Scattering in Susy large N Chern Simons Theory

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References

Based on

- K. Inbasekar, S. Jain, S. Mazumdar, S.M. V. Umesh, S. Yokoyama: to appear soon

Related earlier work

- S. Jain, M. Manglik, S.M. , S. Wadia, T. Takimi, S. Yokoyama : ArXiv 1404.6373
- Y. Dandekar, M. Manglik, S.M. ArXiv 1407.1322

Introduction

- Last three years; fairly intensive study of $U(N)$ Chern Simons theories with fundamental matter.
- Initial motivations: AdS/CFT. Limit of ABJ theories. Vasiliev Duality.
- Field theories turn out to be solvable in the large N limit. Effectively vector like (Chern Simons has matrices but with no degrees of freedom).
- Detailed study has thrown up surprises. First strong weak (Bose-Fermi) duality even without susy. Second, many unusual aspects of dynamics.

Introduction

- In particular: recent intensive study of 2×2 scattering in purely bosonic and purely fermionic theories.
- Results in perfect agreement with duality. However conflict with unitarity unless we assume two modified structural features of S matrices. First: delta function term at forward scattering. Second, modifed rules of crossing symmetry.
- This talk: present explicit computations for the in S matrix of the most general renormalizable $\mathcal{N} = 1$ SUSY Chern Simons theory at all orders in the t' Hooft coupling.
- Will find results in perfect agreement with duality. Unitarity requires the *same* delta function piece at forward scattering and modification of crossing symmetry conjectured for the bosonic and fermionic theories.
- Strong evidence for universality. Generalization to higher susy (ABJ) underway. Expect same features. Along the way make precise expression of the Bose Fermi nature of Giveon Kutasov dualities.

Theories

$$S = -\int d^3x d^2\theta \Bigg[\frac{\kappa}{2\pi} Tr\left(-\frac{1}{4}D_\alpha\Gamma^\beta D_\beta\Gamma^\alpha - \frac{1}{6}D^\alpha\Gamma^\beta\{\Gamma_\alpha,\Gamma_\beta\} - \frac{1}{24}\{\Gamma^\alpha,\Gamma^\beta\}\{\Gamma_\alpha,\Gamma_\beta\}\right) - \frac{1}{2}(D^\alpha\bar{\Phi} + i\bar{\Phi}\Gamma^\alpha)(D_\alpha\Phi - i\Gamma_\alpha\Phi) + m_0\bar{\Phi}\Phi + \frac{\pi w}{\kappa}(\bar{\Phi}\Phi)^2\Bigg], \tag{1}$$

- Gauge multiplet Γ_α. Single fundamental matter multiplet ϕ.
- Parameters: N, κ (integers).w (dimensionless). m_0 (dimensionful). New effectively continuous dimensionless parameter $\lambda = \frac{N}{\kappa}$ in the large N limit.

Conjectured Duality

- Motivated by computations of the thermal partition function on $S^2 \times S^1$, about two years ago Jain, Yokoyama and myself conjectured that this class of theories is self dual.
- Conjectured dualiy map as follows. Define

$$\lambda' = \lambda - \mathrm{Sgn}(\lambda)\ ,\ \ w' = \frac{3-w}{1+w} \quad m_0' = \frac{-2m_0}{1+w}\ . \tag{2}$$

- $$N' = |\kappa| - N + 1, \quad \kappa' = -\kappa$$

  Generalization of level rank duality.
- Turns out pole mass (see below)

$$m = \frac{2m_0}{2 + (-1+w)\lambda\ \mathrm{Sgn(m)}}\ . \tag{3}$$

  Under duality

$$m' = -m.$$

Scattering amplitudes and the duality map

- Would like to understand duality in more detail. Ideally to construct ϕ^D in terms of ϕ. Question imprecise as ϕ^D and ϕ are not gauge invariant.
- While arbitrary insertions of ϕ and ϕ^D are not meaningful, insertions that are taken to infinity along lines of equations of motion - i.e. S matrices - are gauge invariant and well defined. For this reason the map between S matrices is a 'poor mans bosonization' map between these two theories.
- This talk: We indpependently compute the S matrix for ϕ and ϕ^D and examine their interrelationship under duality. Also uncover interesting structural features along the way.

S matrices: Susy

- Solution

$$\Phi(x,\theta) = \int \frac{d^2p}{\sqrt{2p^0(2\pi)^2}} \Big[\Big(a(\mathbf{p})(1 + m\theta^2) + \theta^\alpha u_\alpha(\mathbf{p})\alpha(\mathbf{p}) \Big) e^{ip.x} + \Big(a^{c\dagger}(\mathbf{p})(1 + m\theta^2) + \theta^\alpha v_\alpha(\mathbf{p})\alpha^{c\dagger}(\mathbf{p}) \Big) e^{-ip.x} \Big]$$

- Susy

$$-iQ_\alpha = u_\alpha(\mathbf{p}_i)\left(a\partial_\alpha + a^c\partial_{\alpha^c}\right) + u^*_\alpha(\mathbf{p}_i)\left(-\alpha\partial_a + \alpha^c\partial_{a^c}\right) + v_\alpha(\mathbf{p}_i)\left(a^\dagger\partial^\dagger_\alpha + (a^c)^\dagger\partial_{(\alpha^c)^\dagger}\right) + v^*_\alpha(\mathbf{p}_i)\left(\alpha^\dagger\partial^\dagger_a + (\alpha^c)^\dagger\partial_{(a^c)^\dagger}\right) \quad (4)$$

On Shell Superfield

- $$A_i(\mathbf{p}) = a_i(\mathbf{p}) + \alpha_i(\mathbf{p})\theta_i$$
  $$A_i^\dagger(\mathbf{p}) = a_i^\dagger(\mathbf{p}) + \theta_i\alpha_i^\dagger(\mathbf{p}) \qquad (5)$$

- $$[Q_\alpha, A_i(\mathbf{p}_i, \theta_i)] = Q_\alpha^1 A_i(\mathbf{p}_i, \theta_i)$$
  $$[Q_\alpha, A_i^\dagger(\mathbf{p}_i, \theta_i)] = Q_\alpha^2 A_i^\dagger(\mathbf{p}_i, \theta_i) \qquad (6)$$

- $$Q_\beta^1 = i\left(-u_\beta(\mathbf{p})\frac{\overrightarrow{\partial}}{\partial\theta} - v_\beta(\mathbf{p})\theta\right)$$
  $$Q_\beta^2 = i\left(v_\beta(\mathbf{p})\frac{\overrightarrow{\partial}}{\partial\theta} - u_\beta(\mathbf{p})\theta\right) \qquad (7)$$

Dual Supersymmetry

- Conjectured duality transformation: bose-fermi interchange. $a \leftrightarrow \alpha$
- Action of dual susy on $A, A^\dagger$

$$
\begin{aligned}
Q^1_\beta &= i\left(-v_\beta(\mathbf{p},-m)\theta - u_\beta(\mathbf{p},-m)\frac{\overrightarrow{\partial}}{\partial\theta}\right) \\
Q^2_\beta &= i\left(u_\beta(\mathbf{p},-m)\theta - v_\beta(\mathbf{p},-m)\frac{\overrightarrow{\partial}}{\partial\theta}\right)
\end{aligned}
\tag{8}
$$

- Using

$$u(m,p) = -v(-m,p), \quad v(m,p) = -u(m,p)$$

we see that the dual and original susy generators are proportional to each other. So an S matrix invariant under original susy is automatically invariant under dual susy. Susy compatable with Bose-Fermi interchange.

S matrix in onshell superspace

- Consider the S matrix

$$S(\mathbf{p}_1,\theta_1,\mathbf{p}_2,\theta_2,\mathbf{p}_3,\theta_3,\mathbf{p}_4,\theta_4)(2\pi)^3\delta(p^1+p^2-p^3-p^4) = \langle 0|A_4(\mathbf{p}_4,\theta_4)A_3(\mathbf{p}_3,\theta_3)UA_2^\dagger(\mathbf{p}_2,\theta_2)A_1^\dagger(\mathbf{p}_1,\theta_1)|0\rangle \tag{9}$$

- Invariance under susy implies

$$\Big(\overrightarrow{Q}^1_\alpha(\mathbf{p}_1,\theta_1)+\overrightarrow{Q}^1_\alpha(\mathbf{p}_2,\theta_2) + \overrightarrow{Q}^2_\alpha(\mathbf{p}_3,\theta_3)+\overrightarrow{Q}^2_\alpha(\mathbf{p}_4,\theta_4)\Big)S(\mathbf{p}_1,\theta_1,\mathbf{p}_2,\theta_2,\mathbf{p}_3,\theta_3,\mathbf{p}_4,\theta_4)=0 \tag{10}$$

- We have explicitly solved (10); the solution is given by

$$
\begin{aligned}
&S(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = f_1 + F_2\, \theta_1\theta_2\theta_3\theta_4 + \\
&\left(\frac{1}{2}C_{12} f_1 - \frac{1}{2}C^*_{34} F_2\right)\, \theta_1\theta_2 \\
&+ \left(\frac{1}{2}C_{13} f_1 - \frac{1}{2}C^*_{24} F_2\right)\, \theta_1\theta_3 + \left(\frac{1}{2}C_{14} f_1 + \frac{1}{2}C^*_{23} F_2\right)\, \theta_1\theta_4 \\
&+ \left(\frac{1}{2}C_{23} f_1 + \frac{1}{2}C^*_{14} F_2\right)\, \theta_2\theta_3 + \left(\frac{1}{2}C_{24} f_1 - \frac{1}{2}C^*_{13} F_2\right)\, \theta_2\theta_4 \\
&+ \left(\frac{1}{2}C_{34} f_1 - \frac{1}{2}C^*_{12} F_2\right)\, \theta_3\theta_4
\end{aligned}
\tag{11}
$$

Solution to susy equation

- Where

$$
\begin{aligned}
\frac{1}{2}C_{12} &= -\frac{1}{4m}v^*(\mathbf{p}_1)v^*(\mathbf{p}_2) & \frac{1}{2}C_{23} &= -\frac{1}{4m}v^*(\mathbf{p}_2)u^*(\mathbf{p}_3) \\
\frac{1}{2}C_{13} &= -\frac{1}{4m}v^*(\mathbf{p}_1)u^*(\mathbf{p}_3) & \frac{1}{2}C_{24} &= -\frac{1}{4m}v^*(\mathbf{p}_2)u^*(\mathbf{p}_4) \\
\frac{1}{2}C_{14} &= -\frac{1}{4m}v^*(\mathbf{p}_1)u^*(\mathbf{p}_4) & \frac{1}{2}C_{34} &= -\frac{1}{4m}u^*(\mathbf{p}_3)u^*(\mathbf{p}_4)
\end{aligned}
\tag{12}
$$

- Note that susy determines 6 of the 8 scattering amplitudes in terms of the other two. Indpendent data = four boson and four fermion scattering.

Scattering Amplitudes: Colour Kinematics

- We refer to a quantum (bosonic or ferionic) that transforms in the fundamental of $U(N)$ as a particle and a quantum that tranforms in the antifundamental of $U(N)$ as an antiparticle.
- We study the most general 2×2 scattering process. There are three such processes

$$
\begin{aligned}
&P_i + P_m \to P_j + P_n \\
&P_i + A^j \to P_n + A^n \\
&A^j + A^n \to A^i + A^j
\end{aligned}
\tag{13}
$$

- All these S matrices are contained in appropriate on shell limits of the single correlator

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >$$

Color Kinematics

- It follows from $U(N)$ invariance that

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >= a\delta_i^j \delta_m^n + b\delta_i^n \delta_m^j$$

- Consequently, in each channel we need to compute 2 distinct scattering amplitudes. For particle-particle scattering it is convenient to work in the following basis for $U(N)$ singlets

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >= T_{sym} \left(\delta_i^j \delta_m^n + \delta_i^n \delta_m^j \right) + T_{as} \left(\delta_i^j \delta_m^n - \delta_i^n \delta_m^j \right)$$

$T_{Sym} = \frac{a+b}{2}$ is the S matrix in the channel of symmetric exchange, while $T_{as} = \frac{a-b}{2}$ is the S matrix in the channel of antisymmetric exchange.

Colour Kinematics, $\frac{1}{N}$

- On the other hand for particle - antiparticle exchange the following basis is most useful

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >= T_{Sing} \frac{\delta_i^j \delta_m^n}{N} + T_{Adj} \left(\delta_i^n \delta_m^j - \frac{\delta_i^j \delta_m^n}{N} \right)$$

  $T_{Sing} = Na + b$ is the scattering matrix in the singlet channel, while $T_{Adj} = b$ is the scattering matrix in the adjoint channel.

- It is not difficult to convince oneself that, at leading order in the $\frac{1}{N}$ expansion

$$a \sim b \sim T_{sym} \sim T_{as} \sim T_{Adj} \sim \frac{1}{N}$$

  On the other hand

$$S_{Sing} \sim \mathcal{O}(1)$$

- We have chosen the colour structures multiplying T_{Sing} etc to be simply projectors onto the exchange representations. As projectors square to unity it follows from unitarity that

$$i(T_c - T_c^\dagger) = T_c T_c^\dagger$$

T_c is any one of the four T matrices described above. We have used the fact that e.g. $2 \to 4$ production contributes to unitarity only at subleading order in $\frac{1}{N}$.

- The unitarity equation is rather trivial for T_{sym}, T_{as}, T_{adj}, as the RHS is subleading in $\frac{1}{N}$. However it imposes a highly nontrivial nonlinear constraint on T_{sing}, the most nontrivial of the four scattering matrices.

Scattering: non relativistic Limit

- The S matrix for non relativistic particles interacting via Chern Simons exchange was worked out in the early 90s, most notably by Bak, Jackiw and collaborators.
- The main result is strikingly simple. Consider the scattering of two bosonic particles in representations R_1 and R_2, in exchange channel R. It was demonstrated that the S matrix equals the scattering matrix of a $U(1)$ charged particle of unit charge scattering off a point like flux tube of magnetic field strength $\nu = \frac{c_2(R_1)+c_2(R_2)-c_2(R)}{\kappa}$.
- This quantum mechanical S matrix was computed originally by Aharonov and Bohm and generalized by Bak and Camillio to take account of possible point like interactions between the scattering particles.

Non Relativistic Scattering Amplitude

- $$T_{NR} = -16\pi i c_B \left(\cos\left(\pi\nu\right) - 1\right)\delta(\theta) + 8ic_B \sin(\pi\nu)\mathrm{Pv}\left(\cot\frac{\theta}{2}\right)$$
$$+ 8c_B|\sin\pi\nu|\frac{1 + e^{i\pi|\nu|}\frac{A_{NR}}{k^{2|\nu|}}}{1 - e^{i\pi|\nu|}\frac{A_{NR}}{k^{2|\nu|}}},$$
$$A_{NR} = \frac{-1}{w}\left(\frac{2}{R}\right)^{2|\nu|}\frac{\Gamma(1+|\nu|)}{\Gamma(1-|\nu|)}. \quad (14)$$

- Here $wR^{|}2\nu|$ is a measure of the strength of the Bak-Camillio contact interaction between the scattering particles. In the limit $w(kR)^{|}2\nu| \to 0$ $A_{NR} \to \infty$ and the second line of the scattering amplitude simplifies to
$$8c_B|\sin\pi\nu|,$$
the original Aharonov Bohm result.

The δ function and its physical interpretation

- The non relativistic amplitude above has a very unusual feature, a piece in the scattering amplitude proportional to the $\delta(\theta)$.
- This term in the scattering amplitude was missed in the original paper of Aharonov and Bohm. The amplitude was corrected with the addition of this piece in the early 80s. In the early 90s Jackiw and collabortors emphasized that is term is necessary to unitarize Aharonov Bohm scattering.
- Infact this term has a simple physical interpretation . The physical interpretation makes no reference to the non relativistic limit, so we assume that the δ function piece is present unmodified even in relativistic scattering.

Effective value of ν

- In the large N limit there is a simple formula for the quadratic Casimir of representations with a finite number of boxes plus a finite number of antiboxes.
$$c_2(R) = \frac{N(n_b + n_a)}{2}$$

- using this result in the formula for ν we find
$$\nu_{sym} \sim \nu_{as} \sim \nu_{Adj} \sim \mathcal{O}(1/N)$$
On the other hand
$$\nu_{Sing} = \lambda$$

- Now the non relativistic limit is obtained by taking $k \to 0$ at fixed ν. If $\nu \sim \mathcal{O}(1/N)$ and if we work to leading order in $\frac{1}{N}$ we effectively take $\nu \to 0$ first. In other words the results of Aharonov Bohm Bak Camillio yield a sharp prediction for the non relativistic limit only of T_{sing}.

Exact Propagators: method

- Work in Lorentzian space. Set gauge $\Gamma_- = 0$. In particular implies $A_- = 0$. Supersymmetric generalization of lightcone gauge. Main advantage: No gauge boson self interactions.
- If we want to understand scattering we first have understand free propagation.
- No gauge self interactions plus planarity gives a simple Schwinger Dyson equation (like t' Hooft model). Nonlinear integral equation. Quite remarkably admits simple exact solution

Bare Propagators

- The bare scalar superfield propagator in momentum space is

$$\langle \bar{\Phi}(\theta_1, p)\Phi(\theta_2, -p')\rangle = \frac{D^2_{\theta_1,p} - m_0}{p^2 + m_0^2}\delta^2(\theta_1 - \theta_2)(2\pi)^3\delta^3(p - p') \,. \tag{15}$$

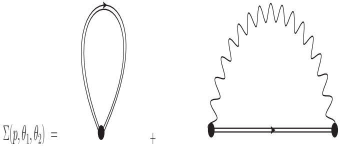
  where m_0 is the bare mass

- The gauge superfield propagator in momentum space is

$$\langle \Gamma^-(\theta_1, p)\Gamma^-(\theta_2, -p')\rangle = -\frac{8\pi}{\kappa}\frac{\delta^2(\theta_1 - \theta_2)}{p_{--}}(2\pi)^3\delta^3(p - p') \,, \tag{16}$$

Equation for Exact Propagator

- Figure : Integral equation for self energy



- $$\Sigma(p,\theta_1,\theta_2) = 2\pi\lambda w \int \frac{d^3r}{(2\pi)^3}\delta^2(\theta_1-\theta_2)P(r,\theta_1,\theta_2)$$
$$- 2\pi\lambda \int \frac{d^3r}{(2\pi)^3} D_-^{\theta_2,-p} D_-^{\theta_1,p}\left(\frac{\delta^2(\theta_1-\theta_2)}{(p-r)_{--}}P(r,\theta_1,\theta_2)\right.$$
$$+ 2\pi\lambda \int \frac{d^3r}{(2\pi)^3}\frac{\delta^2(\theta_1-\theta_2)}{(p-r)_{--}} D_-^{\theta_1,r} D_-^{\theta_2,-r} P(r,\theta_1,\theta_2) \qquad (17)$$

Exact Solution for propagator

- The solution for the exact propagator is incredibly simple. It is given by

$$P(p, \theta_1, \theta_2) = \frac{D^2 - m}{p^2 + m^2} \delta^2(\theta_1 - \theta_2) \tag{18}$$

- $$m = \frac{2m_0}{2 + (-1 + w)\lambda \, \mathrm{Sgn(m)}} \,. \tag{19}$$

Integral equation for four point functions

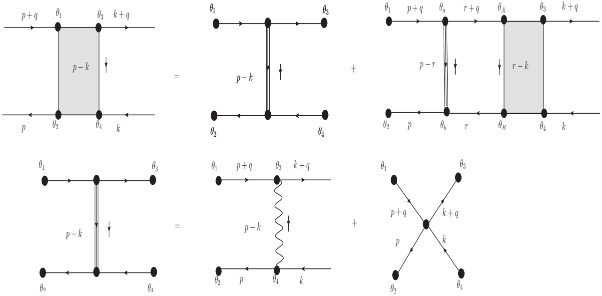


Figure : Four point function: Schwinger-Dyson equation for offshell four point function. The second line represents the tree level contributions from the gauge superfield interaction and the quartic interactions, The first line is the pictorial representation of the integral equation

Restrictions on soln of 4 point function

- Now turn to evaluating the 4 point function needed for computing scattering. Linear integral equation. Unable to solve in general. Exact solution for the special case $q_{\pm} = 0$.
- Defect has different implications in different channels
- In the singlet exchange channel it turns out that q_μ is the centre of mass energy. If $q_{\pm} = 0$ then momentum centre of mass momentum is spacelike. Incompatable with putting onshell. Solution not useful for directly computing T_{sing}.
- In the other three channels q_μ is the momentum transfer. In the scattering of two particles of equal mass, the momentum exchange is always spacelike. Setting $q_{\pm} = 0$ is simply a choice of Lorentz frame. Assuming the answer is covariant, there is no loss of information. Can read off full results for T_{Adj}, T_{sym}, T_{as}.

Results: 4 point function

- Evaluating the onshell 4 pt function subject to this restriction and taking the onshell limit we find
-

$$T_B = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + J_B(|q_3|,\lambda)\ , \qquad (20)$$

$$T_F = \frac{4i\pi}{\kappa}\epsilon_{\mu\nu\rho}\frac{q^\mu(p-k)^\nu(p+k)^\rho}{(p-k)^2} + +J_F(|q_3|,\lambda)\ , \qquad (21)$$

where the J functions are

$$J_B(|q|,\lambda) = \frac{4\pi|q|}{\kappa}\frac{N_1N_2+M_1}{D_1D_2}$$
$$J_F(|q|,\lambda) = \frac{4\pi|q|}{\kappa}\frac{N_1N_2+M_2}{D_1D_2} \qquad (22)$$

Explicit Solution

$$
\begin{aligned}
N_1 &= \left(\left(\frac{2m+iq}{2m-iq} \right)^{-\lambda} (w-1)(2m+iq) + (w-1)(2m-iq) \right) \\
N_2 &= \left(\left(\frac{2m+iq}{2m-iq} \right)^{-\lambda} (q(w+3) + 2im(w-1)) + (q(w+3) \right. \\
&\quad \left. - 2im(w-1)) \right) \\
M_1 &= -8mq((w+3)(w-1) - 4w) \left(\frac{2m+iq}{2m-iq} \right)^{-\lambda} \\
M_2 &= -8mq(1+w)^2 \left(\frac{2m+iq}{2m-iq} \right)^{-\lambda} \\
D_1 &= \left(i \left(\frac{2m+iq}{2m-iq} \right)^{-\lambda} (w-1)(2m+iq) - 2im(w-1) + q(w+3) \right)
\end{aligned}
\qquad (23)
$$

Duality

- It is not too difficult to verify that the S matrices above map to themselves under the duality map, if we also make the identifications
- $$T^B_{Adj} = T^F_{Adj}, \quad T^B_{sym} = T^F_{as}, \quad T^B_{as} = T^F_{sym}$$

  This is the result we should have expected both from level rank duality as well as from basic statisitics.
- This matching is an impressive verification of the duality conjecture.

N=2 Limit

- The supersymmetry in our Lagrangian is enhanced to $\mathcal{N} = 2$ when w is set to unity.
- In this limit the S matrix simplies very dramatically
-

$$T_B^{w=1} = \frac{4i\pi q_3}{\kappa} \frac{(k+p)_-}{(k-p)_-} - \frac{8\pi m}{\kappa} ,$$
$$T_F^{w=1} = \frac{4i\pi q_3}{\kappa} \frac{(k+p)_-}{(k-p)_-} + \frac{8\pi m}{\kappa} \qquad (25)$$

- No loop corrections. Non renormalization in the three channels in which we can directly compute.

Scattering in the singlet channel

- The results in the adjoint, symmetric and antisymmetric channels may be rewritten in a way that makes it manifest that they transform into each under under crossing symmetry. Though we do not have an explicit computation of T_{sing}, according to standard QFT lore one should be able to obtaine it from the naive analytic continuation.
- Performing the naive analytic continuation gives a result that cannot be right. To start with it does not have the delta function piece that we know must be there on physical grounds. Indeed no analytic continuation can give this piece as it is not analytic.

Crossing in the Singlet Channel

- Clearly the rules for crossing symmetry must be modified. Our previous studies of scattering in purely bosonic and fermionic theories led us to conjecture

$$T_{sing} = \frac{\sin(\pi\lambda_B)}{\pi\lambda_B} T^{ac}_{sing} - i(\cos(\pi\lambda_B) - 1) I(p_1, p_2, p_3, p_4).$$

  where T^{ac}_{sing} is the singlet amplitude one obtains from naive analytic continuation
- Two unusual features. δ function. $\frac{\sin(\pi\lambda_B)}{\pi\lambda_B}$. Conjecture for consistency of earlier computations. Physical interpretation of both terms, applies universally. Therefore no wiggle room. Conjecture above is forced on us and cannot further be modified. Either T_{sing} defined passes all consistency tests or it does not.

Explicit conjecture for Singlet Scattering

$$T_B = i4\sqrt{s}\sin(\pi\lambda)\left(\cot\frac{\theta}{2}\right) + -i8\pi\sqrt{s}(\cos(\pi\lambda) - 1)\delta(\theta)$$
$$\frac{\sin(\pi\lambda)}{\pi\lambda} J_B(\sqrt{s}, \lambda)$$
$$T_F = i4\sqrt{s}\sin(\pi\lambda)\left(\cot\frac{\theta}{2}\right) - i8\pi\sqrt{s}(\cos(\pi\lambda) - 1)\delta(\theta) \quad (26)$$
$$\frac{\sin(\pi\lambda)}{\pi\lambda} J_F(\sqrt{s}, \lambda)$$

$$J_B(\sqrt{s}, \lambda) = -4\pi i\lambda\sqrt{s}\frac{N_1 N_2 + M_1}{D_1 D_2}$$
$$J_F(\sqrt{s}, \lambda) = -4\pi i\lambda\sqrt{s}\frac{N_1 N_2 + M_2}{D_1 D_2} \quad (27)$$

Form of J_B and J_F

$$N_1 = (w-1)(2m+\sqrt{s}) + (w-1)(2m-\sqrt{s})e^{i\pi\lambda}\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)^{\lambda}$$

$$N_2 = (-i\sqrt{s}(w+3) + 2im(w-1)) + (-i\sqrt{s}(w+3) - 2im(w-1))e^{i\pi\lambda}\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)^{\lambda} \tag{28}$$

$$M_1 = 8mi\sqrt{s}((w+3)(w-1) - 4w)e^{i\pi\lambda}\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)^{\lambda}$$

$$M_2 = 8mi\sqrt{s}(1+w)^2 e^{i\pi\lambda}\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)^{\lambda} \tag{29}$$

Form of J_B and J_F

$$
\begin{aligned}
D_1 =& i(w-1)(2m+\sqrt{s}) - (2im(w-1) \\
&+ i\sqrt{s}(w+3))e^{i\pi\lambda}\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)^{\lambda} \\
D_2 =& (\sqrt{s}(w+3) - 2im(w-1)) \\
&+ (w-1)(-i\sqrt{s}+2im)e^{i\pi\lambda}\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)^{\lambda}
\end{aligned} \tag{30}
$$

In order to check supersymmetry, it is useful to define the product of two S matrix superfields.

$$S_1 \star S_2 \equiv \int d\Gamma S_1(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{k}_3, \phi_1, \mathbf{k}_4, \phi_2)$$
$$\exp(\phi_1\phi_3 + \phi_2\phi_4) 2k_1^0 (2\pi)^2 \delta^{(2)}(\mathbf{k}_3 - \mathbf{k}_1) 2k_2^0 (2\pi)^2 \delta^{(2)}(\mathbf{k}_4 - \mathbf{k}_2)$$
$$S_2(\mathbf{k}_1, \phi_3, \mathbf{k}_2, \phi_4, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) =$$
$$\exp(\theta_1\theta_3 + \theta_2\theta_4) 2p_3^0 (2\pi)^2 \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_3) 2p_4^0 (2\pi)^2 \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_4) \tag{31}$$

where the measure $d\Gamma$ is

$$d\Gamma = \frac{d^2k_3}{2k_3^0(2\pi)^2} \frac{d^2k_4}{2k_4^0(2\pi)^2} \frac{d^2k_1}{2k_1^0(2\pi)^2} \frac{d^2k_2}{2k_2^0(2\pi)^2} d\phi_1 d\phi_3 d\phi_2 d\phi_4 \tag{32}$$

Unitarity: Properties of star product

- Define

$$I(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{k}_3, \phi_1, \mathbf{k}_4, \phi_2) = \exp(\theta_1\theta_3 + \theta_2\theta_4)$$
$$2p_3^0(2\pi)^2\delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_3)2p_4^0(2\pi)^2\delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_4) \quad (33)$$

- It is easy to check that

$$S \star I = I \star S = S \quad (34)$$

  And I is supersymmetric.

- Also define

$$S^\dagger(\mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2, \mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4) = S^*(\mathbf{p}_3, \theta_3, \mathbf{p}_4, \theta_4, \mathbf{p}_1, \theta_1, \mathbf{p}_2, \theta_2) \quad (35)$$

  If S is susy can show $S^\dagger$ is susy

Unitarity

- With these definitions, unitarity is the equation

$$S \star S^{\dagger} - I = 0$$

- Since the LHS of this equation is supersymmetric, in order to establish it is sufficient to show that the θ^0 and the θ^4 tersm vanish.
- These two conditions are easily written in terms of F and f.

Unitarity

$$\int d\Gamma' \Big[f_1(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_3,\mathbf{k}_4) f_1^*(\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}_3,\mathbf{k}_4)$$

$$- Y(\mathbf{p}_3,\mathbf{p}_4)\Big(f_1(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_3,\mathbf{k}_4) + 4Y(\mathbf{p}_1,\mathbf{p}_2) f_2(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_3,\mathbf{k}_4)\Big)$$

$$\Big(f_1^*(\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}_3,\mathbf{k}_4) + 4Y(\mathbf{p}_3,\mathbf{p}_4) f_2^*(\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}_3,\mathbf{k}_4)\Big)\Big]$$

$$= 2p_3^0(2\pi)^2\delta^{(2)}(\mathbf{p}_1-\mathbf{p}_3)2p_4^0(2\pi)^2\delta^{(2)}(\mathbf{p}_2-\mathbf{p}_4)$$

$$\int d\Gamma' \Big[-16Y^2(\mathbf{p}_3,\mathbf{p}_4) f_2(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_3,\mathbf{k}_4) f_2^*(\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}_3,\mathbf{k}_4)$$

$$+ Y(\mathbf{p}_3,\mathbf{p}_4)\Big(f_1(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_3,\mathbf{k}_4) + 4Y(\mathbf{p}_1,\mathbf{p}_2) f_2(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}_3,\mathbf{k}_4)\Big)$$

$$\Big(f_1^*(\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}_3,\mathbf{k}_4) + 4Y(\mathbf{p}_3,\mathbf{p}_4) f_2^*(\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}_3,\mathbf{k}_4)\Big)\Big] =$$

$$- 2p_3^0(2\pi)^2\delta^{(2)}(\mathbf{p}_1-\mathbf{p}_3)2p_4^0(2\pi)^2\delta^{(2)}(\mathbf{p}_2-\mathbf{p}_4).$$

Unitarity and other consitency checks

- Very complicated nonlinear relation. Since our scattering amplitudes are also quite complicated, having this equation obeyed requires a minor algebraic miracle, which indeed occurs, and unitarity is obeyed. Note that the relation does not work at all without the contact term and the factor of $\frac{\sin \pi\lambda}{\pi\lambda}$.
- Other checks? It can be shown that the conjecture is also consistent with duality.
- Our conjecture also has the correct nonrelativistic limit, as we explain now.

Straight forward non relativistic limit

- The straight forward non relativistic limit is taken by taking $\sqrt{s} \to 2m$. In this limit $F = \mathrm{sgn}(\lambda_B)$ and

$$T_{sing} = 8\pi i\sqrt{s}(1 - \cos(\pi\lambda_B))\delta(\theta) + 4i\sqrt{s}\sin(\pi\lambda_B)\mathrm{Pv}\left(\cot\left(\frac{\theta}{2}\right)\right) + 4\sqrt{s}|\sin(\pi|\lambda_B|)| \tag{36}$$

- This is in perfect agreement with the Aharonov Bohm result. Natural quesiton: is there any way to get the Bak-Camillio modification out of our formula? Ans: yes! There is a second, more sophisiticated non relativistic limit one can take.

Poles in the S Matrix

- Can show that the S matrix (both 4 boson and 4 fermions) have a pole for $w \leq -1$.
- For $w = -1 - \epsilon$ the pole is very near threshhold
- As w is decreased the mass of the pole decreases from $2m$ downto zero. The pole mass hits zero at

$$w_c = -\frac{2}{\lambda} + 1 \tag{37}$$

- As w is decreased further the pole mass rises again, and approaches $2m$ as $w \to -\infty$.
- The appearance of a massless bound state in our aparently massive theory is fascinating. Would be nice to understand further.

Tuned Non Relativistic limit

- Consider the theory at $w = -1 - \delta w$ for δw small. It turns out that $E_B \sim (\delta w)^{1/|\lambda_B|})$ at small δw.
- This observation motivates study of the scaled non relativistic limit

$$\frac{\delta w}{m} \to 0, \quad \frac{k}{m} \to 0, \quad \frac{k}{m}\left(\frac{m}{\delta w}\right)^{\frac{1}{2|\lambda_B|}} = \text{fixed}. \tag{38}$$

- In this limit the 4 boson scattering amplitude may be shown to agree precisely with the Bak-Camillio form with the identification

$$A_{NR} = \frac{\delta w}{2}(2m)^{2\lambda}\,. \tag{39}$$

Physical explanation of $\frac{\sin \pi\lambda}{\pi\lambda}$

- Interesting observation: $\frac{\sin \pi\lambda}{\pi\lambda}$ equals Witten's result for the expectation value of a circular Wilson loop on S^3 in pure Chern Simons theory in the large N limit.
- This observation suggests a possible explanation of the modified crossing symmetry rules (details on white board)
- If this is right it is the tip of the iceberg. Finite N and κ.

Bosons-Fermions?

- Let us accept the conjecture for the moment. How did fermions turn into bosons? Different answers in different channels.
- Adjoint channel. Boring. No phase, no statistics.
- Sym and as channel. No phase but statistics. Level rank duality on Young Tableax of exchange representations
- Singlet channel. Most interesting. Anyonic phase. Neither bosons nor fermions. $e^{i\pi\lambda_B} = e^{i\pi\lambda_F} e^{-i\pi \mathrm{sgn}\lambda_F}$ implies $\lambda_B = \lambda_F - \mathrm{sgn}\lambda_F$. Equating anyonic phases in the singlet channel gives a derivation of the anyonic phase!

Conclusions

- Presented computations and conjectures for all orders scattering matrices in Chern Simons matter theories.
- Results in perfect agreement with duality
- Results are in impressive agreement with the modified crossing symmetry properties conjectured in earlier studies. Current conjectures are large N results. Presumably a consequence of the fact that we are scattering Anyons. Study of crossing at finite N and k a very interesting problem.
- Generalizations to theories with higher susy - like $\mathcal{N} = 6$ ABJ theories with $M \ll N$ - should be straightforward and are currently underway. Hope to soon confront puzzles in these results.