New analytic solutions to binary black-hole dynamics: from spin precession to inspiral

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Abstract

The dynamics of precessing black-hole binaries in the post-Newtonian regime is deeply characterized by a timescale hierarchy: the orbital timescale is very short compared to the spin-precession timescale which, in turn, is much shorter than the radiation reaction timescale on which the orbit is shrinking due to gravitational-wave emission. The binary dynamics is typically studied in an orbit-averaged fashion: one only cares about the orbit itself, not the instantaneous position of each black hole. Here we also average over the precessional time, thus considering the precessional cones "as a whole", without tracking the spin's secular motion. These solutions improve our understanding of spin precession in much the same way that the conical sections for Keplerian orbits provide additional insights beyond Newton's 1/r² law.

Three momenta and three timescales

The dynamics of BH binaries is described by the evolution of three momenta:

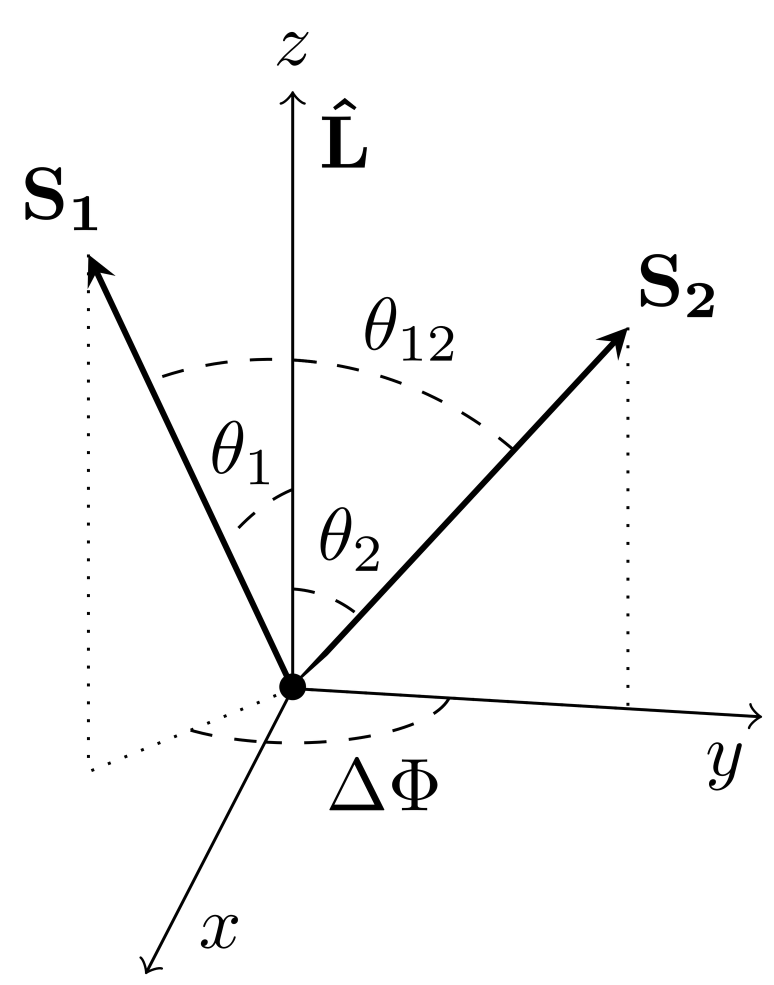
- The orbital angular momentum $\mathbf{L}$
- The spin of the primary BH $\mathbf{S_1}$
- The spin of the secondary BH $\mathbf{S_2}$

Such vectors evolve on three different timescales $t_{\rm orb} \ll t_{\rm pre} \ll t_{\rm rad}$

- The BHs orbit over the smallest timescale $t_{\rm orb} \propto r^{3/2}$
- The spins precess over $t_{\rm pre} \propto r^{5/2}$
- Gravitational waves slowly carry out momentum $t_{\rm rad} \propto r^4$

Usual PN dynamics: orbit-averaged $t_{\rm orb} \ll t \ll t_{\rm pre}$
One does not care about the positions of the BHs along each single orbit, but only study the secular evolution of the orbital parameter

New approach: precession-averaged $t_{\rm pre} \ll t \ll t_{\rm rad}$
We do not track the spins along each precession cycle, but we study the evolution of the precession cones under the effect of radiation reaction



Outlook

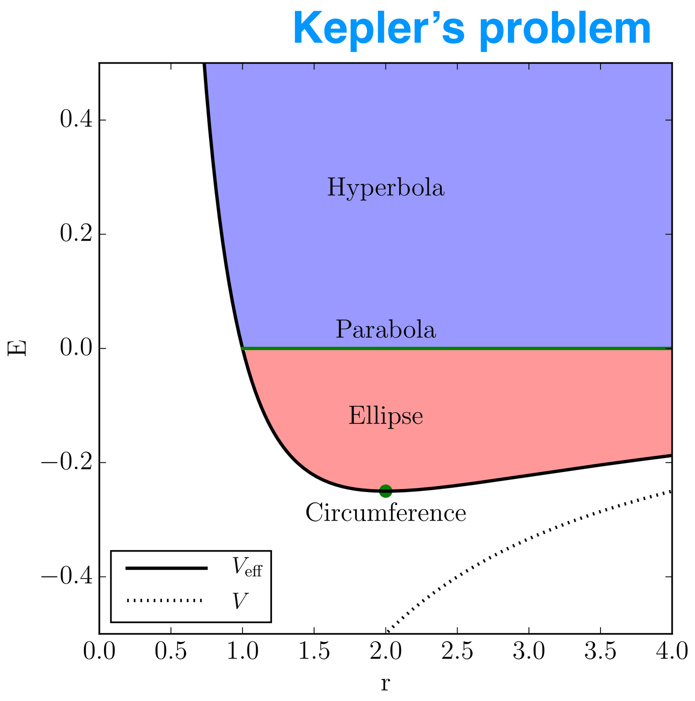
- Deeper understanding of the PN dynamics. Each precession cycle is treated as a whole (like an orbit): binaries can be classified for their precessional morphology.
- Huge computational speed-up. We can timestep in the binary separation (not in time!) with a log scaling. PN inspirals can be performed from **arbitrarily** large separation in $\mathcal{O}(1)$ min with a Python script! Binaries can be transferred from astrophysical separations to the numerical relativity regime.

Effective potentials

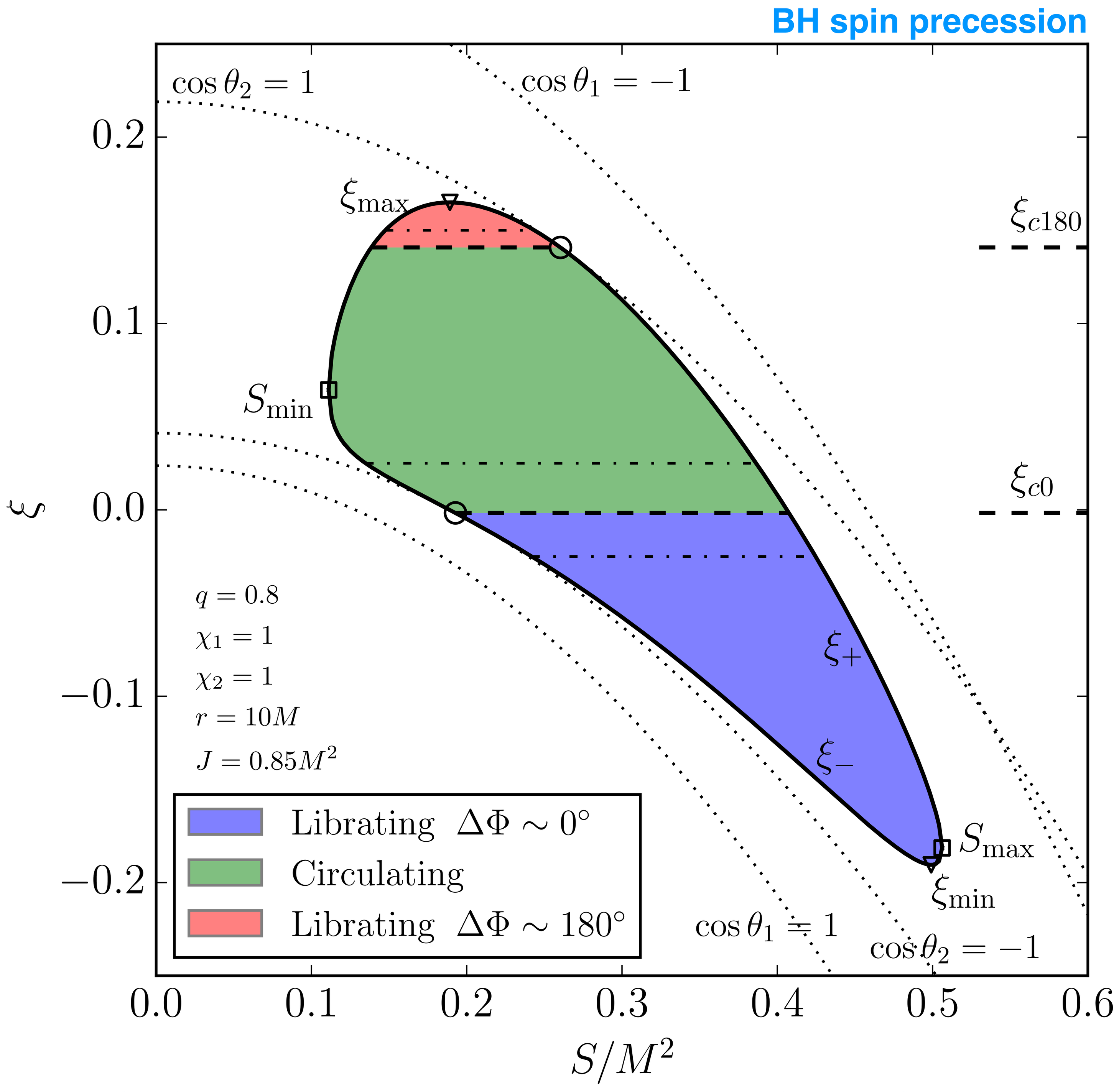
Like in Kepler's two body problem, where conservation of angular momentum and energy let you parametrize the orbital evolution, conservation of total angular momentum J and projected effective spin ξ lets you solve the precessional dynamics analytically in terms of

$$S = |\mathbf{S_1} + \mathbf{S_2}|$$

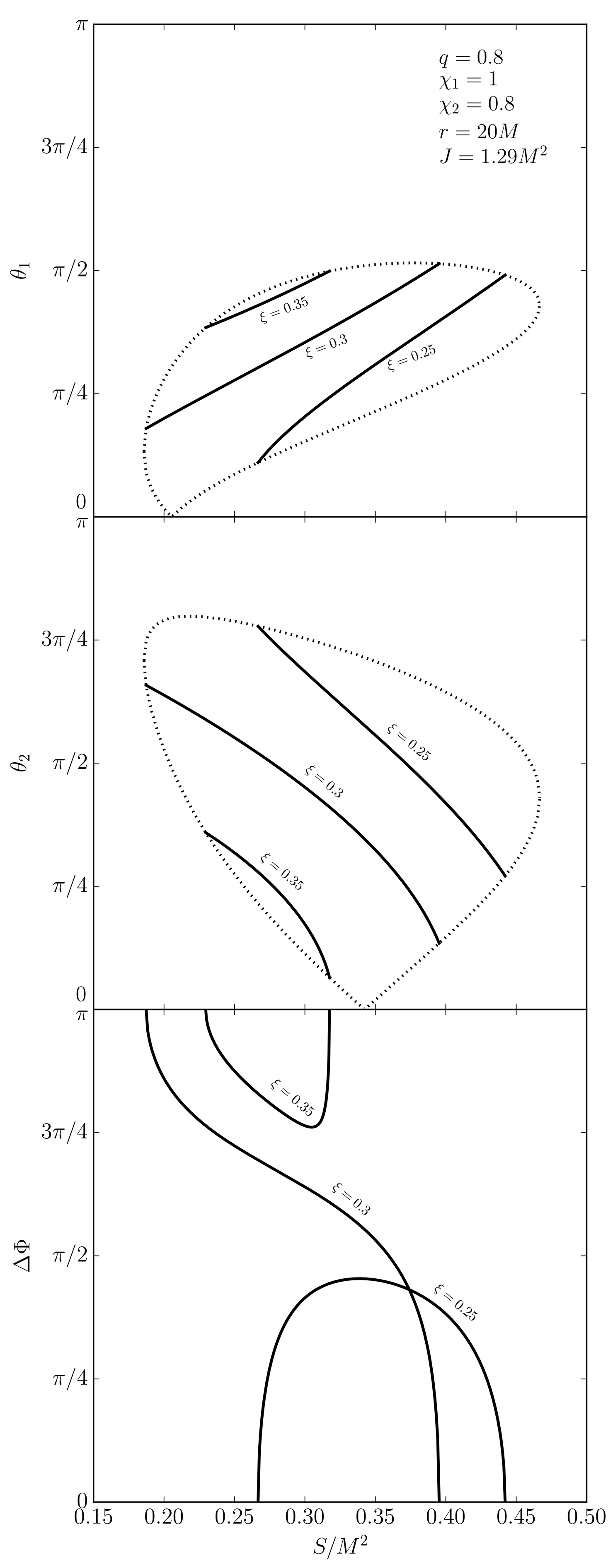
Kepler's problem



BH spin precession



Quasiperiodic solutions: librating and circulating morphologies



These effective potentials allow us to solve the orbit-averaged spin-precession equations analytically for arbitrary mass ratios and spins. Solutions are quasiperiodic functions of time. The precession dynamics of a double-spin BH binary in the PN regime can be cast unambiguously into one of three different morphologies. They are physically set by the mutual orientation of the precession cones of the three momenta, and described by the allowed range in $\Delta\Phi$

Spin-orbit resonances (*Schnittman 2004, Gerosa et al. 2013*) are the limits of the librating morphologies: the precession cones of such librating binaires oscillates about the resonant values.

Phase transitions correspond to cone crossings: the morphology changes whenever there is at least one point in a precession cycle where $\mathbf{L}$ is (anti)aligned with either one of the two spins.

The binary morphology is a distinctive feature encoded in the GW waveform and will be distinguishable with the advanced detectors *(Trifirò, Gerosa et al, in prep)*.

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