Interactions in Higher-Spin Theory from CFT Correlation Functions

Charlotte Sleight

Max-Planck-Institut für Physik, Munich

In collaboration with: X. Bekaert, J. Erdmenger and D. Ponomarev

csleight@mpp.mpg.de

Abstract

The precise nature of quartic and higher-order interactions in higher-spin theories defined on anti-de Sitter space remains elusive. To shed some light on this issue, we apply their conjectured equivalence to certain vector model conformal field theories, through which higher-order interaction vertices can be extracted from the knowledge of relatively simple correlation functions and Witten diagrams involving lower-order interactions. In doing so, we hope to clarify the locality properties of higher-spin interactions, and how they present themselves in the dual CFT. We begin with the scalar quartic vertex in the minimal bosonic higher-spin theory, which requires the computation of four-point exchange diagrams of higher-spin gauge fields between two-pairs of external scalar fields.

Background: The higher-spin / vector model duality

Higher-spin theories on anti-de Sitter space

Higher-spin gauge theories describe massless fields φ_s of spin $s > 2$, exhibiting the linearised gauge symmetry

$$\delta_\xi \varphi_s = \nabla \xi_{s-1}.$$

Theories of *interacting* higher-spin gauge fields can be formulated on (anti)-de Sitter space (AdS space). In their simplest form, for dimensions $d > 2$ they have an infinite spectrum

$$\varphi_s, \qquad s = 0, 1, 2, 3, \ldots$$

The spectrum can also be truncated to fields of only *even* spin, referred to as the *minimal bosonic* higher-spin theory.

Duality with vector model CFTs

Conjecture:

Minimal bosonic higher-spin theory on AdS$_d$ with a parity even scalar φ_0

is equivalent to

The singlet sector of the free/critical $(d-1)$-dimensional scalar $O(N)$ vector model.

[Sezgin, Sundell; Klebanov, Polyakov, '2002]

Vector models: Conformal field theories of fundamental scalars ϕ^i, $i = 1, \ldots, N$.
Singlet sector: scalar $\mathcal{O} = \phi \cdot \phi$ and a tower of operators J_s of even spin s.

Dictionary:

$$\mathcal{O} \quad \leftrightarrow \quad \varphi_0 \quad \text{(scalar in AdS)}$$
$$J_s \quad \leftrightarrow \quad \varphi_s, \quad s = 2, 4, 6 \ldots$$

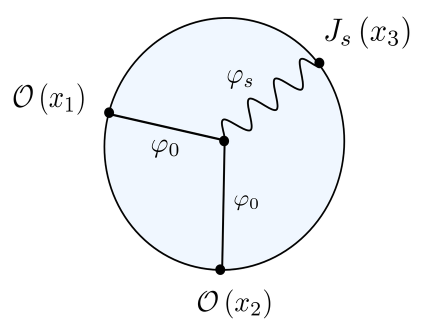
Relevant for us: Correlation functions $\leftrightarrow$ Witten diagrams

Motivation: Interactions, Witten diagrams and CFT correlation functions

The three-point function

$$\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, J_s(x_3) \rangle, \tag{1}$$

can be computed in the dual higher-spin theory from the Witten diagram below.



$$= \langle \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, J_s(x_3) \rangle$$

To compute Witten diagrams:

knowledge of the associated interactions in higher-spin theory is required.

But what if the interactions required are not known in Lagrangian form?

Need interactions of order n in the bulk fields? $\rightarrow$ compute $n-$point correlation functions!

This is the case for the higher-spin/vector model duality:
Interactions beyond cubic order are not known explicitly in higher-spin theory, but correlation functions in the vector models are comparatively straightforward to compute!

Idea: Study the quartic interactions with help from vector model four-point functions

Questions we would like to address:

- It is known that interactions in higher-spin theory are unbounded in their number of derivatives, but to what degree are they non-local?
- How is this reflected in the dual CFT?

Example: extracting the scalar quartic vertex

A good starting point: quartic vertex of the scalar φ_0 in higher-spin theory. This can be found by demanding the equality below.

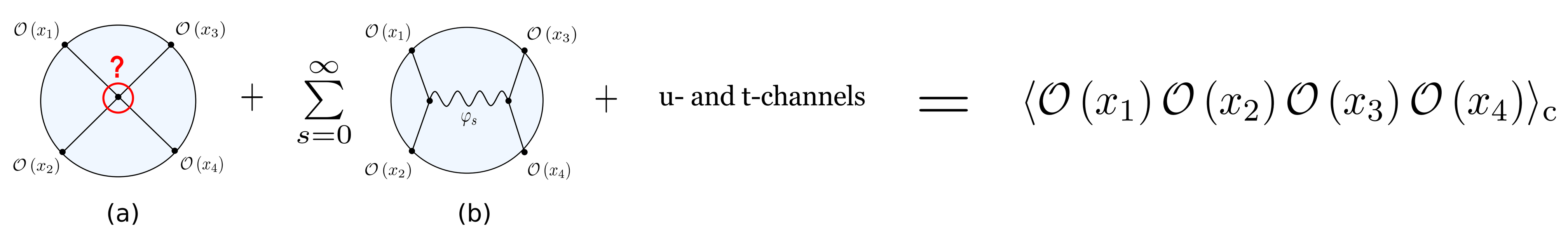


Figure 2: *The quartic interaction of the scalar in higher-spin theory is required to compute the connected four-point function of its dual operator $\mathcal{O}$ holographically.*

First step: Evaluate exchange diagrams (b)

This was completed in [1]. Ingredients:

- Bulk-to-boundary propagators for the four external scalars φ_0.
- The cubic vertex between the two scalars φ_0 and the spin-s gauge field φ_s.
- The bulk-to-bulk propagator for the exchanged spin-s gauge field.

The bulk-to-bulk propagators for gauge fields of any integer spin were also derived in [1], in three different gauges. Their explicit form was previously unknown.

Split-representation of bulk-to-bulk propagators

Computation of the exchange diagrams is facilitated by the *split representation* of the spin-s bulk-to-bulk propagators. Schematically, this is

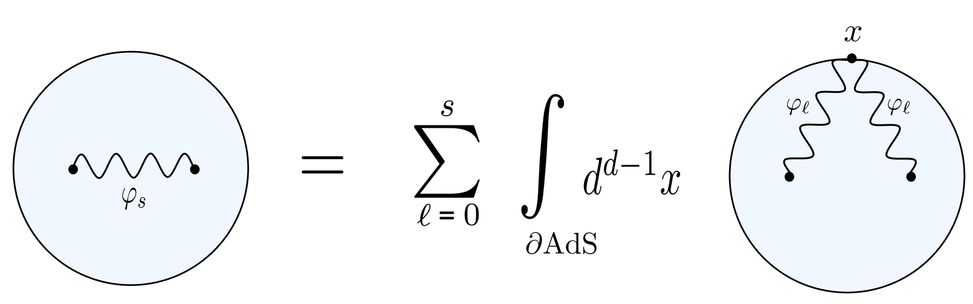
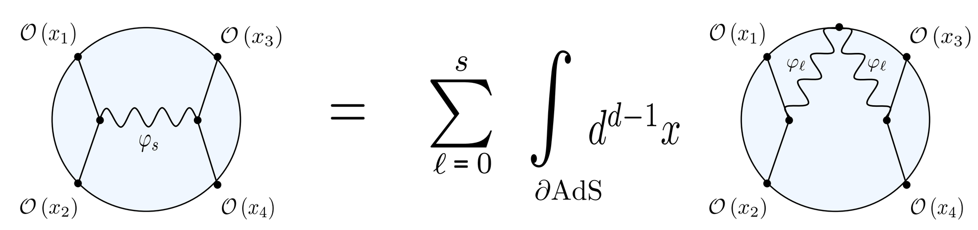


Figure 3: *A spin-s bulk-to-bulk propagator can be decomposed into products of two bulk-to-boundary propagators, integrated over their common boundary point*

Conformal block expansion

Using the split representation, the exchange diagram (b) decomposes into products of three-point functions (1). This is illustrated schematically below



This representation of the exchange diagram is in the form of a conformal block expansion in the dual CFT. This will ease comparison with CFT result, which can also be expressed in this form.

Next step: Demanding the equality in figure 2 holds will tell us about the scalar quartic vertex, and help us answer our questions above. We will report on this in the future.

For more details

[1] X. Bekaert, J. Erdmenger, D. Ponomarev and C. Sleight, "Towards holographic higher-spin interactions: four-point functions and higher-spin exchange", [arXiv: 1412.0016 [hep-th]], to be published in JHEP.