# The four-supercharge bootstrap

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*Based on*:

N. Bobev, S. El-Showk, D. Mazac, MFP
[1502.04124], [1503.02081]

EuroStrings 2015, DAMTP

# Outline

- Motivation
- Superconformal algebras in $D$ dimensions.
- Superconformal Casimir and conformal blocks.
- Bootstrap.

- Lunch!

# Outline

# The Conformal Bootstrap

- Solve theories bottom up requiring only basic ingredients: unitarity, conformal symmetry, crossing.
- Initiated in the 70's with Ferrara, Gatto, Grillo, Parisi and Polyakov.
- Modern approach initiated by Rattazzi, Rychkov, Tonni and Vichi in 2008.

- Constrains and can even determine possible spectra of CFTs. El-Showk, MFP '12
- Provides broad, non-perturbative results.
- Some analytic results (large spin, large $N$), but mostly numerics.

Fitzpatrick,Kaplan,Poland,Simmons-Duffin '12; Komargodski,Zhiboedov '12; Alday, Bissi, Lukowski '14 '15

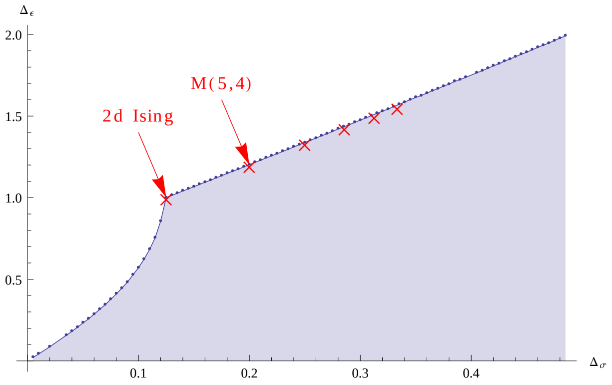
- Progress made possible due to computational as well as important technical advances, most notably the work of F. Dolan and H. Osborn.

Dolan, Osborn (et al) '00 – '11

- Public bootstrap packages are now available! – `JuliBootS` and `SDPB`.

MFP '14; D. Simmons-Duffin '15

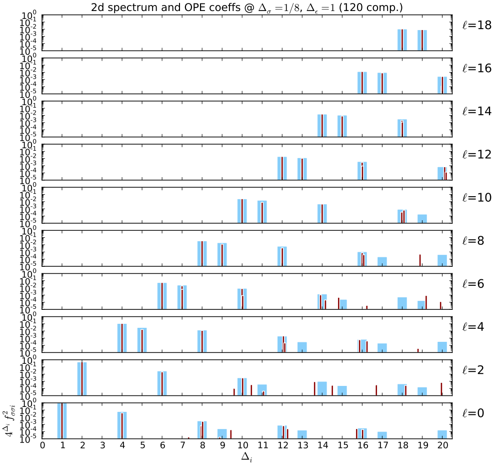
# Exhibit 1: Bounds




- Bounding conformal dimension of leading scalar operator in the operator product expansion:

$$\sigma \times \sigma = \mathbb{1} + \varepsilon + \ldots$$

# Exhibit 2: Spectra



# Adding symmetries

- Both flavour and spacetime (i.e. super) symmetries have been considered.

  Rattazzi, Rychkov, Vichi '10; Poland, Simmons-Duffin, Vichi '10, '11

- Results less general, but in principle more constraining. Technically, symmetry fixes relative OPE coefficients of conformal primaries.
- Supersymmetry allows interplay between exact results from SUSY, and non-pertubative information from bootstrap: e.g. non-renormalization of superpotential (possibly with $a$ or $F$ maximization) can fix dimensions of chiral operators, and in some cases even the stress tensor two point function.
- The promise of bootstrap: to determine the dimensions of unprotected operators to high accuracy. Protected quantities allow us to zoom in on a desired theory, and/or act as a cross-check of results.
- More generally, clearly interesting to constrain the space of superconformal field theories from several perspectives, be they theoretical or phenomenological.

# Bootstrapper's To-Do-List

| | Spacetime | | | |
|---|---|---|---|---|
| SUSY (Q's) | D=1 | D=2 | D=3 | D=4 |
| 0 | ✓ | ✓ | ✓ | ✓ |
| 2 | ✗ | ✗ | ✓ | – |
| 4 | ✗ | ✗ | ✗ | ✓ |
| 8 | ✗ | ✗ | ✗ | ✓ |
| 16 | ✗ | ✗ | ✓ | ✓ |

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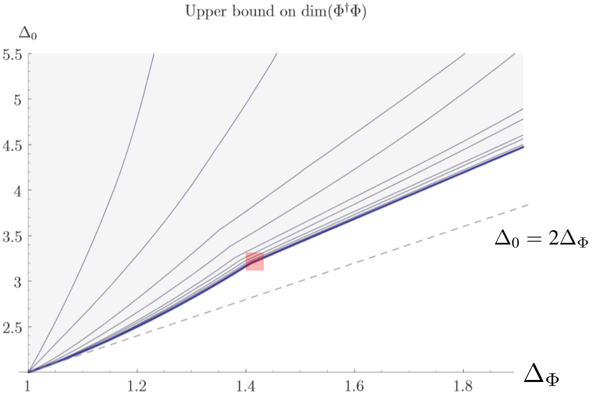
| 8 | ✗ | ✗ | ✗ | ✓ |
| 16 | ✗ | ✗ | ✓ | ✓ |

# Bootstrapper's To-Do-List

| | Spacetime | | | | | | |
|---|---|---|---|---|---|---|---|
| SUSY (Q's) | D=1 | 1<D<2 | D=2 | 2<D<3 | D=3 | 3<D<4 | D=4 |
| 0 | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| 2 | ✗ | ✗ | ✗ | ✗ | ✓ | – | – |
| 4 | ✗ | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ |
| 8 | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| 16 | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ | ✓ |

# Motivation: $D = 4$




- In $D = 4$ a mysterious kink appears around $\Delta_\Phi = 1.41$. Poland, Simmons-Duffin, Vichi '11
- We will determine spectrum of chiral operators of this theory as well as its central charge, and examine the fate of the kink as $D \to 2$.

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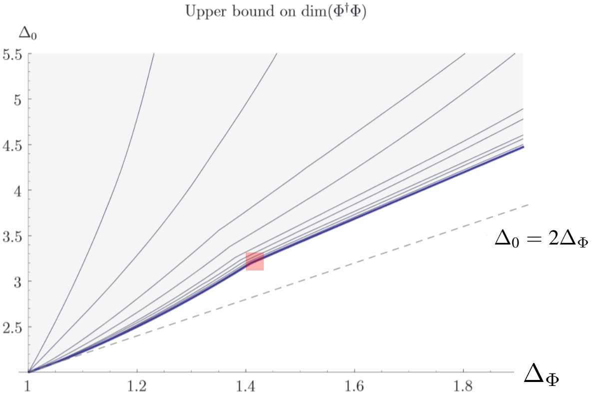


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- We will determine spectrum of chiral operators of this theory as well as its central charge, and examine the fate of the kink as $D \to 2$.
- Spoiler: we still don't know what it is!

# Motivation: $D = 2$

- In $D = 2$, we explore NS-NS sector of $\mathcal{N} = (2,2)$ theories.
- Infinite family of minimal models, superpotential $\mathcal{W} = \Upsilon^{k+2}$, $\Upsilon|_{\theta=0} = \Phi$.

$$c = \frac{3k}{k+2}$$

- In general ($k \geq 2$), have a family

$$\Delta_\Phi = \frac{1}{k+2}, \qquad \Delta_{[\Phi\bar{\Phi}]} = \frac{4}{k+2}, \qquad \Rightarrow \qquad \Delta_{[\Phi\bar{\Phi}]} = 4\Delta_\Phi$$

- For $k = c = 1$, minimal model is special. Virasoro primaries include identity and single chiral $\Phi$, with

$$\Delta_\Phi = 1/3, \qquad \Phi \times \bar{\Phi} = \mathbb{1}$$

- First (quasi)-primary in OPE above is scalar with dimension 2 inside identity multiplet.
- If we recover the minimal models with our methods this will act as a cross-check on other results!

# Motivation: $D = 3$

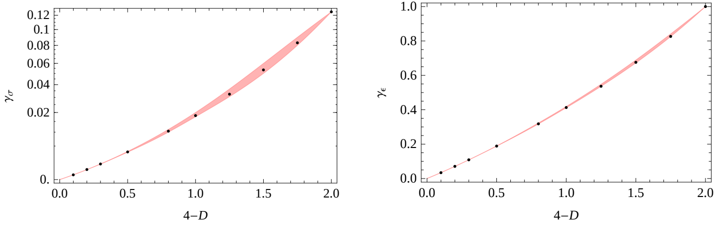
- Our work will constrain the general landscape of $\mathcal{N} = 2$ SCFT theories in $D = 3$.
- There is huge set of such theories! But a particularly simple example is the Wess-Zumino model with cubic superpotential – the $\mathcal{N} = 2$ 3d Ising model.
- This model has become of interest as it can seemingly be realized in nature. It describes a certain quantum critical point on the surface of 3d topological insulators. S.-S. Lee, '06; P. Ponte and S.-S Lee '14; Grover,Sheng, Vishwanath '13
- We will be able to determine unprotected quantities in this model, which determine approach to criticality – i.e. critical exponents.
- Theory of a single chiral field, cubic superpotential:

$$[\Phi] = \frac{D-1}{3}, \quad \Phi^2 = 0, \quad [\Phi\bar{\Phi}] = 2 + \mathcal{O}(\epsilon^2)$$

- From our point of view this is a useful goal in what should be thought of as initial investigations of SCFTs in $D = 3$, one that in particular is accessible perturbatively via $\epsilon$-expansion.

# Why arbitrary $D$?

- Conformal blocks, and indeed bootstrap naturally allows for any $D$.
- In principle, theories could be defined in these dimensions – via systems on fractal lattices. Golden, MFP '14; Mandelbrot et al '80, '84.
- Three birds ($D = 2, 3, 4$) with one stone. Actually, $\infty$ birds.
- Fractional $D$ allows us to track theories across dimensions, compare with $\epsilon$-expansion. El-Showk,MFP,Poland,Rychkov,Simmons-Duffin,Vichi '13




- Anomalous dimensions of $\sigma$ and $\varepsilon$ for Wilson-Fisher fixed point – bootstrap in black, 5-loop Borel resummed $\epsilon$-expansion in orange.

# Outline

# Superconformal algebra in any $D$.

- Our construction relies on a (somewhat) formal construction of the superconformal algebra for arbitrary $D$.
- Similar constructions exist (trivially) for non-supersymmetric case (it's just $so(d+1,1)$), and also for super-Poincaré algebras.
- Our results are consistent at the level of traces, and reproduce correct algebras in integer dimensions – at the end of the day you may argue this is what matters.
- The superalgebra allows for determination of representations in OPE of chiral operators; unitarity bounds; construction of the conformal Casimir. This information is required for the bootstrap.
- A non-trivial cross-check is that using the (super)-conformal blocks obtained using the algebra, one can decompose (generalized) free field correlation functions in any dimension with *positive* coefficients – much like in the non-susy case.

# Superconformal algebra in any $D$.

The list of generators includes:

- $P_i, K_i, D, M_{ij}, i, j = 1, \dots D$ – conformal group generators.
- The $U(1)$ R-charge generator $R$.
- Poincaré supercharges $Q^+_\alpha, Q^-_{\dot\alpha}$.
- Superconformal charges $S^{\dot\alpha +}, S^{\alpha -}$.
- $M_{\hat{i}\hat{j}}, \hat{i}, \hat{j} = D+1, \dots, 4$ rotations in internal space (act as R-symmetries).

The $+, -$ supercharges can transform a priori under different spinor representations of $SO(D) \times SO(4-D)$, which however should be two-dimensional (indices run from 1 to 2).

# Superconformal algebra in any $D$.

- The starting point is the anticommutators

$$\{Q^+_\alpha, Q^-_{\dot\alpha}\} = \Sigma^i_{\alpha\dot\alpha} P_i\,,$$
$$\{S^{\dot\alpha +}, S^{\alpha -}\} = \bar\Sigma_i^{\dot\alpha\alpha} K_i\,,$$

  The $\Sigma$, $\bar\Sigma$ play the role of gamma matrices in generic dimensions.

- The supercharges transform under rotations as

$$[M_{ij}, Q^+_\alpha] = (m_{ij})_\alpha^{\ \beta} Q^+_\beta\,,$$
$$[M_{ij}, Q^-_{\dot\alpha}] = (\tilde m_{ij})^{\dot\beta}_{\ \dot\alpha} Q^-_{\dot\beta}\,,$$
$$[M_{ij}, S^{\dot\alpha +}] = -(\tilde m_{ij})^{\dot\alpha}_{\ \dot\beta} S^{\dot\beta +}\,,$$
$$[M_{ij}, S^{\alpha -}] = -(m_{ij})_\beta^{\ \alpha} S^{\beta -}\,,$$

# Superconformal algebra in any $D$.

- Jacobi identities for the triplets $[P_i, K_j, Q^+_\alpha]$ and $[P_i, K_j, Q^-_{\dot\alpha}]$ imply

$$\Sigma_j \bar\Sigma_i = \delta_{ij} + 2\mathrm{i} m_{ij} \,,$$
$$\bar\Sigma_i \Sigma_j = \delta_{ij} + 2\mathrm{i} \tilde m_{ij} \,.$$

- Taking the symmetric parts implies that the $\Sigma_i$ tensors satisfy the Clifford algebra

$$\Sigma_i \bar\Sigma_j + \Sigma_j \bar\Sigma_i = 2\delta_{ij} \,,$$
$$\bar\Sigma_i \Sigma_j + \bar\Sigma_j \Sigma_i = 2\delta_{ij} \,,$$

- Taking the antisymmetric parts leads to explicit formulas for the rotation generators in terms of $\Sigma_i$

$$m_{ij} = -\frac{\mathrm{i}}{4}\left(\Sigma_j \bar\Sigma_i - \Sigma_i \bar\Sigma_j\right) ,$$
$$\tilde m_{ij} = -\frac{\mathrm{i}}{4}\left(\bar\Sigma_i \Sigma_j - \bar\Sigma_j \Sigma_i\right) .$$

- We will take the tensors with hatted indices to satisfy the same relations, so that different algebras relate by dimensional reduction.

# Superconformal algebra for any $D$

- It remains to fix $\{S,Q\}$. General form consistent with all Jacobis except those involving three fermions:

$$\{S^{\alpha-},Q^{+}_{\beta}\} = \delta^{\alpha}{}_{\beta}(\mathrm{i}D - aR) + (m_{ij})_{\beta}{}^{\alpha}M_{ij} + b(m_{\hat{i}\hat{j}})_{\beta}{}^{\alpha}M_{\hat{i}\hat{j}}\,,$$
$$\{S^{\dot{\alpha}+},Q^{-}_{\dot{\beta}}\} = \delta^{\dot{\alpha}}{}_{\dot{\beta}}(\mathrm{i}D + aR) + (\tilde{m}_{ij})^{\dot{\alpha}}{}_{\dot{\beta}}M_{ij} + b(\tilde{m}_{\hat{i}\hat{j}})^{\dot{\alpha}}{}_{\dot{\beta}}M_{\hat{i}\hat{j}}\,,$$

- From $[Q^+,Q^-,S^-]$ we get

$$\bar{\Sigma}_i^{\dot{\alpha}\alpha}\Sigma^i_{\beta\dot{\beta}} = \frac{2a+1}{2}\delta^{\alpha}{}_{\beta}\delta^{\dot{\alpha}}{}_{\dot{\beta}} + (m_{ij})_{\beta}{}^{\alpha}(\tilde{m}_{ij})^{\dot{\alpha}}{}_{\dot{\beta}} + b(m_{\hat{i}\hat{j}})_{\beta}{}^{\alpha}(\tilde{m}_{\hat{i}\hat{j}})^{\dot{\alpha}}{}_{\dot{\beta}}\,.$$

- Taking trace and using Clifford algebra we get

$$\mathrm{tr}(\Sigma^i\bar{\Sigma}_i) = 2d\,, \qquad a = \frac{d-1}{2}\,.$$

- Similar considerations involving $[Q^+,Q^+,S^-]$ fix $b=-1$

# Superconformal algebras in any $D$

We can check the algebra in integer $D$:

- In $D = 4$ we obtain by construction the super algebra $sl(4|1)$. The value $a = (D-1)/2 = 3/2$ leads to the usual chirality condition $\Delta = 3q/2$.
- In $D = 3$ we obtain $osp(2|4)$, with the identification $\Sigma^i_{\alpha\dot{\alpha}} = (\sigma_i)_\alpha^{\ \dot{\alpha}}$ the Pauli matrices. Dotted and undotted indices are equivalent.
- In $D = 2$ obtain NS-NS sector of $\mathcal{N} = (2,2)$ superalgebra, $sl(2|1)_l \times sl(2|1)_r$. Rotation in 3,4 become extra R-symmetry.
- Bonus: in $D = 1$ obtain $psu(1,1|2)$. $U(1)$ R-symmetry may be projected out, but rotations in $2,3,4$ become $SU(2)$ R-symmetry.

# Outline

# Crossing and conformal blocks

- Conformal symmetry fixes kinematic dependence of four-point function.

$$\langle\Phi_1(x_1)\Phi_2(x_2)\Phi_3(x_3)\Phi_4(x_4)\rangle = \frac{|x_{24}|^{\Delta_{12}}|x_{14}|^{\Delta_{34}}}{|x_{14}|^{\Delta_{12}}|x_{13}|^{\Delta_{34}}}\frac{g(z,\bar{z})}{|x_{12}|^{\Delta_1+\Delta_2}|x_{34}|^{\Delta_3+\Delta_4}}$$

  with $z\bar{z} = x_{12}^2 x_{34}^2/x_{13}^2 x_{24}^2$, $(1-z)(1-\bar{z}) = x_{14}^2 x_{23}^2/x_{13}^2 x_{24}^2$.
- On the other hand, the Operator Product Expansion gives

$$\Phi_1 \times \Phi_2 \simeq \sum_{\mathcal{O}} c_{12}^{\mathcal{O}}\mathcal{O}$$

- Primary operators and their derivatives (descendants) come together in *conformal blocks*:

$$g(z,\bar{z}) = \sum_{\mathcal{O}} c_{12}^{\mathcal{O}} c_{34}^{\mathcal{O}} G_{\mathcal{O}}(z,\bar{z})$$

- For instance, in $D=2$ blocks are *known* products of hypergeometric functions.

# The superconformal Casimir

- Using the superconformal algebra one constructs the Casimir operator.

$$C_2 = C_2^b - \frac{1}{2} M_{\hat{i}\hat{j}} M_{\hat{i}\hat{j}} - \frac{d-1}{4} R^2 + \frac{1}{2} \left( \left[ S^{\dot{\alpha}+}, Q^-_{\dot{\alpha}} \right] + \left[ S^{\alpha -}, Q^+_{\alpha} \right] \right) .$$

  with $C_2^b$ the non-SUSY Casimir. Coefficients above are fixed by the superconformal algebra.
- We consider its action on a correlation function involving two chiral fields, $\Delta_{1,3} = \frac{D-1}{2} q_{1,3}$:

$$\langle \Phi_1(x_1) \Phi_2(x_2) \Phi_3(x_3) \Phi_4(x_4) \rangle = \frac{|x_{24}|^{\Delta_{12}} |x_{14}|^{\Delta_{34}}}{|x_{14}|^{\Delta_{12}} |x_{13}|^{\Delta_{34}}} \frac{g(z, \bar{z})}{|x_{12}|^{\Delta_1 + \Delta_2} |x_{34}|^{\Delta_3 + \Delta_4}}$$

- Chirality condition is necessary so that action of Casimir does not map to correlator of fermions. Dimensions are otherwise arbitrary.

# The superconformal blocks

- Conformal block expansion in (13) channel is usual one (non-SUSY), but in (12), (14) we get superconformal blocks.

$$g(z,\bar{z}) = \sum_{\mathcal{O}} c_{12}^{\mathcal{O}} c_{34\mathcal{O}}\, \mathcal{G}_{\Delta_{\mathcal{O}},s_{\mathcal{O}}}^{\Delta_{12},\Delta_{34}}(z,\bar{z}).$$

- Conformal blocks are eigenfunctions of Casimir operator. Action of Casimir takes form of second order differential operator:

$$\tilde{D}_2(z,\bar{z},\partial_z,\partial_{\bar{z}})\mathcal{G}_{\Delta_{\mathcal{O}},s_{\mathcal{O}}}^{\Delta_{12},\Delta_{34}}(z,\bar{z}) = 0$$

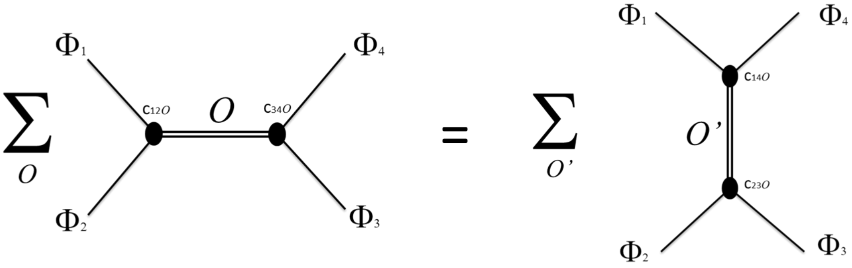
- Can be solved:

$$\begin{aligned} \mathcal{G}_{\Delta,s} &= G_{\Delta,s} + a\, G_{\Delta+1,s+1} + b\, G_{\Delta+1,s-1} + c\, G_{\Delta+2,s} \\ [\mathcal{O}]_{\text{s.c.}} &= [O] + [Q\bar{Q}O] + [Q\bar{Q}O] + [Q^2\bar{Q}^2 O] \end{aligned}$$

- Matches previously known results in $D = 2,4$. Also, leads to positive decomposition of generalized free field correlation function in arbitrary dimension.

# Outline

# Crossing Symmetry




- Crossing, or associativity of the OPE implies sum of blocks in one of the channels matches sum in another one.
- Non-trivial constraint on both OPE coefficients and spectrum.
- Bootstrap: solve these constraints numerically in a Taylor series.

# Crossing Equations

- Equality of the OPEs in the three channels (12), (13), (14) leads to three crossing equations:

$$\sum_{\mathcal{O}} |c^{\mathcal{O}}_{\Phi\bar{\Phi}}|^2 \begin{pmatrix} \mathcal{F}^{\Delta_\Phi}_{\Delta,s} \\ \tilde{\mathcal{F}}^{\Delta_\Phi}_{\Delta,s} \\ \tilde{\mathcal{H}}^{\Delta_\Phi}_{\Delta,s} \end{pmatrix} + \sum_{\mathcal{P}} |c^{\mathcal{P}}_{\Phi\Phi}|^2 \begin{pmatrix} 0 \\ F^{\Delta_\Phi}_{\Delta,s} \\ -H^{\Delta_\Phi}_{\Delta,s} \end{pmatrix} = 0\,,$$

with the operators

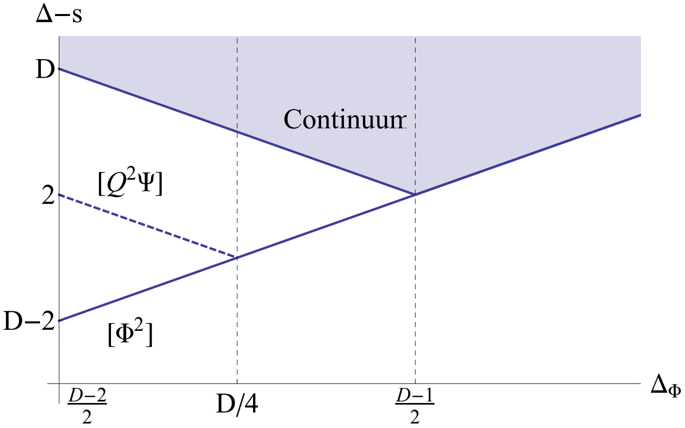
$$\mathcal{O}: \qquad \Delta \geq s+d-2 \qquad\qquad s=0,1,2,\ldots\,,$$

$$\mathcal{P}: \begin{cases} \Delta = 2\Delta_\Phi + s\,, & s=0,2,\ldots\,, \\ \Delta = d-2\Delta_\Phi\,, & s=0, \quad \Delta_\Phi \leq d/4\,, \\ \Delta \geq |2\Delta_\Phi-(d-1)|+s+(d-1)\,, & s=0,2,\ldots\,. \end{cases}$$

- $F, H, \tilde{F}, \tilde{H}$ are kinematically determined functions which are related to (super)conformal blocks.
- Structure above relies on the precise form of the superconformal algebra.

# Charged OPE

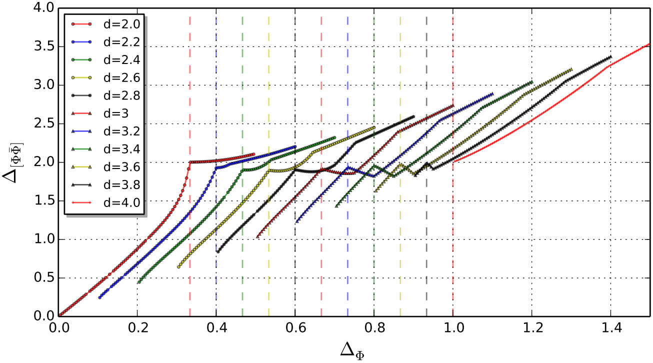
- $\mathcal{P}$ operators appearing in $\Phi \times \Phi$ OPE (dashed operator only appears for $s = 0$).




- Expect something interesting may happen at $D/4$ and $(D-1)/2$.

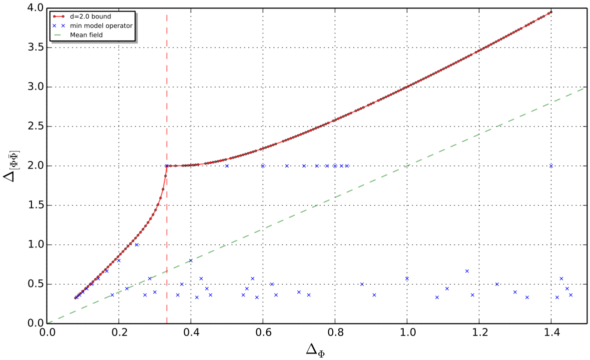
# Scalar operator bounds

- These are bounds from analysing correlator of (anti-)chirals $\langle \Phi\bar{\Phi}\Phi\bar{\Phi}\rangle$, with OPE $\Phi \times \bar{\Phi} = 1 + [\Phi\bar{\Phi}] + \ldots$.



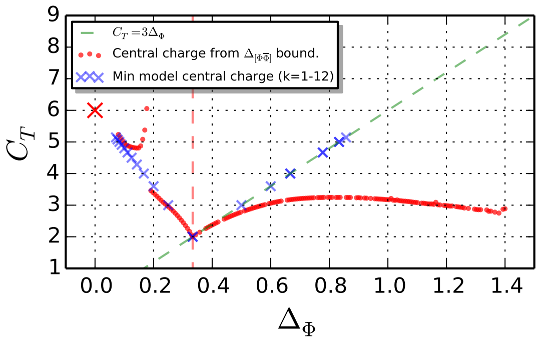

- The dashed vertical lines correspond to $\Delta_\Phi = \frac{D-1}{3}$. This is dimension of chiral in WZ model with cubic superpotential. Lines perfectly line up with kinks in the bounds.

# Closer look: $D = 2$



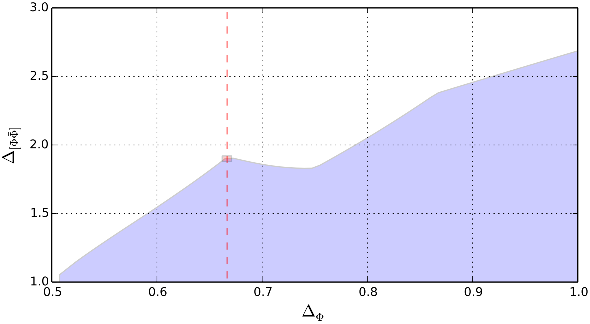

- The blue crosses mark the exact dimensions of operators from various superconformal minimal models. The cross at $(\frac{1}{3}, 2)$ corresponds to the super-Ising model (i.e. the $k = 1$ super-Virasoro minimal model). Close to the origin the bound approaches $\Delta_{\bar{\Phi}\phi} = 4\Delta_\Phi$ as for superminimal models.

# Closer look: $D = 2$



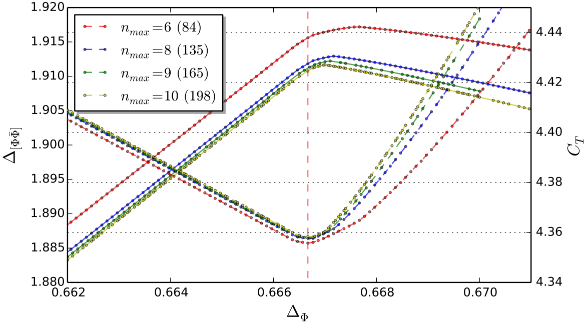

- Perfect agreement for $k = 1, k = 2$ minimal models. For others agreement is only asymptotic.
- Leading OPE coefficients can be also be checked for $k = 1$ model, with good agreement. We're on the right track!

# Closer look: $D = 3$



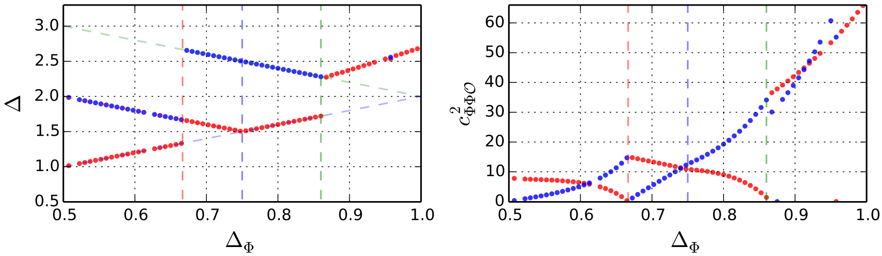

- Three kinks: first we conjecture to be the critical Wess-Zumino model. Second occurs at $\Delta_\Phi = D/4 = 3/4$. Third is the continuation of the mysterious kink in $D = 4$.
- The value $D/4$ coincides with the decoupling of a possible operator in the charged OPE, namely $Q^2\Psi$ ($\Psi$ becomes free field).

# Closer look: $D = 3$



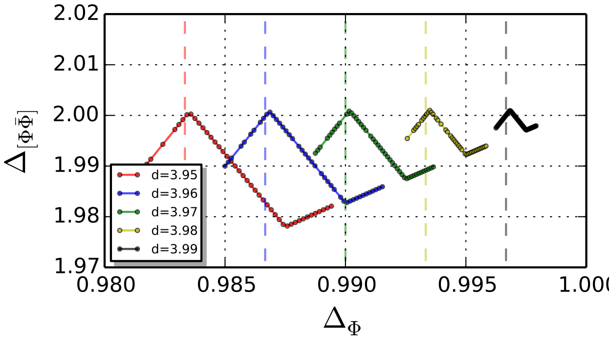

- Hills: upper bounds on $[\Phi\bar{\Phi}]$. Valleys: central charge of the solution saturating the bound. As accuracy is increased, both sets of curves converge, with a sharp transition at $2/3 = (D-1)/3$

# Closer look: $D = 3$




- The operator $\Phi^2$ decouples from OPE precisely at $\Delta_\Phi = 2/3$. Consistent with chiral ring relation $\Phi^2 = 0$ for WZ model.
- There is also decoupling at the location of the third kink $\Delta_\Phi \sim 0.86$. Suggests this might be a physical solution.

# $\epsilon$-expansion




$$\Delta_{[\Phi\bar{\Phi}]} - 2 = -0.283\,\epsilon^2 + 7.76 \times 10^{-3}\epsilon + 7.17 \times 10^{-5}$$

- Results consistent with $\Delta_{\Phi\bar{\Phi}} = 2 + \mathcal{O}(\epsilon^2)$ derived from $\epsilon$ expansion.

# Summary: Wess-Zumino model in $D = 3$

- These are our most accurate results so far:

| | |
|---|---|
| $\Delta_\Phi = 1/2 + \eta/2$ | $\frac{2}{3}$ (exact) |
| $\Delta_{[\bar{\Phi}\Phi]} = 3 - 1/\nu$ | $1.9098(20)$ |
| $\Delta_{[Q^4\bar{\Phi}\Phi]} = 3 + \omega$ | $3.9098(20)$ |
| $\Delta_{[\bar{\Phi}\Phi]'}$ | $5.3(1)$ |
| $\Delta_{J'}$ | $5.25(25)$ |
| $C_T$ | $4.3591(20)$ |

- Our $C_T$ is consistent with exact results from localization on a squashed sphere:

Imamura, Yokoyama '11

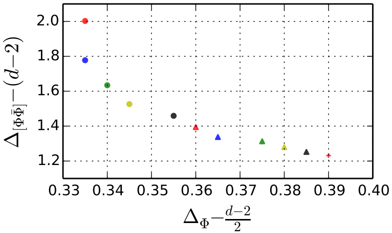
$$\left.\frac{C_T}{C_T^{\text{free}}}\right|_{\text{exact}} = 0.7268, \qquad \left.\frac{C_T}{C_T^{\text{free}}}\right|_{\text{bootstrap}} = 0.72652(33)$$

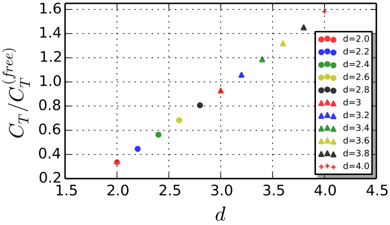
- Furthermore we have agreement with $\epsilon$-expansion, and the relation $\Phi^2 = 0$.

# Second kink?

- Properties: chiral of dimension $D/4$; decoupling of field $(Q^2\Psi)$ from OPE; central charge becomes that of *three* chiral fields in $D=4$
- Suggests there exists weakly coupled fixed point in $\epsilon$ expansion.
- Non-trivial check on a possible guess: central charge can be computed in $D=3$ given superpotential.
- We were able to find precise agreement only for strange choice: field theory actually contains *five* chiral superfields, but one of them has the wrong sign kinetic term, so that in terms of $C_T$, they effectively appear as three chiral superfields.
- Superpotential $W=(X^2+Z^2+W^2+V^2)Y$, $F$-maximization leads to $\Delta_Y<1/2$, which is below the unitarity bound, and signals that the field $Y$ is actually free $\Rightarrow \Delta_X=\Delta_Z=\Delta_W=\Delta_V=3/4$.
- Admittedly odd, but at the fixed point non-unitary field decouples so maybe ok... (someone) should check $\epsilon$-expansion.

# The third kink




- Theory corresponds to local minima of $C_T$ – same behaviour as say Ising model.
- $\Phi^2 = 0$ decouples.
- In $D = 2$ seems to merge onto $k = 1$ minimal model. $2 + \epsilon$ expansion?
- Several hints that this may be physical. $\Phi^2 = 0$ is important hint, but we haven't been able to make a good guess for the theory.

# Conclusions

- We constructed superconformal algebra in any $D$.
- We have conjecturally found the Wess-Zumino model in any $D$, and determined low lying spectrum.
- We have found a possible new (interesting?) theory accessible in perturbation theory. More generally, idea of exploring non-unitarily driven fixed points is interesting.
- Our approach to superalgebras generalizes to 8 supercharge case – bootstrap $(4,4)$ in $D=2$, $\mathcal{N}=4$ in $D=3$, $\mathcal{N}=2$ in $D=4$, $\mathcal{N}=1$ in $D=5$ and $(1,0)$ in $D=6$.
- What is the third kink? Is it physical?
- In $D=3$ many interesting theories for which we know $C_T$ and protected spectrum. Imposing extra constraints may lead us to find them in our bounds, determine unprotected spectrum.