Higher-Spins and Matter Interactions in 3D

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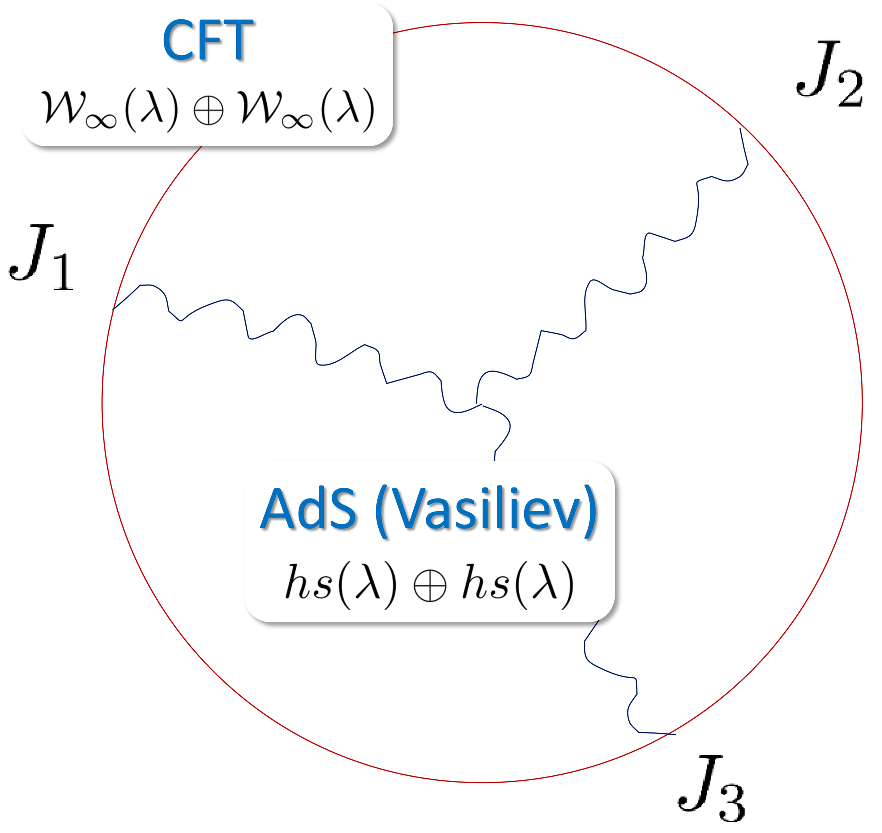

Goals

- Understand the coupling of higher-spins and scalar matter in 3d
- Extract the associated couplings from Vasiliev's equations (beyond Giombi and Yin)
- Link Vasiliev's equation to an ordinary Lagrangian formulation
- Test Gaberdiel-Gopakumar conjecture beyond symmetry considerations
- Understand non-localities in Vasiliev's theory

Caveats

- Vasiliev's equations are not manifestly covariant
- Need to solve part of the equations to find the physical equations
- Non-locality: functional class not specified
- Twisted sector: no CFT counterpart so far

$$[y_\alpha,y_\beta]_\star=2i\epsilon_{\alpha\beta}\qquad [z_\alpha,z_\beta]_\star=-2i\epsilon_{\alpha\beta}$$

$$\phi^2=1\qquad \psi^2=1\qquad \{\phi,\psi\}=0$$

$$f(y,z;\phi,\psi)=f_0(y,z;\phi)+\psi f_1(y,z;\phi)$$

$$\boldsymbol{W}\to\frac{1}{2}\varpi^{\alpha\alpha}L^s_{\alpha\alpha}-\frac{i}{4}h^{\alpha\alpha}\,\phi y_\alpha y_\alpha+\boldsymbol{W}$$

Trick: field redefinition to achieve manifest covariance

$$L^s_{\alpha\beta}\equiv-\frac{i}{2}(y_\alpha y_\beta-z_\alpha z_\beta)-\boldsymbol{T}_{\alpha\beta}$$

Vasiliev's equations

$$d\boldsymbol{W}=\boldsymbol{W}\star\boldsymbol{W}$$
$$d\boldsymbol{T}_{\alpha\beta}=[\boldsymbol{W},\boldsymbol{T}_{\alpha\beta}]_\star$$
$$d\boldsymbol{S}_\alpha=[\boldsymbol{W},\boldsymbol{S}_\alpha]_\star$$
$$\frac{i}{4}\{\boldsymbol{S}_\alpha,\boldsymbol{S}_\beta\}_\star=\boldsymbol{T}_{\alpha\beta}\qquad \text{Osp(1|2)}$$
$$[\boldsymbol{T}_{\alpha\beta},\boldsymbol{S}_\gamma]_\star=\boldsymbol{S}_\alpha\epsilon_{\beta\gamma}+\boldsymbol{S}_\beta\epsilon_{\alpha\gamma}$$

(Prokushkin & Vasiliev; Alkalaev, Grigoriev, Skvortsov)

The Vasiliev's eqs. in physical space

$$D\omega=0\quad\Rightarrow\quad\text{Flat connection}$$

$$\widetilde{D}\tilde{\omega}=\frac{1}{8}(h\wedge h)^{\alpha\alpha}(y_\alpha+i\partial^w_\alpha)(y_\alpha+i\partial^w_\alpha)C(w,\phi)\Big|_{w=0}$$

$$D\widetilde{C}=0$$

(Vasiliev, 1999)

$$\widetilde{D}C=0\quad\Rightarrow\quad(\Box+\tfrac{3}{4})\Phi(x)=0$$

$$\widetilde{D}C^{(2)}=[\omega,C]_\star+[\mathcal{V}(h,C,C)]_{d>3}$$
$$D\widetilde{C}^{(2)}=\widetilde{\mathcal{V}}(h,C,C)$$
$$\widetilde{D}\tilde{\omega}^{(2)}=\widetilde{\mathcal{V}}(h,\omega,C)$$
$$D\omega^{(2)}=\omega\star\omega+\mathcal{V}(h,h,C,C)$$

So far only one cocycle has been studied (symmetry based)

(Giombi & Yin; Ammon, Kraus & Perlmutter)

The presence of ψ twists the covariant derivative in the translational part:

Twisted Sector entangled

Not clear interpretation in CFT

$$L_{\alpha\beta}=-\frac{i}{2}y_\alpha y_\beta\qquad P_{\alpha\beta}=\phi L_{\alpha\beta}$$
$$D=\nabla-\frac{1}{2}h^{\alpha\alpha}[L_{\alpha\alpha},\bullet]=\nabla-\phi h^{\alpha\alpha}y_\alpha\partial^y_\alpha$$
$$\widetilde{D}=\nabla-\frac{1}{2}h^{\alpha\alpha}\{L_{\alpha\alpha},\bullet\}=\nabla+\frac{i}{2}\phi h^{\alpha\alpha}(y_\alpha y_\alpha-\partial^y_\alpha\partial^y_\alpha)$$

Canonical currents

$$J_{\mu_1\dots\mu_s}\approx\Phi^*\overleftrightarrow{\nabla}_{\mu_1}\dots\overleftrightarrow{\nabla}_{\mu_s}\Phi+\mathcal{O}(\Lambda)$$

Solving torsion

$$\Box\phi_{\mu_1\dots\mu_s}+\dots=g_sJ_{\mu_1\dots\mu_s}$$

Vasiliev backreaction

$$J^{gl}_{\mu_1\dots\mu_s}=g'_s\sum_l\nabla_\mu\dots\nabla_\mu\overbrace{\nabla_\nu\dots\nabla_\nu}^{l}\Phi^*\nabla^\nu\dots\nabla^\nu\nabla_\mu\dots\nabla_\mu\Phi$$

$$S_{cur}=\sum_s g_s\int\phi_{\mu(s)}J^{\mu(s)}\quad\Rightarrow\quad\delta^{(s)}\Phi=ig_s\,\epsilon_{\mu(s-1)}(-2i\nabla^\mu)^{s-1}\Phi$$

Vasiliev's cubic action(s)

$$\delta\Phi=tr\{\xi_e,C\}_\star=\sum_s(1+(-)^s)\frac{(-)^{s-1}}{(2s-2)!}\xi_e^{\alpha(2s-2)}C_{\alpha(2s-2)}$$

$$g_s=\frac{1}{[2(s-1)]!}$$

Results

- The redefinition that removes the linear backreaction on the twisted sector is non-local and ambiguous
- The above ambiguity can be fixed by the requirement that the second order backreaction on the twisted sector is exact in cohomology (this is a test of the conjectured duality)
- Another redefinition is required to the next order and it is also non-local (explicit form found)
- The source to the Fronsdal equations is found upon solving the torsion constraint
- The result differs from the canonical currents at the action level in an infinite non-local tail
- Detailed analysis of couplings, improvements and cohomology
- Remarkably: CC cohomology at form degree 2 is empty

$$\mathbf{J}=D\mathbf{U}$$

$$\int_{AdS}tr\,(\omega\wedge D\mathbf{U})=-\int_{\partial AdS}tr\,(\omega\wedge\mathbf{U})$$

Boundary non-trivial Interaction

Outlook

- Functional class of acceptable redefinitions not yet understood (non-locality)
- Possible interpretation at the action level as redefinition linking bulk and boundary
- Next step: test functional class by finding the redefinition that maps Vasiliev's non-local currents to canonical s-derivative currents
- PV theory not in the right field frame (other theory needed like the d-dimensional theory at d=3)