Euclidean Supergravity

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Motivation

Physics

1. Related to Lorentzian theories by **dim. reduction/lifting over time.**

2. Construct and (geometrically) classify soliton and instanton solutions.

3. New types of compactifications, extend concepts of 'stringy' geometry.

Mathematics

1. Interesting para-complex and para-quaternionic special geometry.

2. Maps between special geometries induced by dimensional reduction.

3. Geometric structure of solutions, e.g. Einstein, Kähler and $SU(n)$ strs.

4D Euclidean Supergravity

The action of 4D, $\mathcal{N} = 2$ supergravity coupled to vector-multiplets with spacetime signature $(\varepsilon + + +)$ is given by [1]

$$\mathcal{L}_4 \sim R - \tfrac{1}{2} g_{A\bar{B}} \partial_\mu z^A \partial^\mu \bar{z}^B - \tfrac{1}{4} \mathcal{I}_{IJ} F^I_{\mu\nu} F^{J\mu\nu} - \tfrac{1}{4} \mathcal{R}_{IJ} F^I_{\mu\nu} \tilde{F}^{J\mu\nu} \qquad (*)$$

The scalar fields z^A are complex if $\varepsilon = -1$ and para-complex if $\varepsilon = +1$.

Definition: Para-complex numbers C *(a.k.a. split-complex)*

$$C = \mathbb{R}\underline{1} + \mathbb{R}\underline{e} \qquad \text{where} \qquad \underline{1}.\underline{1} = \underline{1} \qquad \underline{1}.\underline{e} = \underline{e}.\underline{1} = \underline{e} \qquad \underline{e}.\underline{e} = \underline{1}$$

Remark: Para-cx numbers contain zero-divisors, e.g. $(\underline{1} + \underline{e}).(\underline{1} - \underline{e}) = 0$.

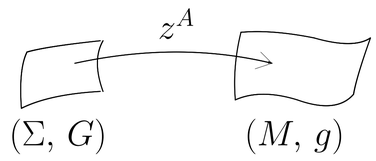
Remark: One can identify $C = \mathbb{R} \oplus \mathbb{R}$ through $x\underline{1} + y\underline{e} \mapsto (x + y, x - y)$.

Special Geometry

Theories with extended supersymmetry ($\mathcal{N} \geq 2$) have two types of geometry:

1) Geometry of spacetime Σ

2) 'Special' geometry of scalar target space M



Interesting types of special geometry occur in $D = 3, 4, 5$, $\mathcal{N} = 2$ supergravity coupled to vector-multiplets, with Lorentzian or Euclidean spacetime sigs. Note: in $D = 3$ vector-multiplets are dual to hyper-multiplets.

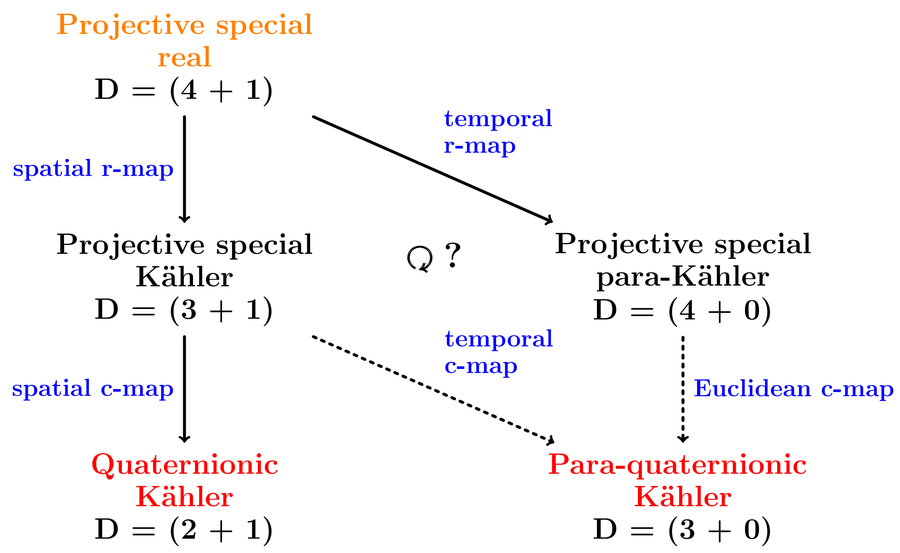


FIGURE 1: Maps between various types of special geometry induced by dim. reduction.

Theorem: (Cortés, Dempster, Mohaupt & **OV**, 2015 [2])
Manifolds in the images of the temporal and Euclidean c-maps are para-quaternionic Kähler.

Open Problems:

- Does the diagram in figure 1 commute?
- Construction of para-HQ/para-QK correspondence?

New 4D Solitons and Instantons

Idea:

Reduce 4D, $\mathcal{N} = 2$ supergravity $(*)$ over timelike or spacelike S^1 to obtain 3D Euclidean supergravity. Spacetime metrics related by

$$G_4 = \varepsilon e^{\phi} (dt + w)^2 + e^{-\phi} G_3 \, .$$

The 3D equations of motion are easier to solve (vectors dualise to scalars). Solutions correspond to geodesics in para-quaternionic Kähler target mf. New 3D instanton solutions lift to either new soliton or instanton solutions in 4D (depending on the signature of 4D spacetime).

New constructions:

1. Non-extremal AdS black holes in gauged supergravity [4, 5, 6].

2. Euclidean analogues of AdS black holes and dS spaces. Generalise single-centre Gibbons-Hawking instantons.

10D to 4D Euclidean Supergravity over a CY_3

Theorem: (Sabra & **OV**, 2015 [3])
Reduction of 11D supergravity over a timelike circle followed by a CY_3 results in 4D, $\mathcal{N} = 2$ Euclidean supergravity coupled to vector- and hyper-multiplets.

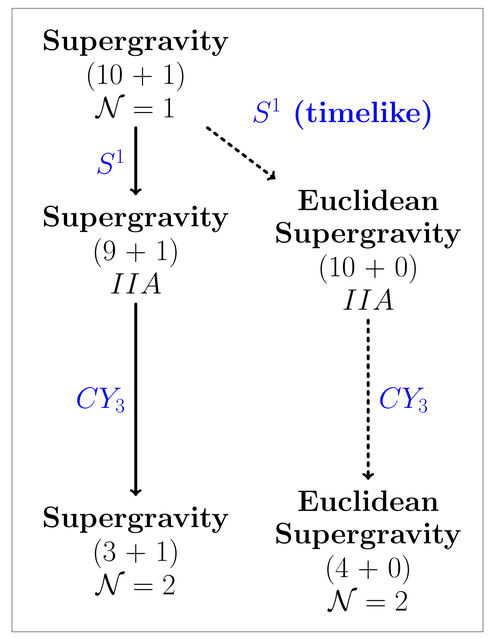


FIGURE 2: Possible reductions of 11D supergravity to 4D.

Open Problems

- How does this fit into generalised geom.?
- New instanton solutions of string theory, and soliton solutions of M-theory?
- Euclidean string theories?
- Euclidean gauge/gravity duality?

References

[1] V. Cortés and T. Mohaupt, JHEP 0907 (2009) 066 [arXiv:0905.2844].

[2] V. Cortés, P. Dempster, T. Mohaupt and **OV**, in preparation.

[3] W. A. Sabra and **OV**, [arXiv:1503.05095].

[4] D. Klemm and **OV**, JHEP 1301 (2013) 053 [arXiv:1207.2679].

[5] D. Klemm and **OV**, Class. Quant. Grav. 30 (2013) 065003 [arXiv:1211.1618].

[6] A. Gnecchi, K. Hristov, D. Klemm, C. Toldo and **OV**, JHEP 1401 (2014) 127 [arXiv:1311.1795].

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