Quantum Dynamics & Instantons in Supergravity

UNIVERSITY OF CAMBRIDGE

Carl Turner

DAMTP

Gravitational instantons

Quantum gravity is an excellent effective theory. Wick rotated, it has instanton solutions, which often have different topology:

$$Z = \sum_{\mathcal{M}} \int \mathcal{D}g \; \exp\left(- \int \mathrm{d}^4 x \, \mathcal{R} \right)$$

Just as in QCD, these are the saddle points of the action in the landscape,

and they are usually also self-dual, meaning ${}^{\star}\mathcal{R}_{\mu\nu\rho\sigma} = \pm \mathcal{R}_{\mu\nu\rho\sigma}$.

The analogy with QCD is actually very good. There are gravitational anomalies for fermions, for example, and a corresponding θ angle, which fit into this framework perfectly. More intriguingly, as we will see in the next column, there is also an equivalent of Λ_{QCD}.

▶ How does perturbative physics interact with instantons?

A hidden scale

In gravity, we can include a Gauss-Bonnet term

$$S_\alpha = \frac{\alpha}{32\pi^2} \int \mathrm{d}^4 x \, {}^{\star}\mathcal{R}^{\star}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma}$$

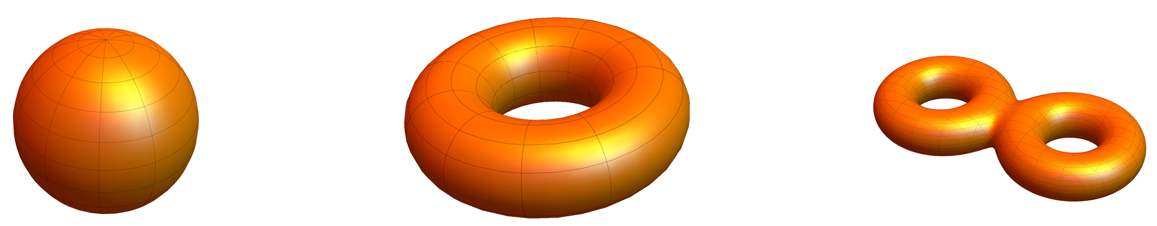
where the highlighted expression is the Euler characteristic χ.

But at one loop, this coupling runs logarithmically. Gravity is not one loop finite. We find a β function with coefficients known given the matter content, and have to let $\alpha \to \alpha(\mu)$. But just like in QCD, this introduces a new RG-invariant scale into the theory,

$$\boxed{\Lambda_{\mathrm{grav}} \equiv \mu \exp\left(- \frac{\alpha(\mu)}{2\beta} \right)}$$

It can naturally be exponentially small!

What's more, wherever gravitational instantons appear, they should come with this scale Λ_{grav}. Schematically:



$$Z = \left(\Lambda_{\mathrm{grav}}^{2\beta} \right)^2 Z_{\chi=2} + \left(\Lambda_{\mathrm{grav}}^{2\beta} \right)^0 Z_{\chi=0} + \left(\Lambda_{\mathrm{grav}}^{2\beta} \right)^{-2} Z_{\chi=-2} + \cdots$$

As a side note, it is also naturally complex, with the phase being given by the corresponding θ angle.

▶ What is an example where this scale appears?

A concrete example

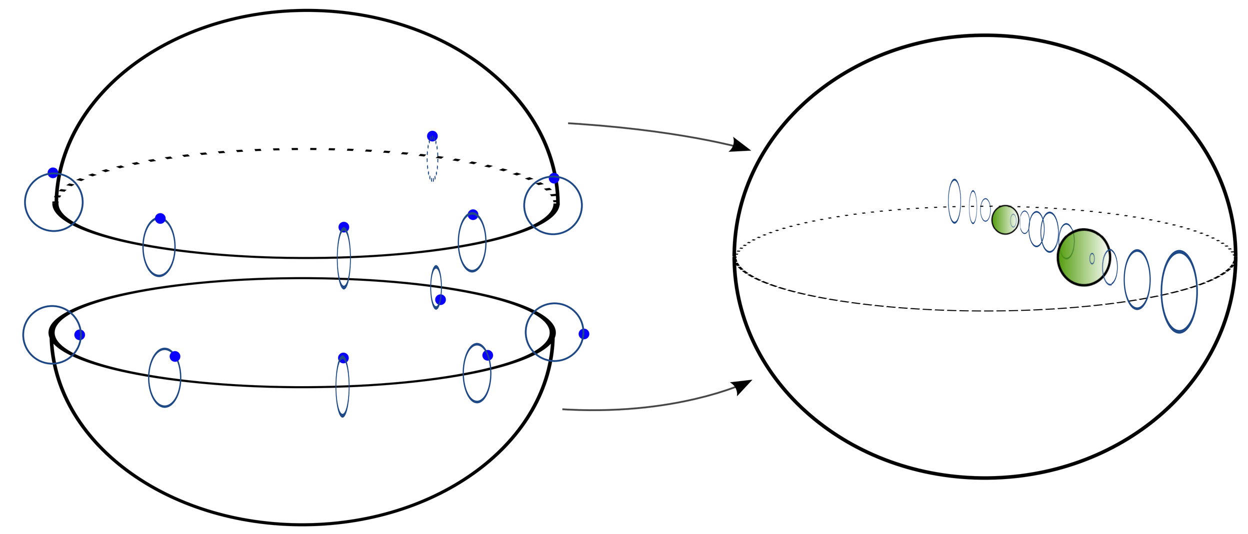
Imagine we live in $\mathbb{R}^3 \times S^1$, for simplicity with $\mathcal{N} = 1$ supergravity. Dimensionally reducing gives a gravitational multiplet and a chiral multiplet containing R, the radius of the circle:

$(g_{(4)}, \Psi)$ → $(g_{(3)}, \psi)$

$(g_{(4)}, \Psi)$ → $(R^2 + i\sigma, \chi)$

Perturbatively, we see no potential for R, so the theory looks stable.

But there are an infinite number of well-controlled instantons also contributing to the path integral: the Taub-NUT manifolds. The S^1 shrinks smoothly to a point at a NUT.



The one-instanton contribution gives rise to a non-zero correlator $\langle \chi\chi \rangle \propto e^{-S}$, which implies a superpotential for the chiral multiplet — we calculate this exactly at one loop, giving a scalar potential

$$V(R) \propto \left(\Lambda_{\mathrm{grav}} R \right)^{41/24} \exp\left(-4\pi^2 M_{\mathrm{pl}}^2 R^2 \right)$$

The compactification is unstable!

Supergravity determinants

An interesting technicality: we can exactly evaluate one loop determinants in our k-instanton backgrounds.

$$\frac{\text{det fermions}}{\text{det bosons}} = C \, \mu^{41k/24} \, R^{-7k/24} \int_{\text{moduli}} \mathrm{d}X$$

where C is a known constant, and μ is an ultraviolet cutoff. In particular the determinants fail to cancel.

Things to take away

▶ There is an **undetermined scale** Λ_{grav} in generic gravitational theories.

▶ We can perform well-defined, **quantitative** calculations in quantum gravity.

▶ Compactifying $\mathbb{R}^4$ on a circle is **unstable** even with supersymmetry.

Open questions

▶ How do these manipulations generalize to $\mathcal{N} = 2$?

▶ Where does Λ_{grav} crop up in cosmology and beyond?

▶ What instantons are relevant in, say, wormhole physics?

Image credit: Davide De Martin (ESA/Hubble)

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c.p.turner@damtp.cam.ac.uk, d.tong@damtp.cam.ac.uk