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Positivity, Negativity and Entanglement

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Holography and Entanglement

different arguments suggest that the internal structure of entanglement of the boundary state might be crucial for the reconstruction of the bulk:

- entanglement entropy is not enough to capture the full structure
- other measures give additional information but in general they are difficult to compute
- negativities offer an interesting option, but the physical interpretation is not completely clear

Some interesting facts to look at

- the value of the ratio between negativity and entanglement entropy
- the entanglement plateaux and the saturation of Araki-Lieb inequality
- the monogamy of holographic mutual information
- the ER=EPR conjecture in the multiple boundaries set-up

Questions

- what is the interpretation of the ratio between negativity and entropy?
- how is the pattern of entanglement measured by the negativity related to the saturation of Araki-Lieb inequality?
- how special is the monogamy of mutual information?
- how is it related to the internal structure of entanglement?

Qubits toy models

- simple systems where these quantities and properties can be explored looking for some intuition about the behaviour of larger systems
- the possible structures of entanglement for 3 qubits have been completely classified
- some classification is known also for 4 qubits
- at present no complete classification is known for larger systems but one can still investigate generic states and some particular subsets

Some definitions

partial transpose of a density matrix: $\langle i^A l^B|\,\tilde{\rho}\,|j^A m^B\rangle = \langle i^A m^B|\,\rho\,|j^A l^B\rangle$

the *negativity* is essentially the trace norm of the partial transpose: $\mathcal{N} = \dfrac{\|\tilde{\rho}\| - 1}{2}$

for pure states the negativity is half of the *robustness of entanglement*, a measure of the amount of noise required to disrupt the entanglement: $\rho \to \dfrac{1}{1+s}\left(\rho + s\rho_s\right)$

tripartite information:
$$I3(A:B:C) = S(A) + S(B) + S(C) - S(AB) - S(AC) - S(BC) + S(ABC)$$

mutual information is monogamous if: $I3\,(A:B:C) \leq 0$

the *tangle* is a measure of quantum multipartite entanglement: τ_4

Entanglement structures for 4 qubits

States of four qubits systems have been classified under operations known as SLOCC (stochastic local operation and classical communication)

DEFINITION OF THE CLASSES

$$|\blacksquare\rangle = \frac{a+d}{2}(|0000\rangle + |1111\rangle) + \frac{a-d}{2}(|0011\rangle + |1100\rangle) + \frac{b+c}{2}(|0101\rangle + |1010\rangle) + \frac{b-c}{2}(|0110\rangle + |1001\rangle)$$

$$|\blacksquare\rangle = \frac{a+b}{2}(|0000\rangle + |1111\rangle) + \frac{a-b}{2}(|0011\rangle + |1100\rangle) + c(|0101\rangle + |1010\rangle) + |0110\rangle)$$

$$|\blacksquare\rangle = a(|0000\rangle + |1111\rangle) + b(|0101\rangle + |1010\rangle) + |0110\rangle + |0011\rangle)$$

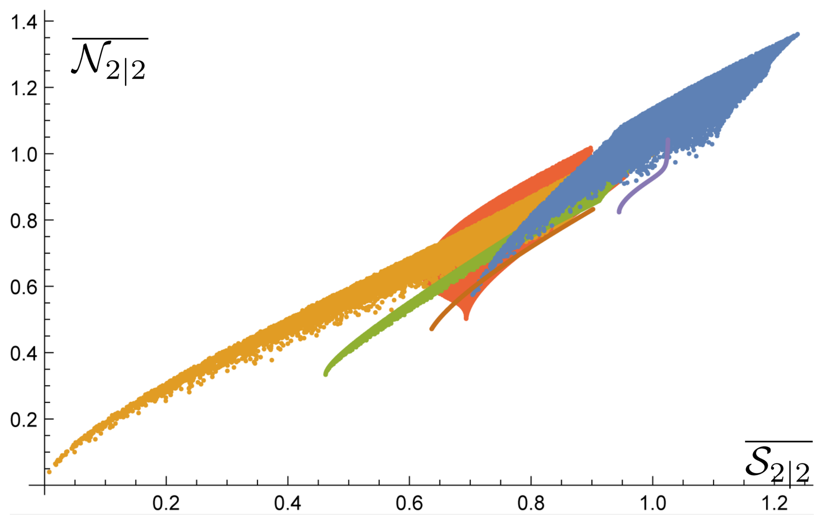
$$|\blacksquare\rangle = a(|0000\rangle + |1111\rangle) + \frac{a+b}{2}(|0101\rangle + |1010\rangle) + \frac{a-b}{2}(|0110\rangle + |1001\rangle) + \frac{i}{\sqrt{2}}(|0001\rangle + |0010\rangle + |0111\rangle + |1011\rangle)$$

$$|\blacksquare\rangle = a(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle) + i\,|0001\rangle + |0110\rangle - i\,|1011\rangle$$

$$|\blacksquare\rangle = a(|0000\rangle + |1111\rangle) + |0011\rangle + |0101\rangle + |0110\rangle$$

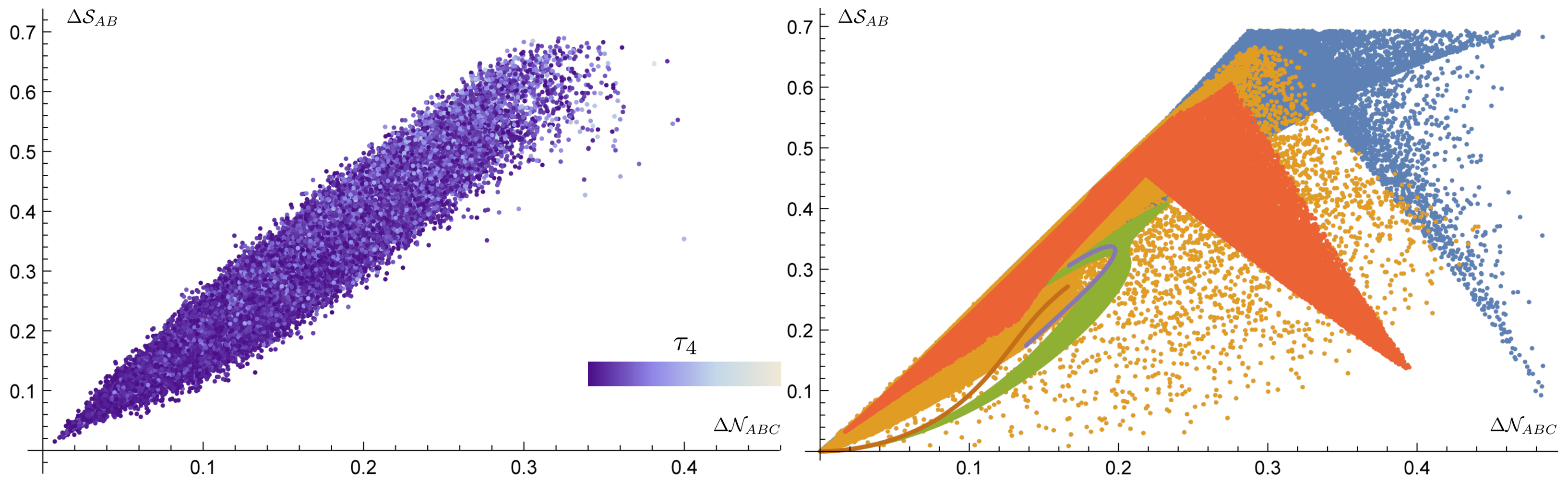
(three other classes are uninteresting as they only contain a single state) [quant-ph/0109033]

DISTINGUISHING THE CLASSES WITH ENTROPY AND NEGATIVITY



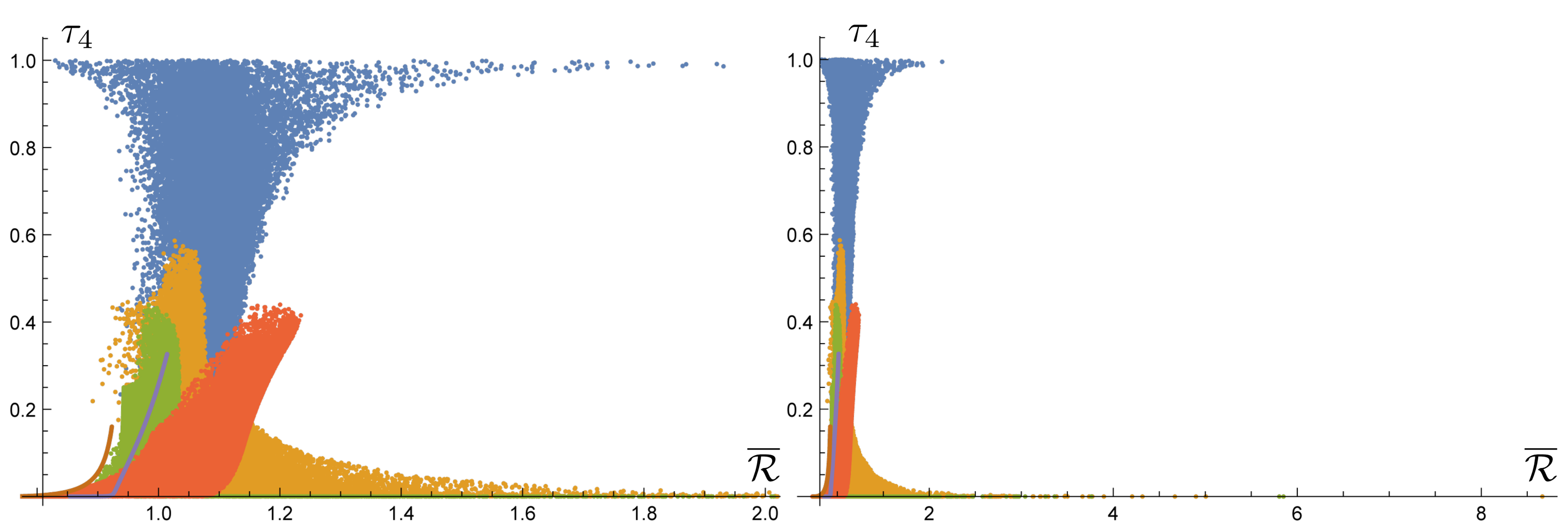
Combined information from negativity and entropy allows some resolution of the classes. The two measures were computed for the largest bipartition (2 qubits per subsystem) and averaged over the three different bipartitions of that kind.

DISENTANGLING THEOREM AND SATURATION OF ARAKI-LIEB

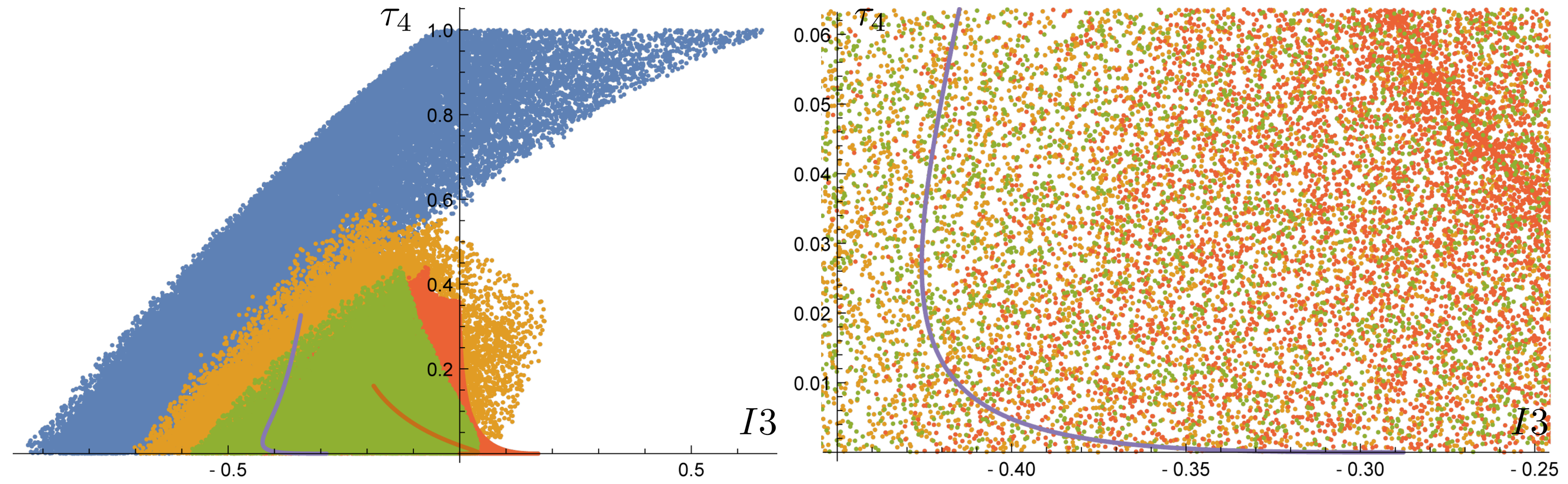


Some states almost saturate the Araki-Lieb inequality but this is not captured by the disentanglement of internal d.o.f. as seen by the negativity.

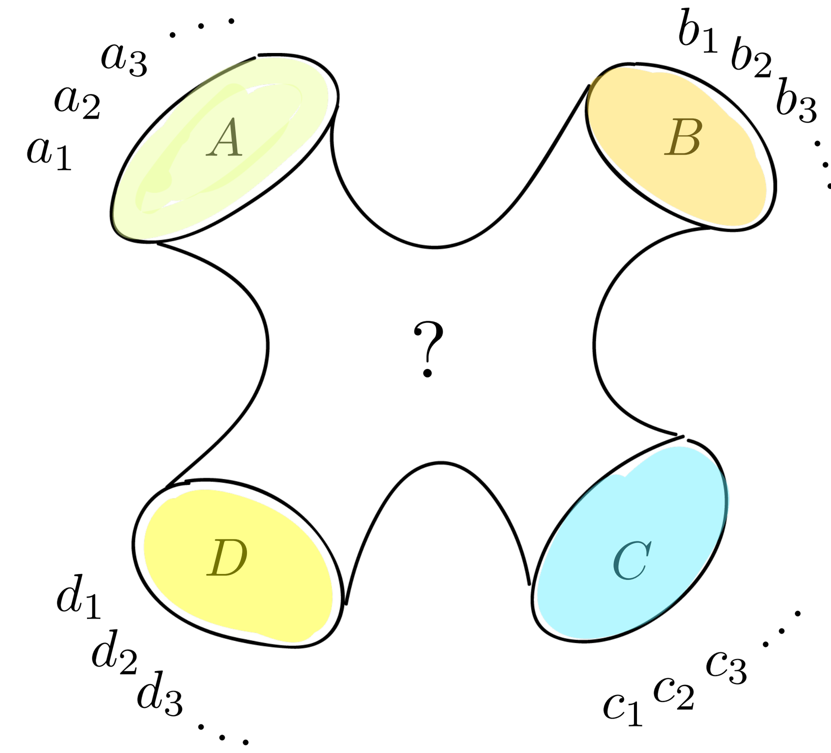
AVERAGE RATIO BETWEEN NEGATIVITY AND ENTROPY



MONOGAMY OF MUTUAL INFORMATION



There is only one class which always respect the monogamy of MI!



$$|\Psi\rangle = |abcd\rangle^{\otimes N}$$

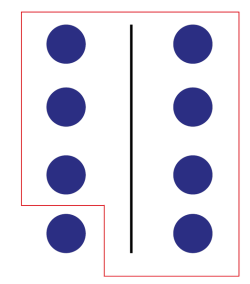
The interior can be a smooth geometry only if the mutual information is monogamous. This for example does not happen for a GHZ state [arXiv:1308.0289].

Even if the mutual information of a particular state is monogamous, monogamy can be violated acting on the state with SLOCC. To what extent is the geometry stable against these operations?

Interpreting the ratio: 8 qubits

the ratio between negativity and entropy is a measure of the robustness of entanglement for a fixed value of the entropy. It depends on the internal pattern of entanglement of the state.

Entanglement between sets of qubits across the "entangling surface"



Entanglement between qubits inside one susystem