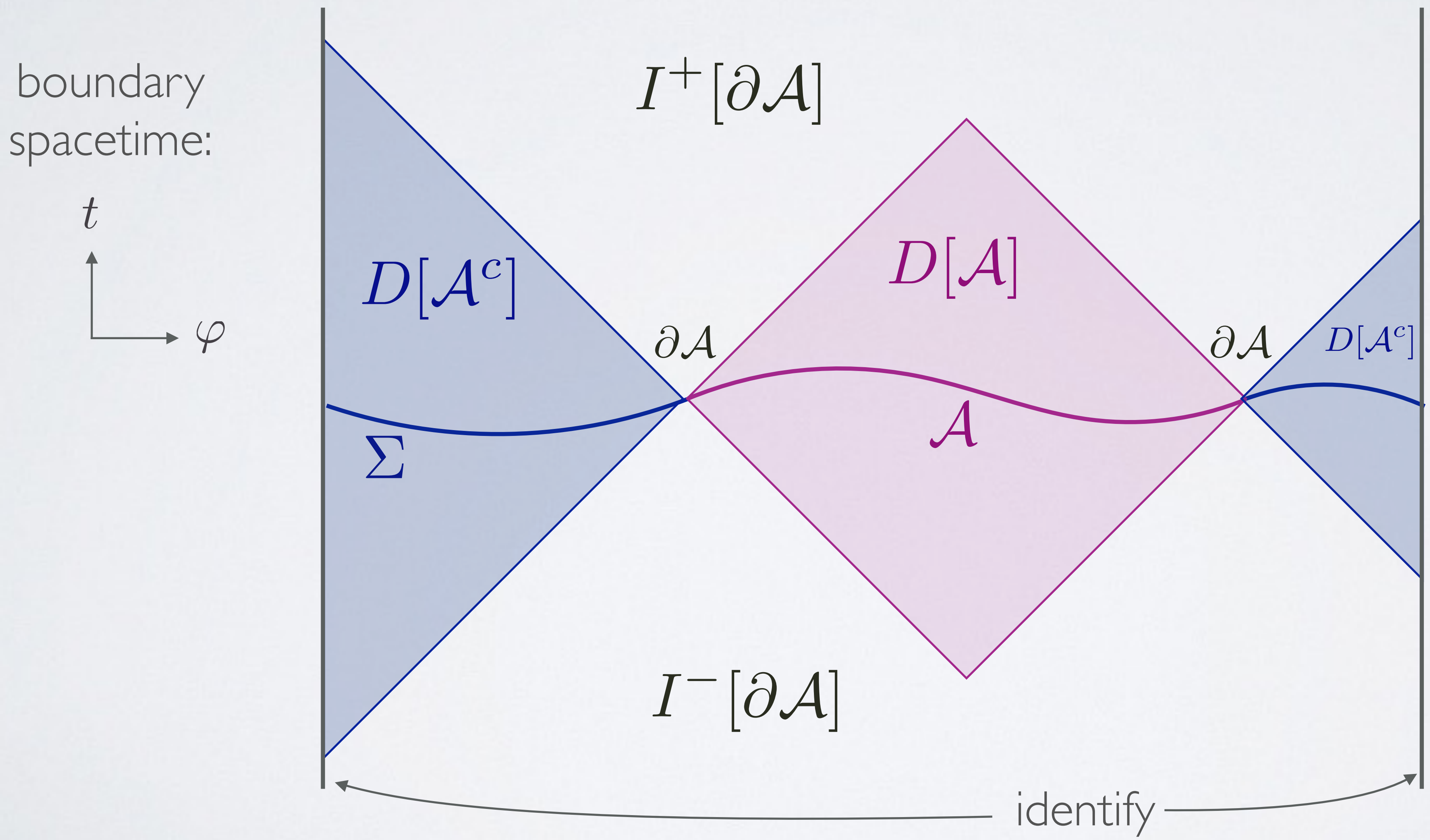
But we can't use Euclidean techniques for proof…

?: Is HRT prescription consistent with CFT constraints, e.g. causality?

CFT causal restriction

- Entanglement entropy $S_{\mathcal{A}}$ only depends on $D[\mathcal{A}]$ and not on Σ.
- Natural separation of boundary spacetime into 4 regions:

$$\partial\mathcal{M} = D[\mathcal{A}] \cup D[\mathcal{A}^c] \cup I^-[\partial\mathcal{A}] \cup I^+[\partial\mathcal{A}]$$



- EE should not be influenced by any change to state within $D[\mathcal{A}]$ or $D[\mathcal{A}^c]$.

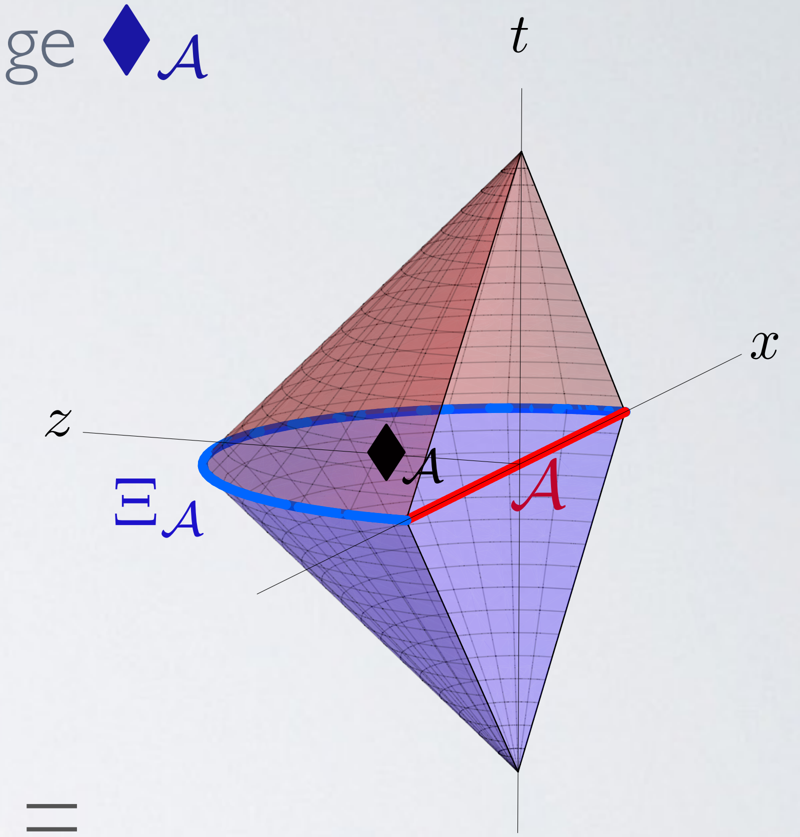
CFT causal requirement on bulk

- Extremal surface cannot lie within the bulk causal wedge $\blacklozenge_{\mathcal{A}}$

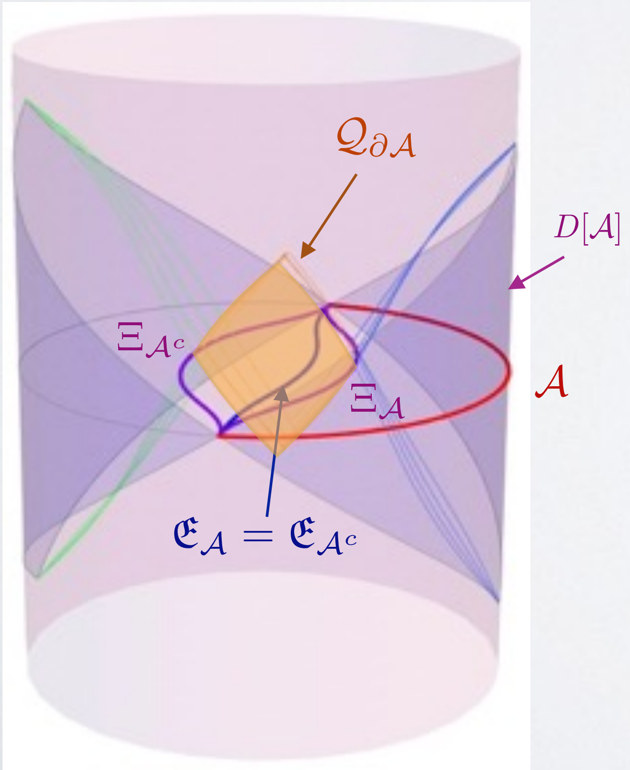
  $\blacklozenge_{\mathcal{A}} \equiv J^{-}[D[\mathcal{A}]] \cap J^{+}[D[\mathcal{A}]]$

  = { bulk causal curves which begin and end on $D[\mathcal{A}]$ }

  shown in [VH, Rangamani '12]



- In fact it must lie in the causal shadow $\mathcal{Q}_{\partial\mathcal{A}}$



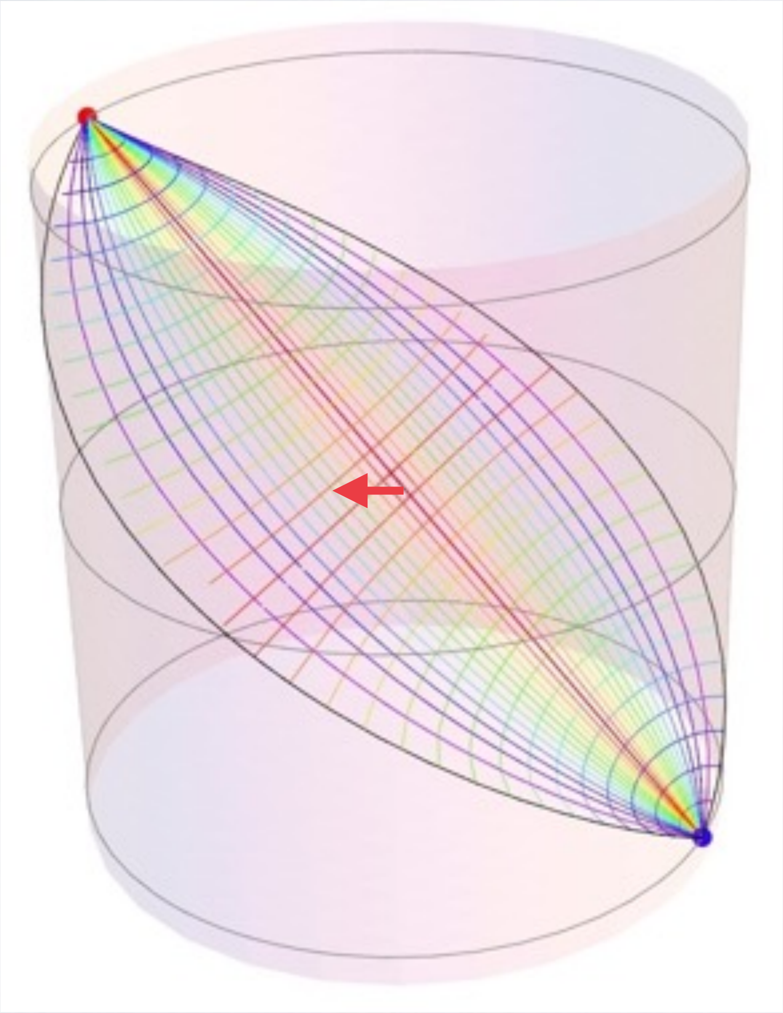
$\mathcal{Q}_{\partial\mathcal{A}}$ = causal shadow = bulk region which is causally disconnected from both $\mathcal{A}$ and $\mathcal{A}^c$

- Shown in [Headrick, VH, Lawrence, Rangamani '14]
- Non-trivial condition on holographic EE

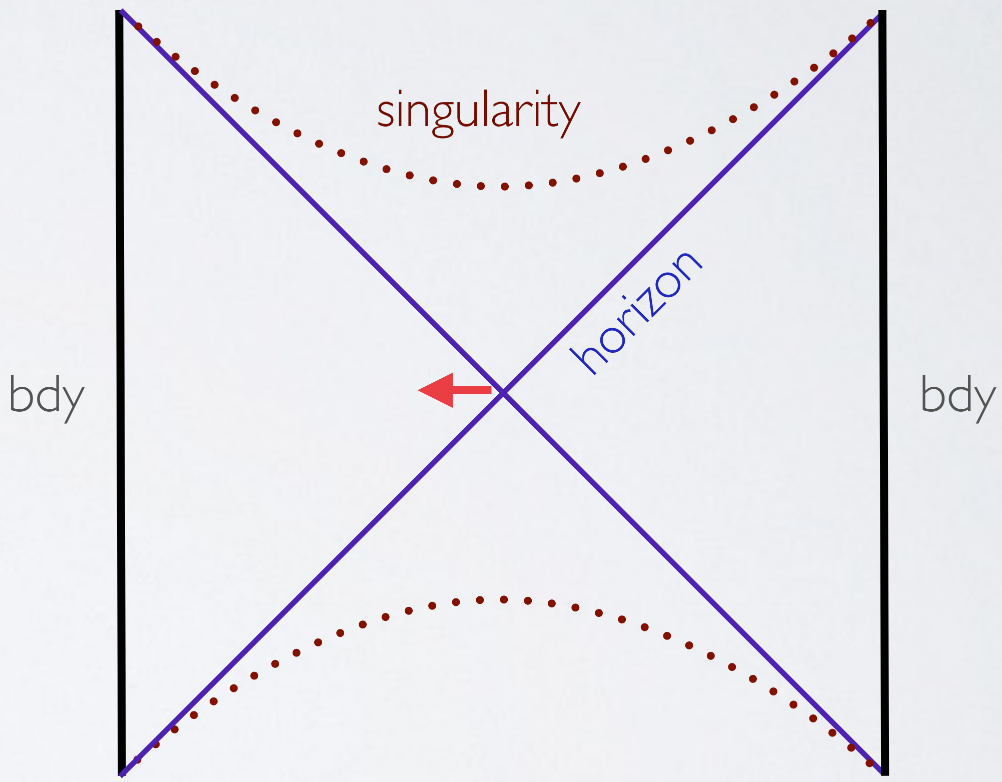
Marginal for static case…

- In static situations where RT applies, causality is upheld just marginally

pure AdS



Schwarzschild-AdS black hole

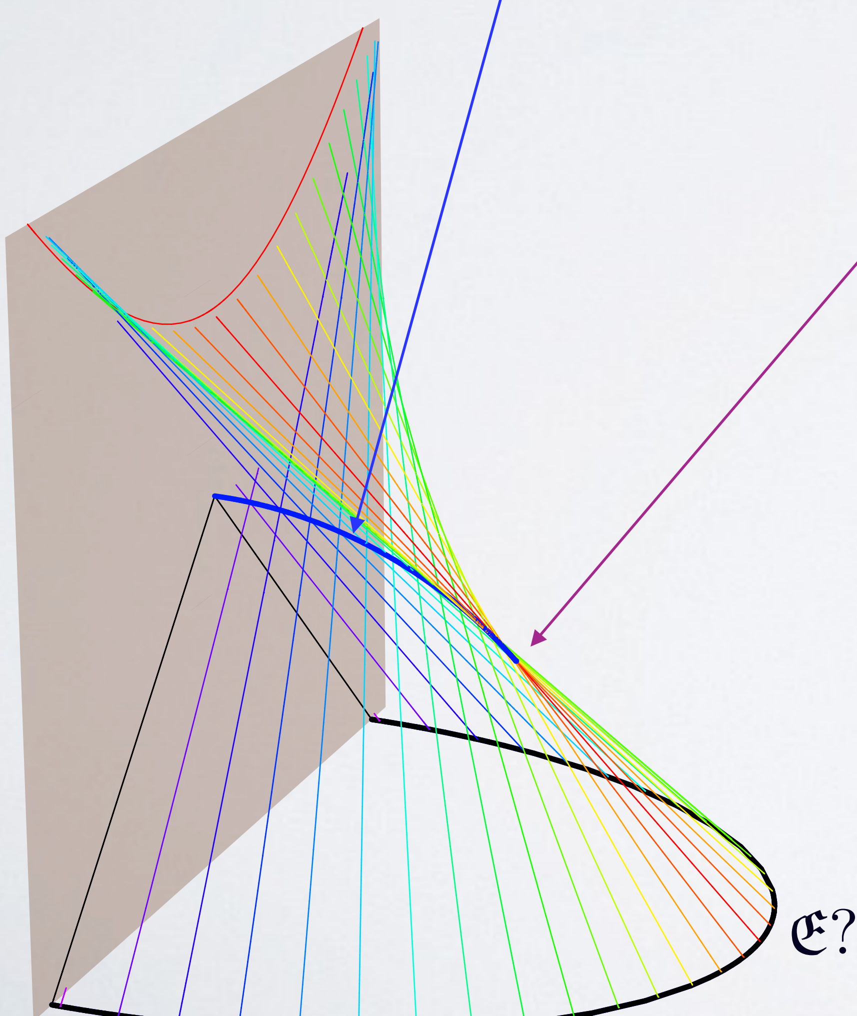


- Danger: arb. small deformation of extremal surface could violate causality!

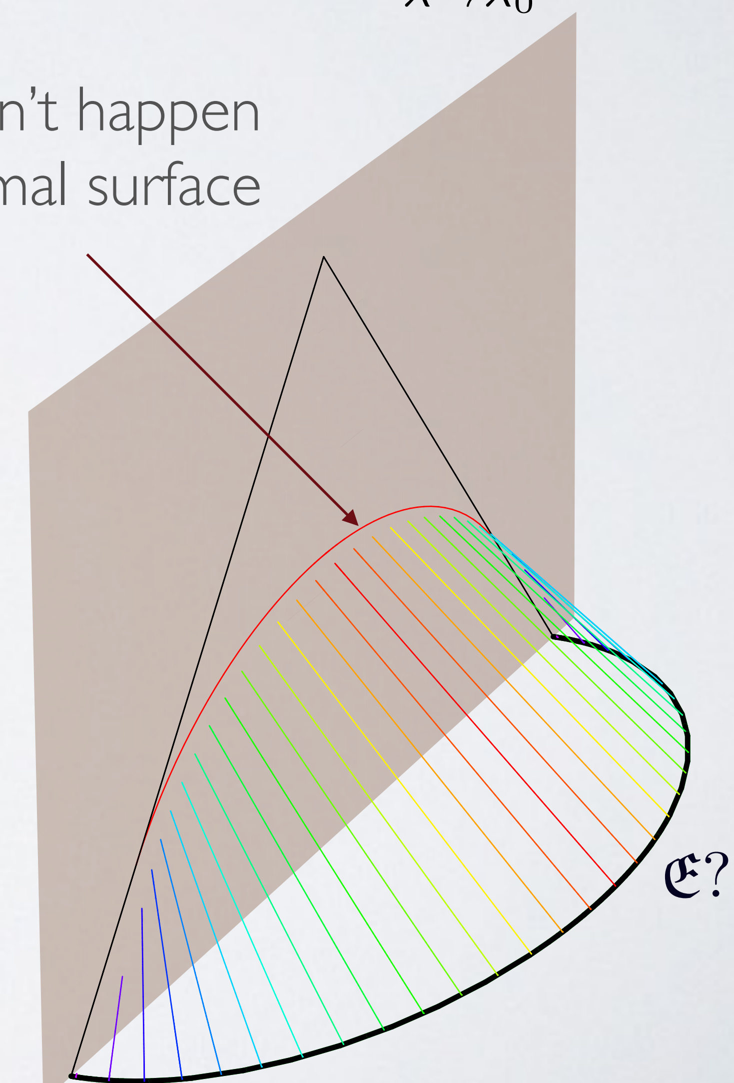
Structure of null congruences

- The proof of causality assumes NEC & uses structure of null geodesics.
- Only for special cases do null normals from $\mathfrak{E}$ reach boundary.
- In general, the generators terminate at a crossover seam (ending with a caustic).

$$\Theta_{\lambda=0} = 0 \;\Rightarrow\; \Theta_{\lambda>0} \leq 0$$
$$\Rightarrow\; \Theta_{\lambda\to\lambda_0} \to -\infty$$



i.e. this can't happen
for extremal surface



Utility of null congruences

- In generic Lorentzian spacetime, null congruences which define a causal set provide useful characterization of "natural" bulk regions.

2 options:

…starting from bdy:

$D[\mathcal{A}] \rightsquigarrow$ Causal Wedge: $\blacklozenge_{\mathcal{A}}$

…continued past Ξ: $\rightsquigarrow$ Causal Shadow

…starting from bulk:

$\mathfrak{E} \rightsquigarrow$ Entanglement Wedge: $\mathcal{W}_E[\mathcal{A}]$

- We can prove the inclusion property [Headrick, VH, Lawrence, Rangamani '14]

$$\blacklozenge_{\mathcal{A}} \subset \mathcal{W}_E[\mathcal{A}]$$

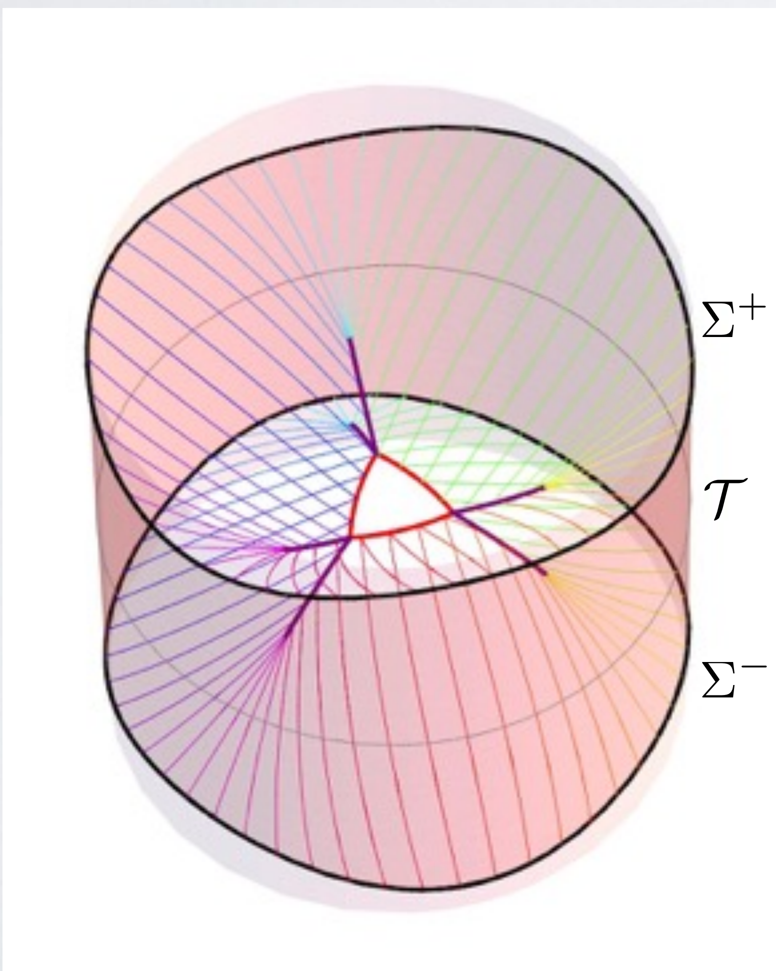
Generalization of inclusion property

Aside: cf. Covariant Residual Entropy proposal [VH,'14]

cf. differential entropy [Balasubramanian, Chowdhury, Czech, de Boer, & Heller, `13]

Generalization of Causal Wedge:

boundary $\mathcal{T}$ $\rightsquigarrow$ Strip Wedge:

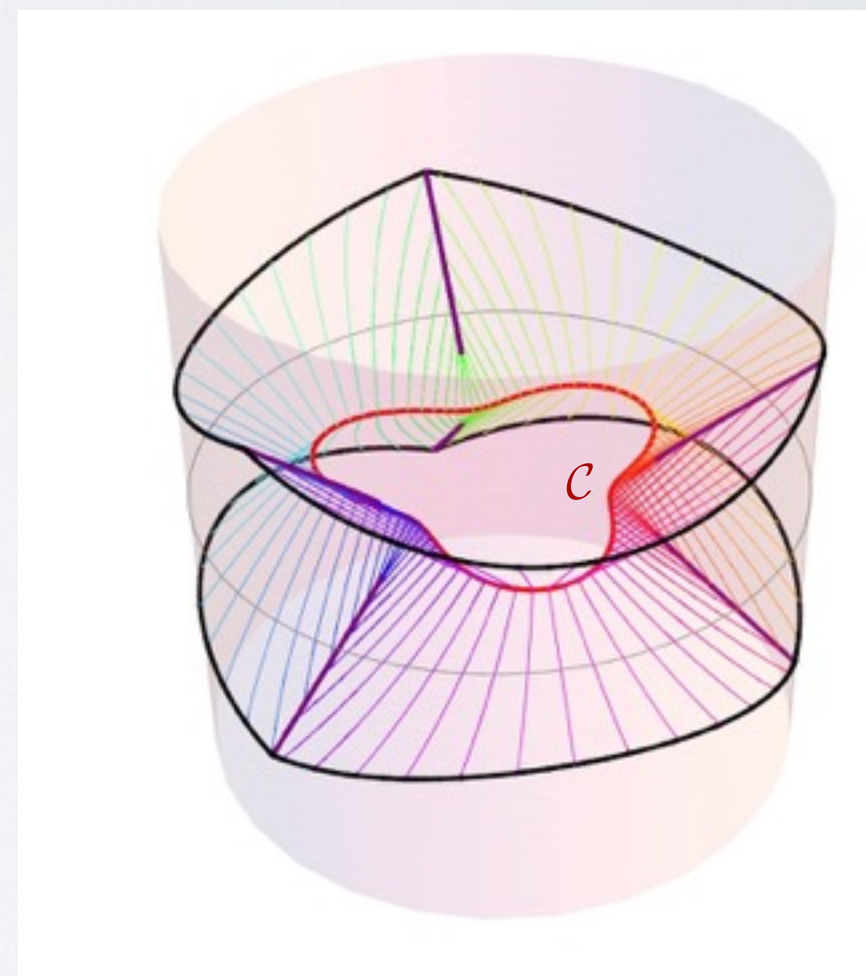


in reverse-construction:

$\subset$

Generalization of Entanglement Wedge:

bulk $\mathcal{C}$ $\rightsquigarrow$ Rim Wedge:



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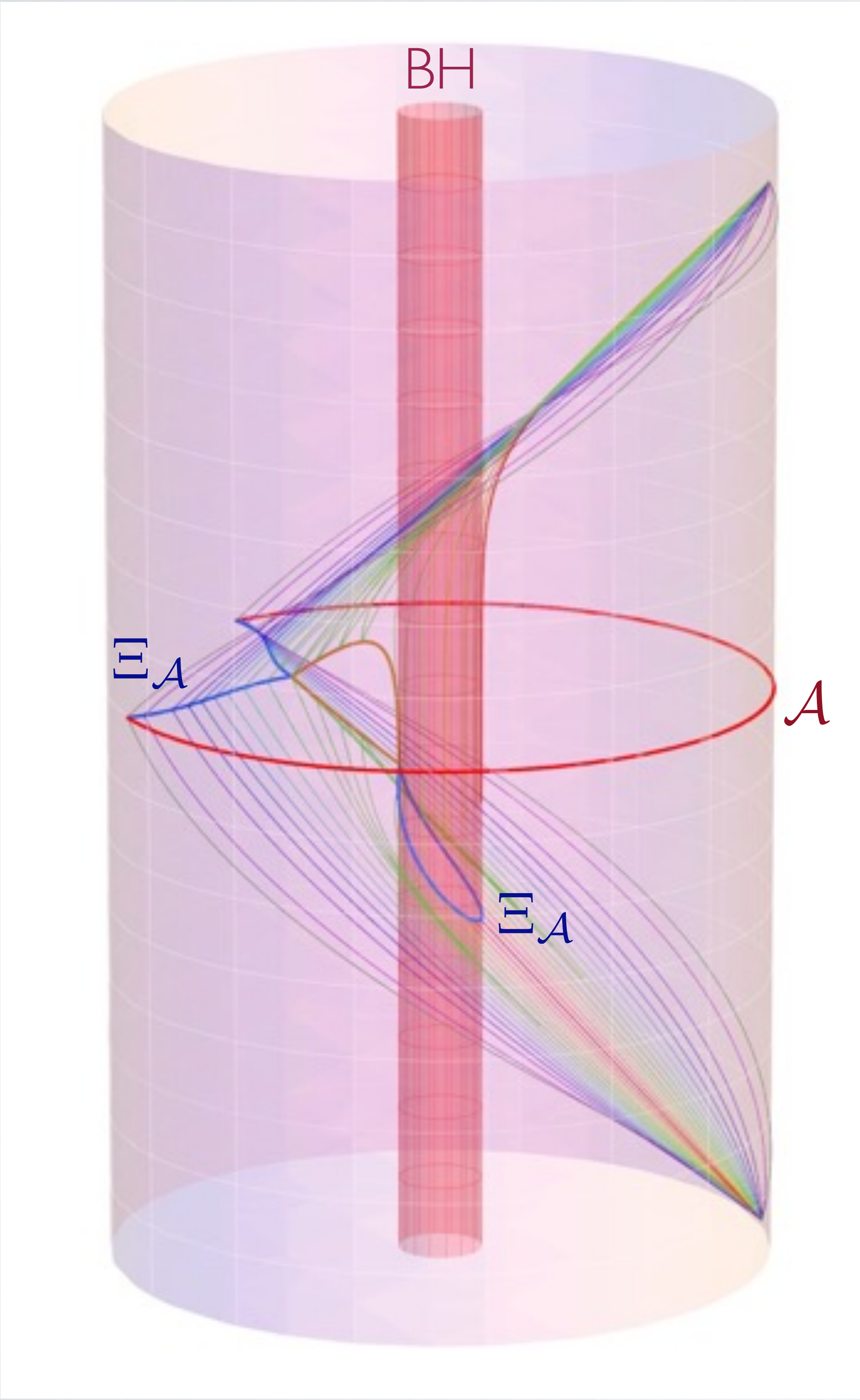
...continued past Ξ: $\rightsquigarrow$ Causal Shadow

- We can prove the inclusion property [Headrick, VH, Lawrence, Rangamani '14]

$$\blacklozenge_{\mathcal{A}} \subset \mathcal{W}_E[\mathcal{A}]$$

- Consequences:
  - HRT is consistent with CFT causality ✔
  - Entanglement plateaux [VH, Maxfield, Rangamani, Tonni, '13]

Entanglement plateaux from CW



- Causal wedge can have holes…
- Important implication for entanglement:
  - whenever $\mathcal{A}$ is large enough for $\Xi_{\mathcal{A}}$ to have two disconnected pieces, there cannot exist a single connected extremal (minimal) surface $\mathfrak{E}_{\mathcal{A}}$ homologous to $\mathcal{A}$!
  - in such cases, $\Rightarrow \; S_{\mathcal{A}} = S_{\mathcal{A}^c} + S_{\rm BH}$

    (saturates Araki-Lieb inequality)

    → *entanglement plateau*

    [VH, Maxfield, Rangamani, Tonni, '13]

    → two components to entanglement
  - Causal wedge argument guarantees this even for generic time-dependent BHs.

Entanglement wedge

- Boundary spacetime separation:

$$\partial\mathcal{M} = D[\mathcal{A}] \cup D[\mathcal{A}^c] \cup I^-[\partial\mathcal{A}] \cup I^+[\partial\mathcal{A}]$$

- This naturally induces a corresponding separation into 4 bulk regions:

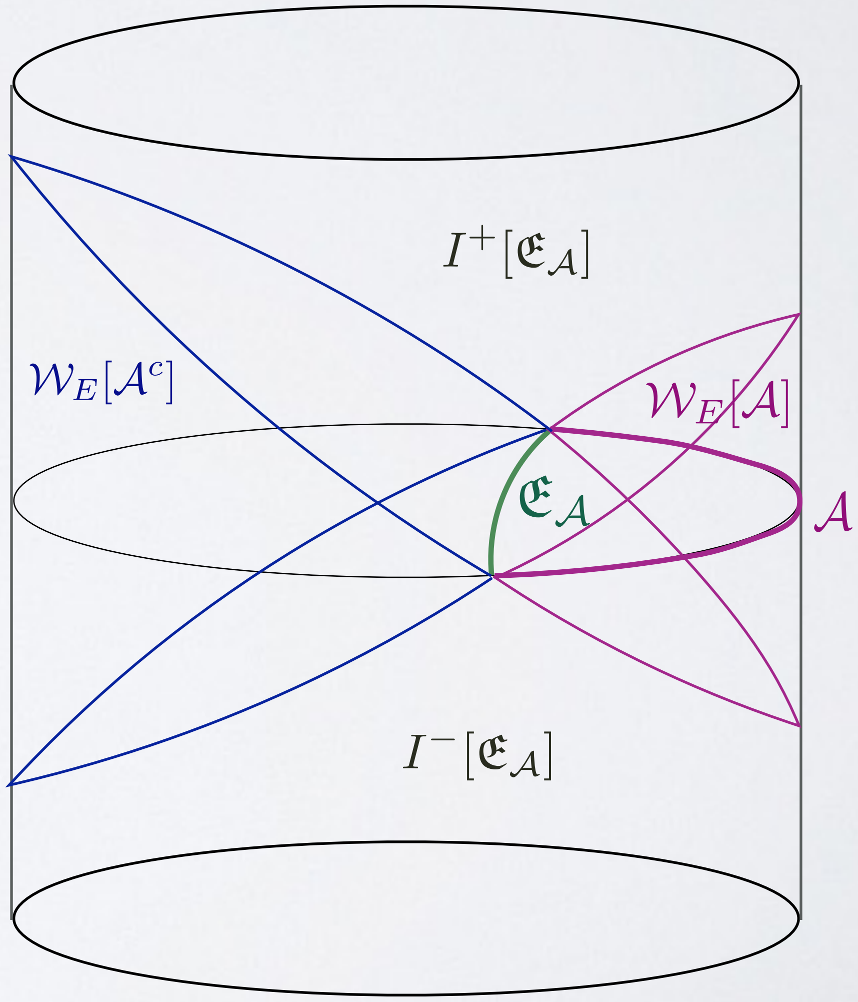
$$\mathcal{M} = \mathcal{W}_E[\mathcal{A}] \cup \mathcal{W}_E[\mathcal{A}^c] \cup I^-[\mathfrak{E}_\mathcal{A}] \cup I^+[\mathfrak{E}_\mathcal{A}]$$

(for pure state)

entanglement wedge of $\mathcal{A}$

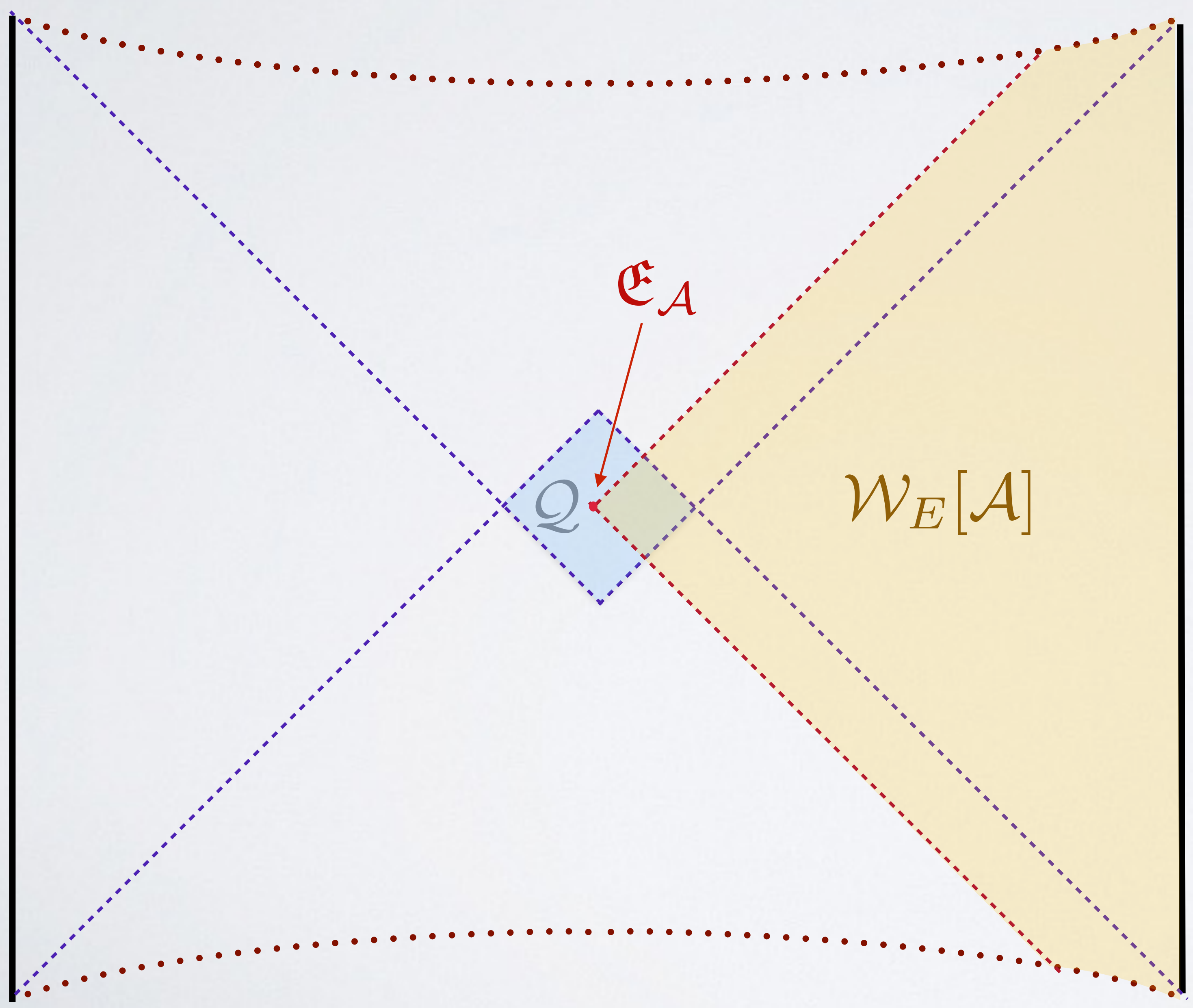
- $\mathcal{W}_E[\mathcal{A}]$ ends on $D[\mathcal{A}]$
- contains the causal wedge $\blacklozenge_\mathcal{A}$
- generated by null geodesics normal to $\mathfrak{E}_\mathcal{A}$

$\Rightarrow$ natural 'dual' of $\rho_\mathcal{A}$



Entanglement wedge in deformed SAdS

In deformed eternal Schw-AdS, (compact) extremal surface corresponding to $\mathcal{A} = \Sigma_L$ or $\mathcal{A} = \Sigma_R$ must lie in the 'shadow region' $\mathcal{Q}$



i.e. causally disconnected from both boundaries...

(for static Schw-AdS, shadow region = bifurcation surface)

⇒ Entanglement wedge extends past event horizon

Curious properties of EE:

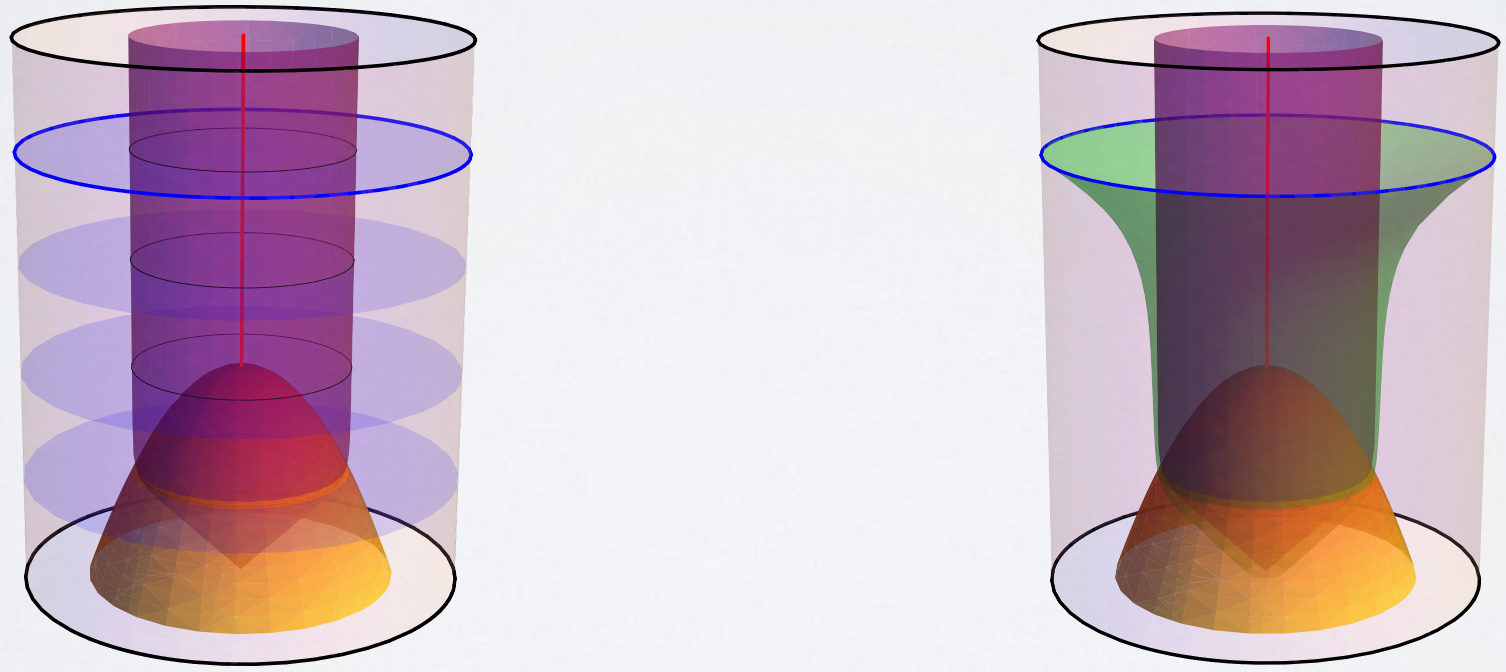
- EE satisfies nontrivial causality constraints
- Entanglement plateaux ($\delta S_{\mathcal{A}}$ saturates to S_{ρ_Σ} for large enough $\mathcal{A}$)
- EE has two separate components
- EE is a 'fine-grained' observable

These are all easy to see from the holographic dual!

EE is fine-grained observable!

Example: black hole formed from a collapse

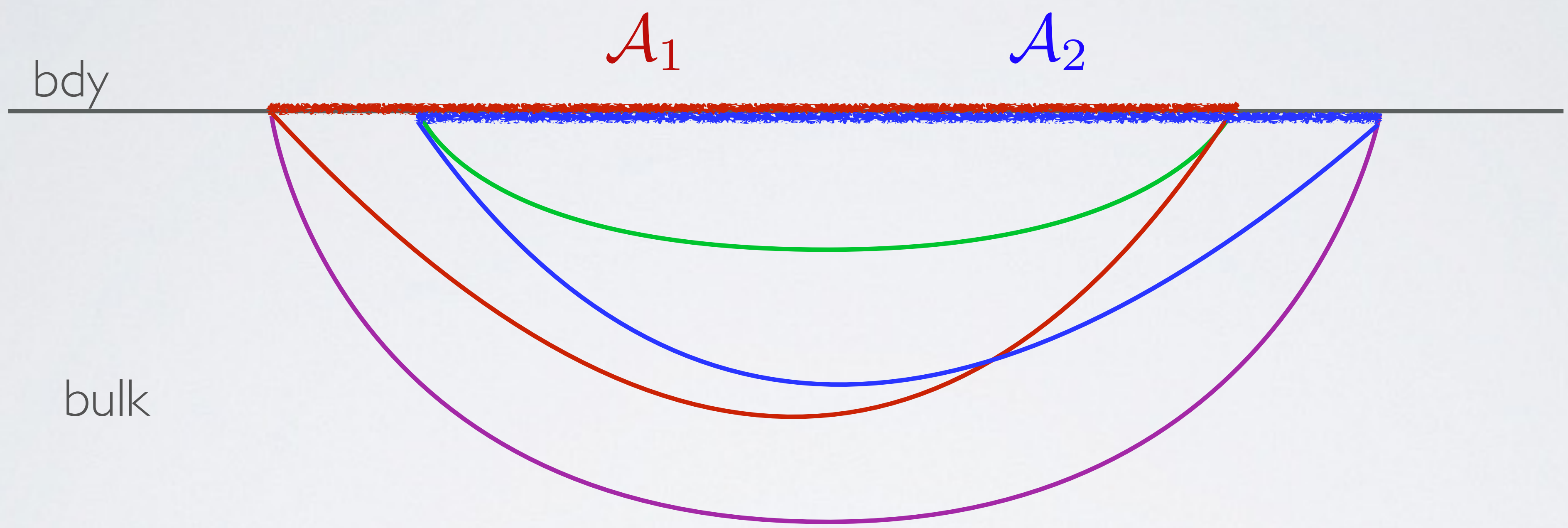
- In contrast to the static (i.e. eternal) black hole, for a collapsed black hole, there is no non-trivial homology constraint on extremal surfaces. [cf. Takayanagi & Ugajin]



- Hence we always have $S_{\mathcal{A}} = S_{\mathcal{A}^c}$ as for a pure state.

Role of SSA?

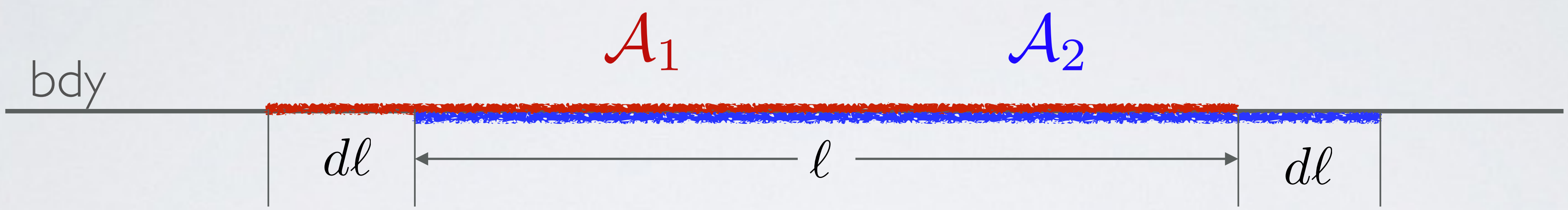
- Strong subadditivity: $S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \geq S_{\mathcal{A}_1 \cup \mathcal{A}_2} + S_{\mathcal{A}_1 \cap \mathcal{A}_2}$



- Easy to prove for RT [Headrick&Takayanagi], harder for HRT [Wall]
- Much harder in CFT directly!
- Profound property (cf. 2nd Law of Thermodynamics)

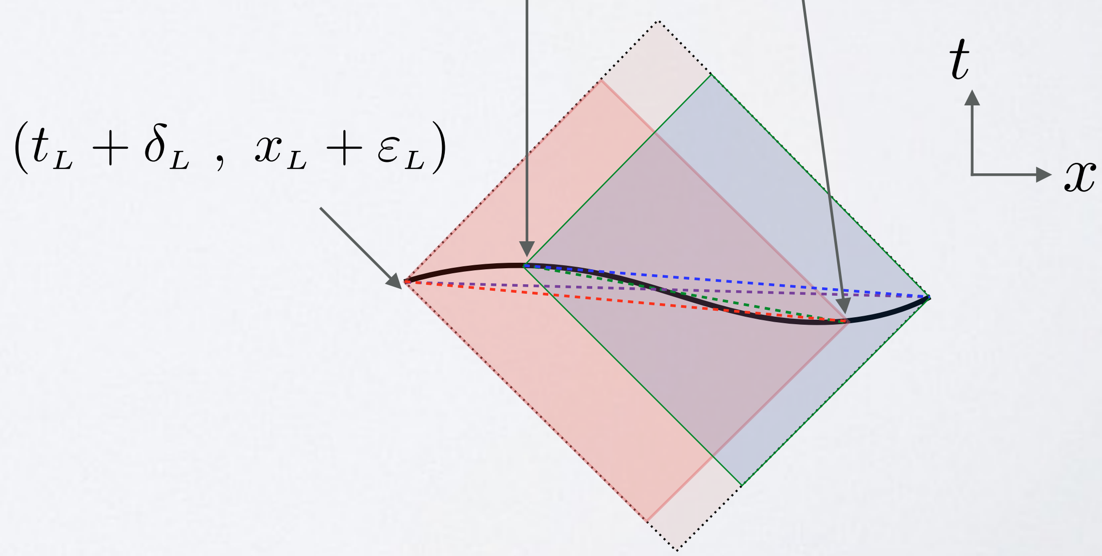
Role of SSA?

- Strong subadditivity: $S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \geq S_{\mathcal{A}_1 \cup \mathcal{A}_2} + S_{\mathcal{A}_1 \cap \mathcal{A}_2}$



- differential version of SSA (dSSA): e.g. $S''(\ell) \leq 0$
- we can generalize this to bi-local 2nd order differential expression

$$\left(\delta_{L}\, \partial_{t_L} + \varepsilon_{L}\, \partial_{x_L}\right) \left(\delta_{R}\, \partial_{t_R} + \varepsilon_{R}\, \partial_{x_R}\right) S(t_{L}, x_{L}, t_{R}, x_{R}) \leq 0$$



- SSA = convexity of EE
- dSSA ⇔ SSA

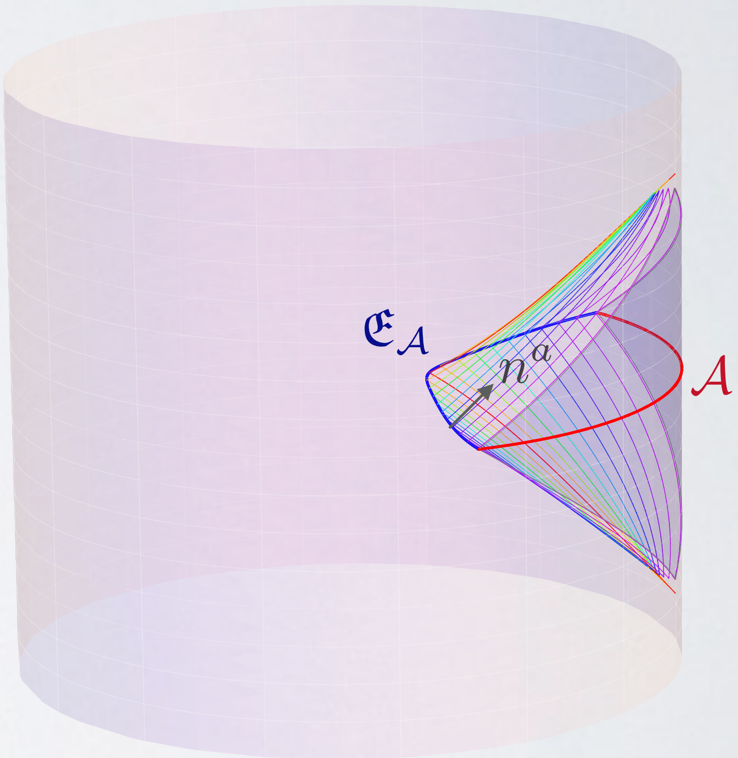
Bulk dynamics from EE?

- We can in principle decode the bulk geometry from $\{S_{\mathcal{A}}\}$ for a suitable set of $\mathcal{A}$'s.
- But can we extract bulk dynamics more directly?
  - Use the strong subadditivity property of EE:

$$\delta^2_{\mathcal{A}} S_{\mathcal{A}} \sim \int_{\mathfrak{E}_{\mathcal{A}}} E_{ab}\, n^a\, n^b \geq 0$$

cf. Null Energy Condition

specific 2nd order variation of region



  - proved at linearized level in 3-d, but conjectured to hold more generally…

[Bhattacharya, Hubeny, Rangamani, Takayanagi, '14]
cf. [Lashkari, Rabideau, Sabella-Garnier, Van Raamsdonk]

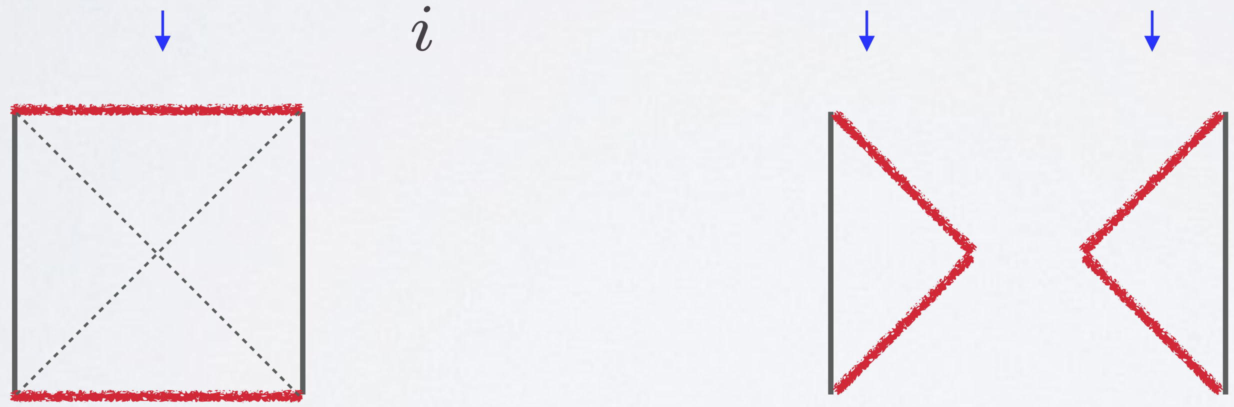
Spacetime from entanglement?

How does bulk spacetime emerge in the first place?

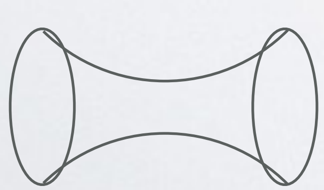
- Some connected spacetimes emerge as superpositions of disconnected spacetimes [Van Raamsdonk; Swingle]

eg. eternal AdS black hole as thermofield double:

$$|\psi\rangle = \sum_i e^{-\frac{\beta\, E_i}{2}}\, |E_i\rangle \otimes |E_i\rangle$$



- Entanglement builds bridges: ‘ER = EPR’ [Maldacena, Susskind]



Einstein-Rosen bridge

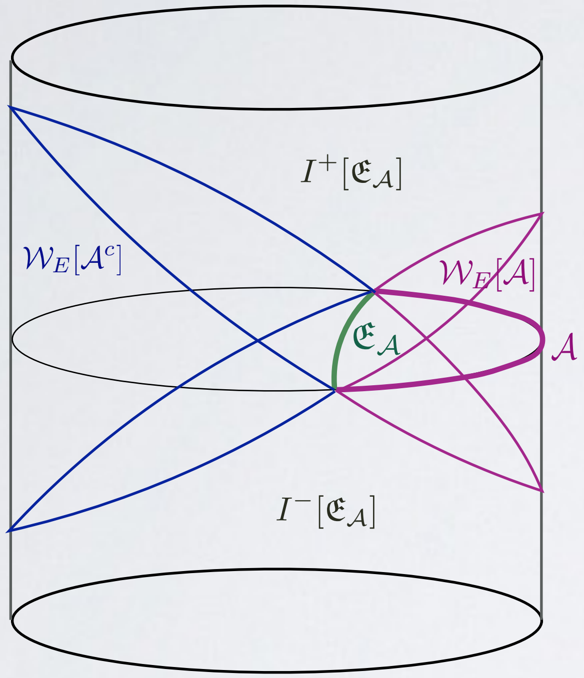
Einstein-Podolsky-Rosen entanglement

Summary & Outlook

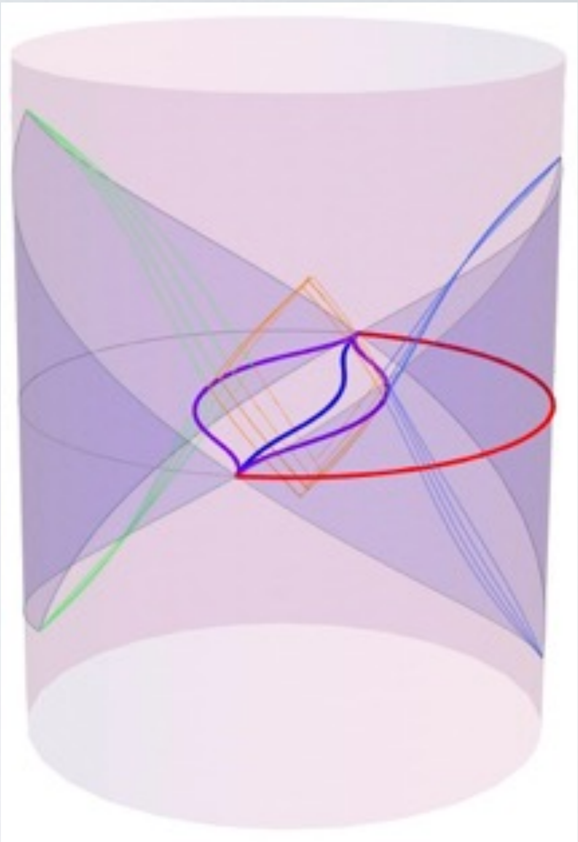
- General covariance is a powerful guiding principle
  - Motivated entanglement wedge, causal wedge, …
  - (In what sense) is entanglement wedge the 'dual' of $\rho_{\mathcal{A}}$?
  - What is the CFT dual of causal wedge (from first principles) & causal holographic information χ?

- HRT construction nontrivially upholds CFT causality & SSA
  - Can we prove HRT directly?
  - Does dSSA determine HRT?

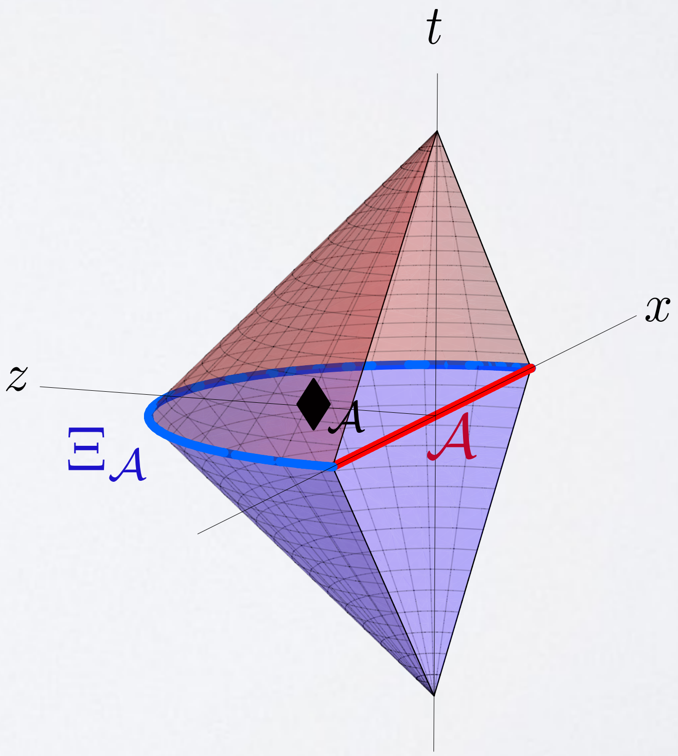
- SSA plays important role in holography
  - How constraining is dSSA on bulk geometry?
  - How constraining is dSSA on bulk EoMs?



$I^+[\mathfrak{E}_\mathcal{A}]$
$\mathcal{W}_E[\mathcal{A}^c]$
$\mathcal{W}_E[\mathcal{A}]$
$\mathfrak{E}_\mathcal{A}$
$\mathcal{A}$
$I^-[\mathfrak{E}_\mathcal{A}]$



Thank you



t
x
z
$\blacklozenge_\mathcal{A}$
$\Xi_\mathcal{A}$
$\mathcal{A}$