Geometry and Quantum Noise

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CMS

Cambridge

a panorama of previous work...

Peres (1984)

Deutsch (1991)

Srednicki (1994)

Maldacena (2001)

Dyson, Kleban & Susskind (2002)

Birmingham, Sachs & Solodukhin (2003)

Barbon & Rabinovici (2003)

Kleban, Porrati & Rabadan (2004)

Festuccia & Liu (2007)

RECENTLY...

Marolf & Polchinski
Shenker & Stanford
Susskind
Balasubramanian, Berkooz, Ross & Simon
Barbon & Rabinovici

- **Introduction- What Does(does not) Geometry capture?**
- **Geometry Topology and Quantum Noise I - QFT, BH Information**
- **VERY(!) long time correlations. VERY small.**
- **Quantum Noise II- ב מועד-Firewalls?**
- **Geometry and Quantum Noise II**
- **Discussion**

Round I

N=4 describes also a theory of a string moving in a background a AdS_5 X S^5 And a black hole in AdS_5 X S^5

The AdS/CFT Correspondence

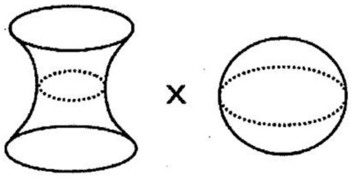
[Maldacena '97]

D=4, N=4 , SUSY Y.M. SU(N)

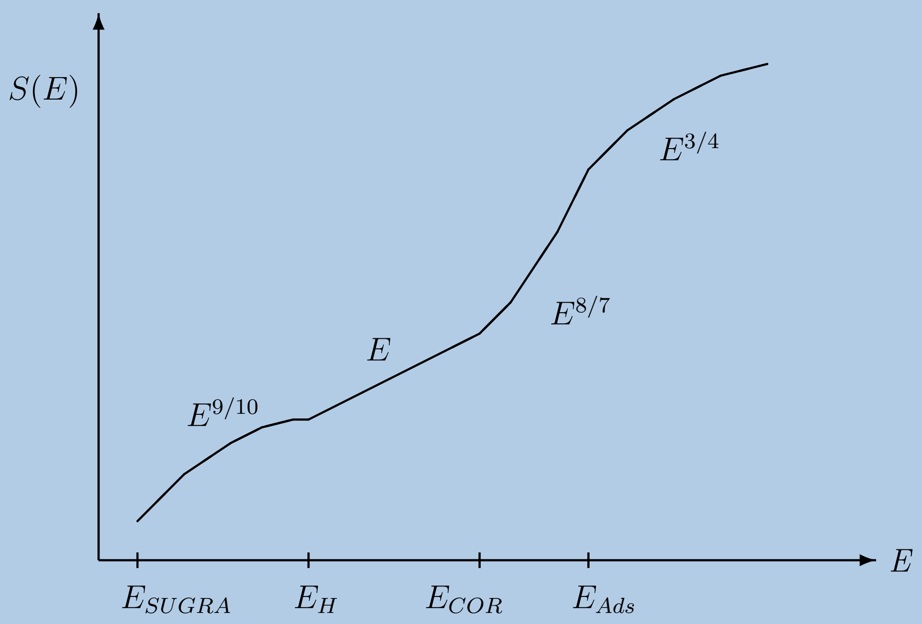
't Hooft coupling: $\lambda = N g_{\mathrm{YM}}^2$ 1/color number: $\frac{1}{N}$ theta angle: θ_{YM}

$\mathcal{N} = 4$ SYM was conjectured to be dual to a string theory:

IIB Superstrings on $AdS_5 \times S^5$



tension: $\frac{R^2}{\alpha'} = \sqrt{\lambda}$ coupling: $g_s = \frac{\lambda}{4\pi N}$ axion: $\langle C \rangle = \theta_{\mathrm{YM}}$


$S(E)$
$E^{3/4}$
$E^{8/7}$
E
$E^{9/10}$
E
E_{SUGRA}
E_H
E_{COR}
E_{Ads}

- AdS_5 metric

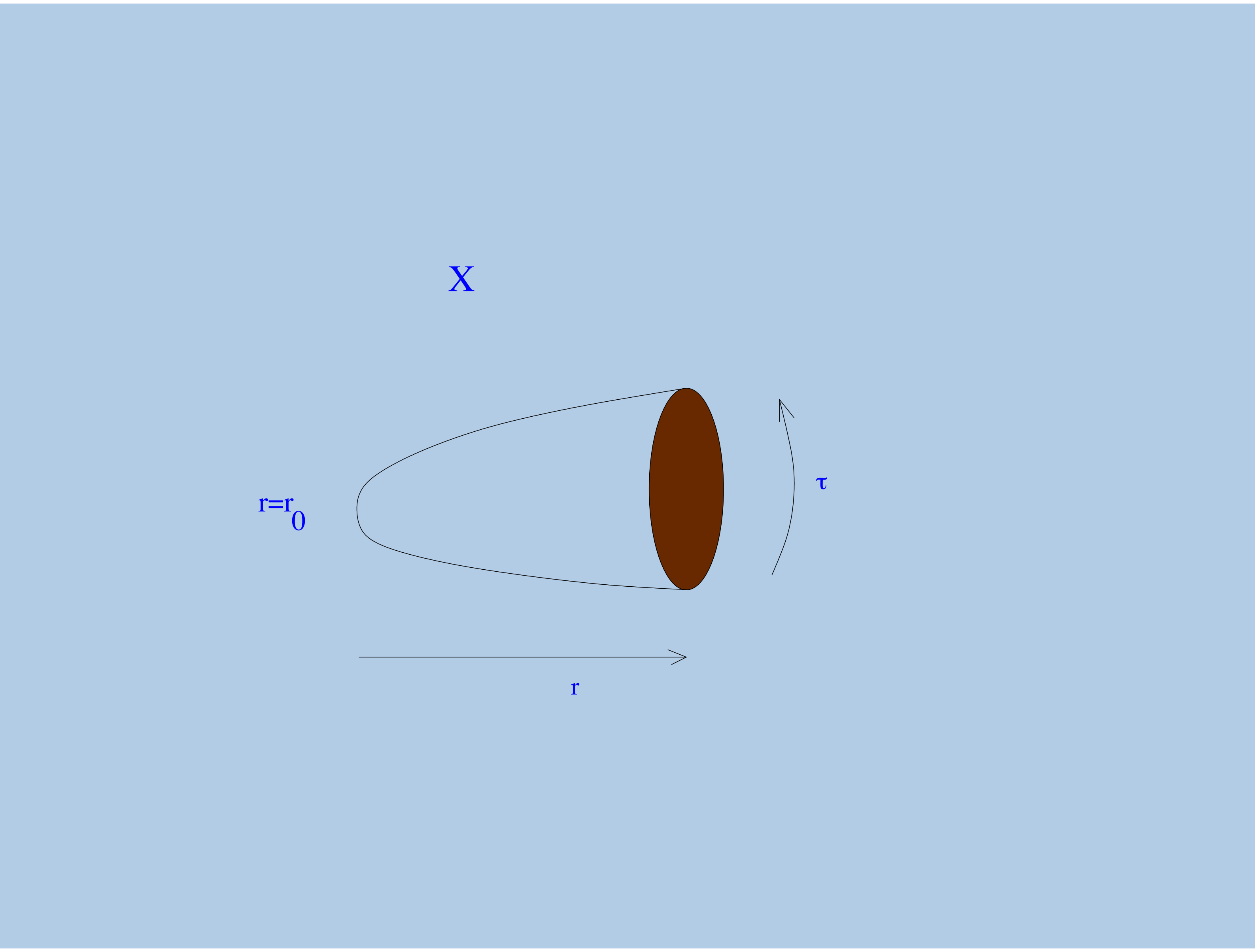
$$ds^2 = -\left(1 + \frac{r^2}{R^2}\right) dt^2 + \left(1 + \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2 \, d\Omega_3^2 + R^2 \, d\Omega_5^2$$

- Effective temperature

$$T(r) = \frac{T(0)}{\sqrt{1 + r^2/R^2}}$$

- Black Hole in AdS_5 metric

$$\begin{aligned} ds^2 \; &= \; -\left(1 + \frac{r^2}{R^2} - \frac{M}{Cr^2}\right) dt^2 + \left(1 + \frac{r^2}{R^2} - \frac{M}{Cr^2}\right)^{-1} dr^2 \\ &+ \; r^2 \, d\Omega_3^2 + R^2 \, d\Omega_5^2 \end{aligned}$$


X
r=r0
τ
r

- For $T < 1/R$
  Only thermal AdS

- For $T \gtrsim 1/R$
  Thermal AdS plus BH in AdS,
  (actually two Black Holes)
- For $T > 1/R$
  BH dominates

Black Hole Information Paradoxes

- **BH formation paradox**
- **Eternal BH paradox** (Maldacena)
  Tool for CFT $\Longrightarrow$ AdS

- **In Principle**

  Initial bulk state $\Longrightarrow$ Initial CFT state

  . $\Downarrow$

  Final bulk state $\Longleftarrow$ Final CFT state

  Instead consider slight deviation from thermal equilibrium on the field theory side

Consider

$$G(t) = Tr\left[\rho A(t) A(0)\right]$$

For very large time scale

$$C(t) = \sum_{mn} \rho_m \, B_{mn} \, B_{nm} \, e^{i(E_m - E_n)t} \ ,$$

Aspects of Long Time Scales in Field Theory

Classical		Quantum
Compact Phase Space	$\Longleftrightarrow$	Discrete Spectrum
Volume Conservation	$\Longleftrightarrow$	Unitarity

Then, If

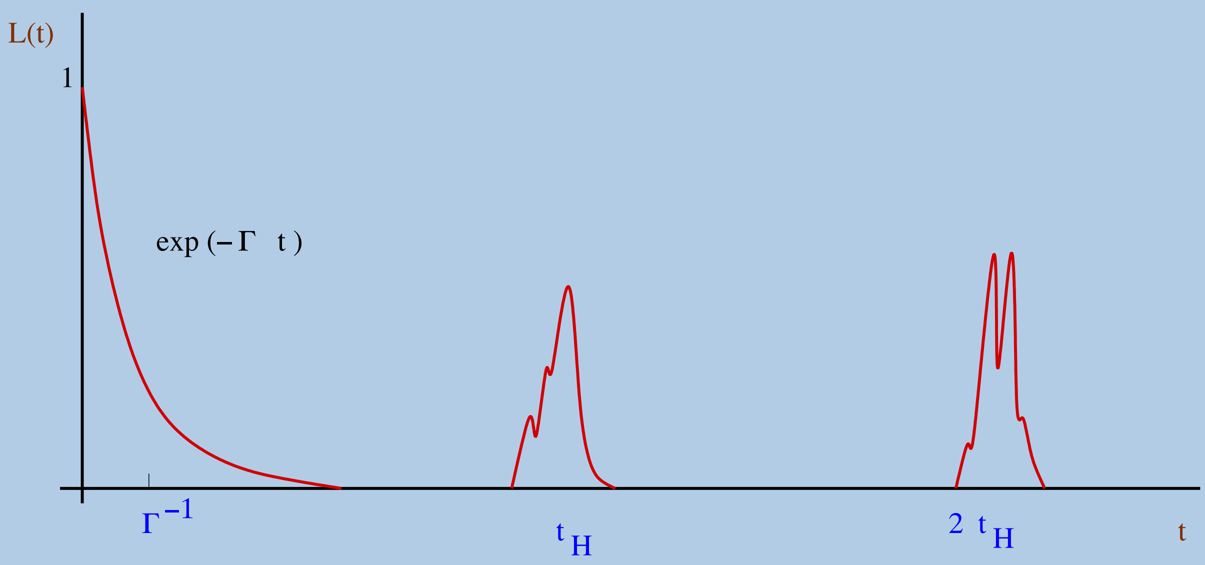
$$G(t_0) =< \theta_1(t_0, x_1)|\theta_2(0, x_2) >$$

for any ϵ there is a $t^P(\epsilon)$ such that

$$|G(t^P(\epsilon)) - G(t^0)| < \epsilon$$

You See It All!

$$C(t) = \sum_{mn} \rho_m \, B_{mn} \, B_{nm} \, e^{i(E_m - E_n)t} \ ,$$



L(t)
1
exp (− Γ t)
Γ −1
t H
2 t H
t

$$\overline{C(t)} \equiv \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau dt\, C(t) = \sum_m \rho_m |B_{mm}|^2 \; .$$

Generically positive. For B with no diagonal terms average the square.

An estimate gives a normalisation Exp(-S) times

a number. So the decay must stop, the discrete

nature of the spectrum felt and the magnitude

is Exp(-S) *

NOISE

The Noise is defined by

$$|\mathrm{noise}| \equiv \left[\overline{|C(t)|^2}\right]^{1/2}$$

$$\overline{|C(t)|^2} = \sum_{mnrs} \rho_m \rho_r |B_{mn}|^2 \, |B_{rs}|^2 \, \overline{e^{i(E_m - E_n + E_s - E_r)t}} \, .$$

B has no diagonal elements so

$$E_m = E_r \text{ and } E_n = E_s$$

$$|\text{noise}| = \left[\sum_{mn} \rho_m^2 \, |B_{mn}|^4\right]^{1/2}$$

$$|\text{peak}| \sim |C(t)|_{\max} = \sum_{mn} \rho_m \, |B_{mn}|^2$$

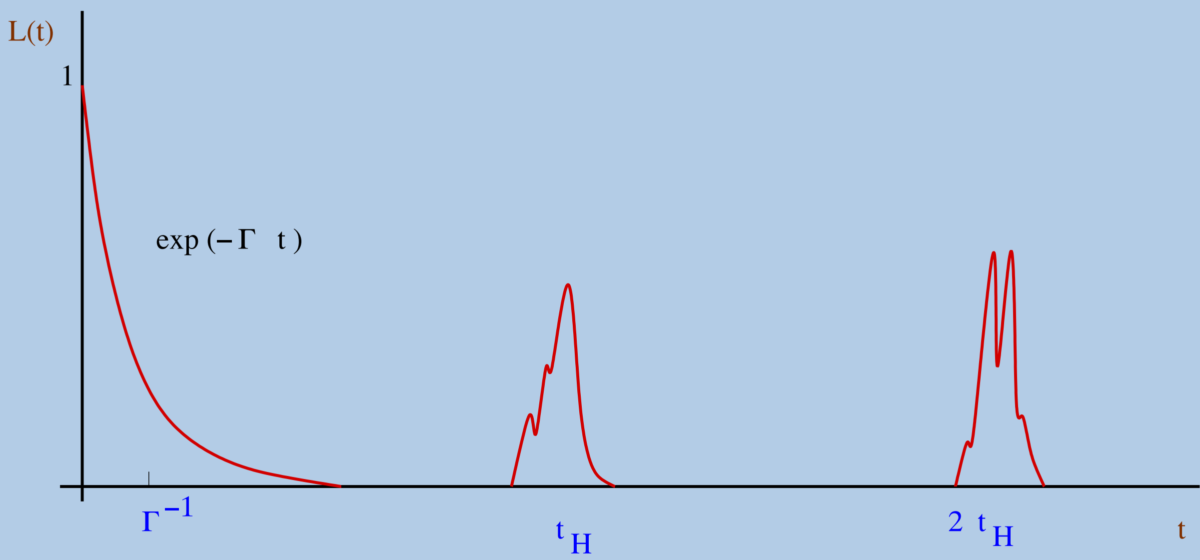
$$\frac{|\text{noise}|}{|\text{peak}|} = \left[\frac{\sum_{mn} \rho_m^2 \, |B_{mn}|^4}{\left(\sum_{mn} \rho_m \, |B_{mn}|^2\right)^2}\right]^{1/2} \, .$$

= Exp(-S)

SQRT{Exp(2S)/Exp(4S)}

The Time is takes to reach the average if the decay is exponential is

t ~ S



L(t)
1
exp (−Γ t)
Γ −1
t H
2 t H
t

Γ is not universal

$$t_H = \frac{1}{< w >} \qquad < w > = < E_i - E_j >$$

$$< w > \sim \frac{\Gamma}{\Delta n_\Gamma},$$

Δn_Γ is the number of states in a band of width Γ.

$$t_H \sim \frac{1}{\Gamma} \exp(S(\beta))$$

$$t^P(\epsilon) \sim \exp(f(\epsilon) \exp S) \qquad |G(t^P(\epsilon)) - G(0)| < \epsilon$$

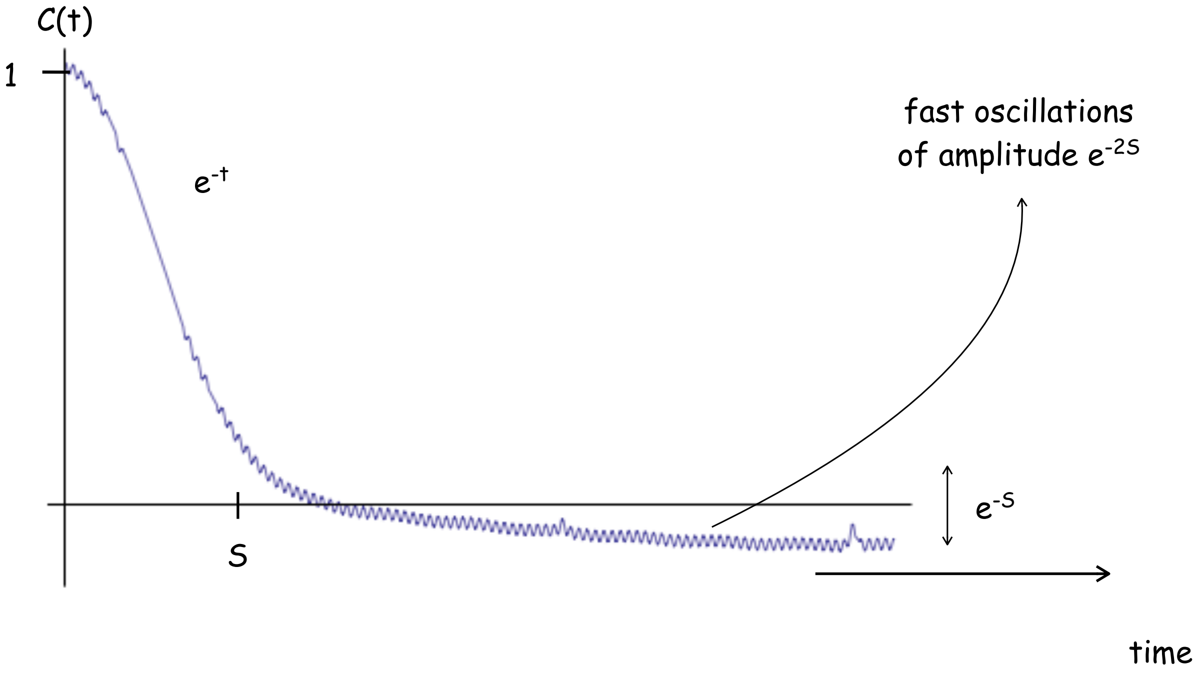
$$C(t) = \sum_{mn} \rho_m \, B_{mn} \, B_{nm} \, e^{i(E_m - E_n)t} \ ,$$

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



Page time scale

Estimate of Poincare Time

Consider “clocks” Exp(iEt)

$t=1/\nu$ $\nu = (\Delta\alpha/2\pi)^{Neff}$

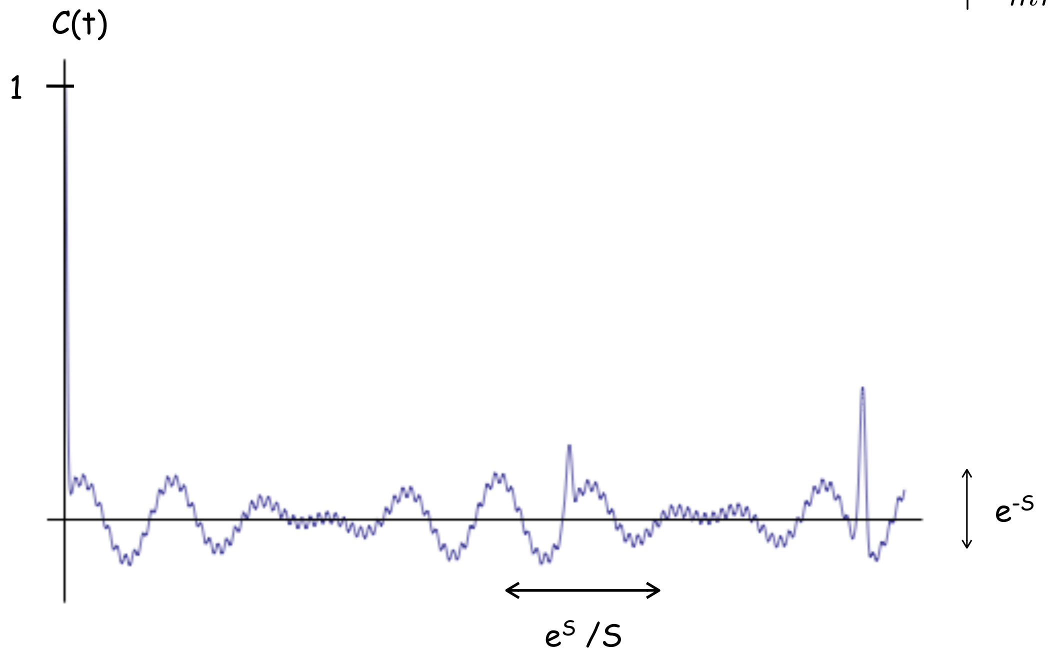
$Et \sim \exp(Neff \log(2\pi/\Delta\alpha)) \sim \exp(\exp(S^* \log(2\pi/\Delta\alpha)))$

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



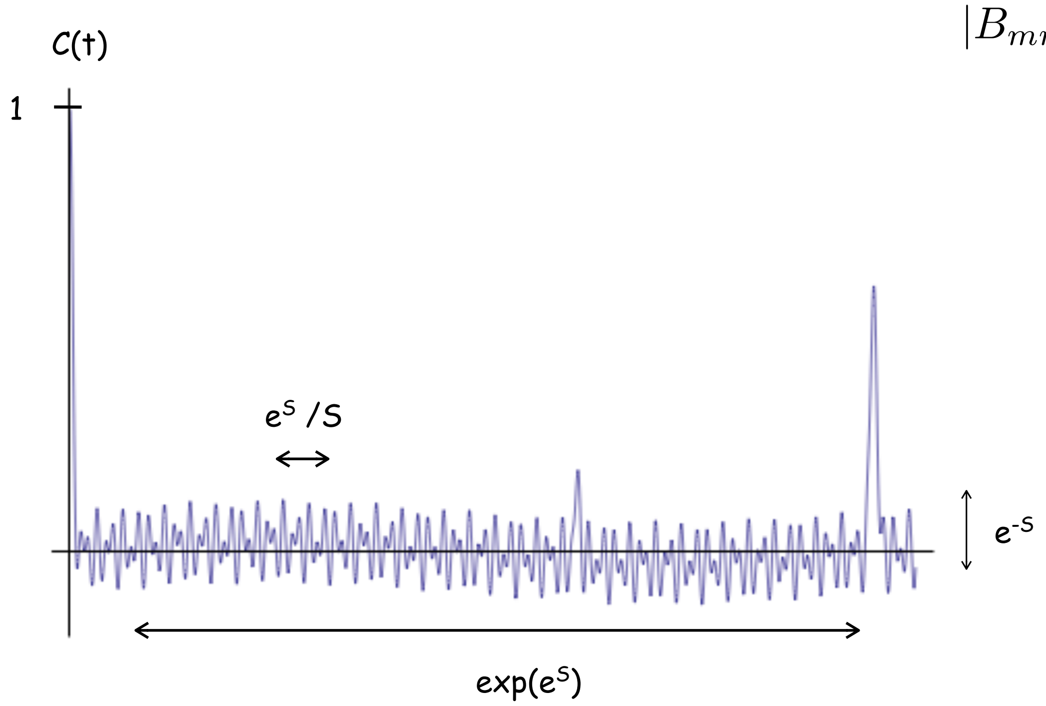
Heisenberg time scale

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



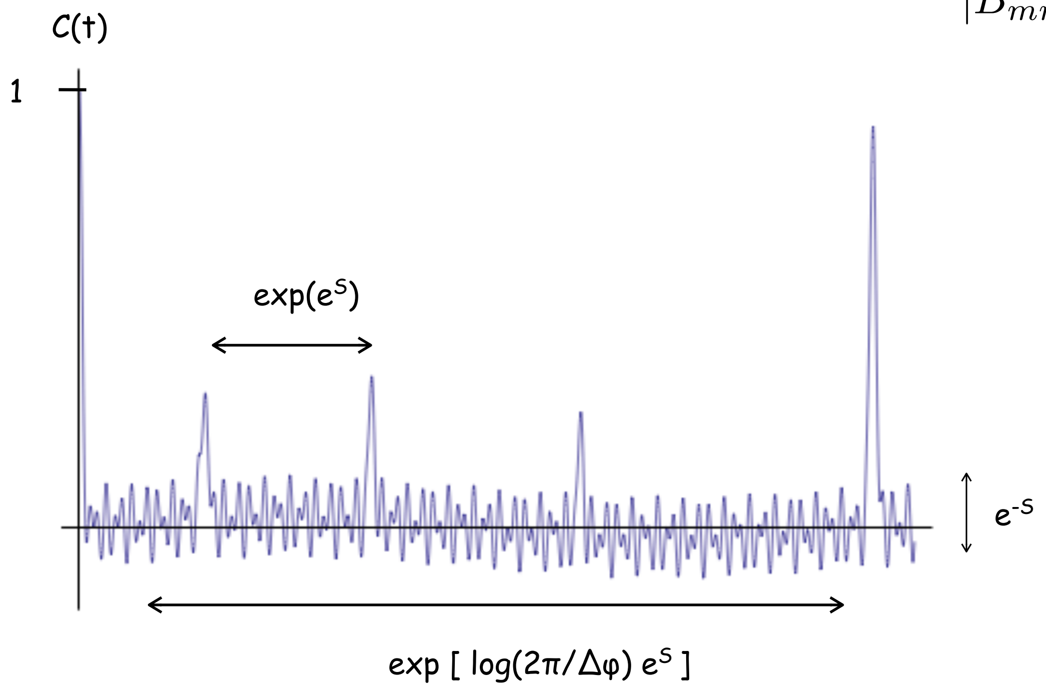
Poincaré time scale

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



detailed Poincaré time scale

Some Proportion -Page Times S

In units of the Universe's Life UL

Page time for a BH the size of a proton 10^10 ULs

Page time of a BH in 10^9 sm Quasar 3 km 10^87 ULs

This is just S!!!

One reaches for Poincare 10 to the 10 five times…

Summary:

Time Scales related by Log

Log S - Scrambling time BH, 1/S boundary(UP,T)

S-Page time, end of decay

Exp(S)- Heisnberg time

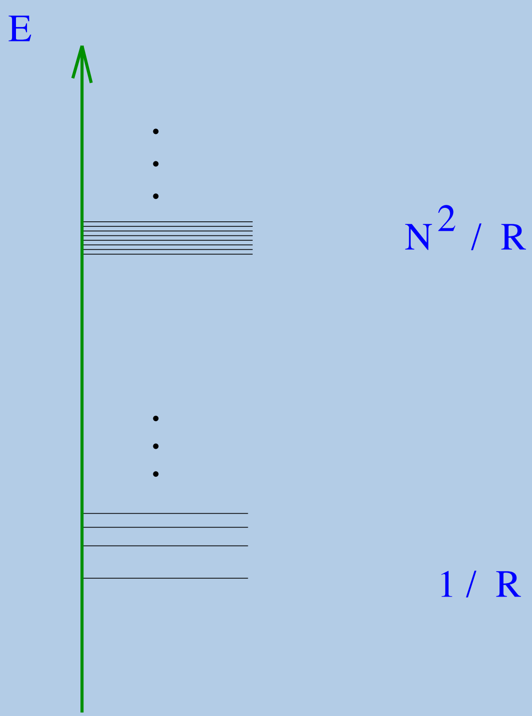
Exp(Exp(#S))- Poincare time

Consider

$$L(t) = \left| \frac{G(t)}{G(0)} \right|^2$$

$$\bar{L} = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt\, L(t)$$

The CFT is unitary and has a Gap


E
N2 / R
1 / R

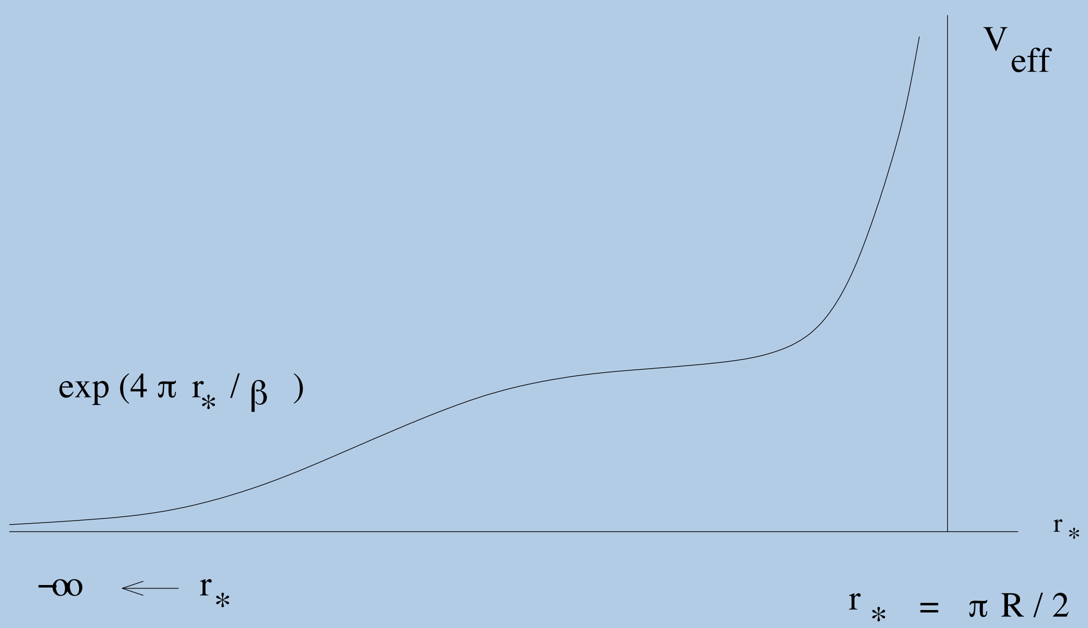
$$\bar{L} \sim \frac{\Delta L}{\Gamma t_H} \sim \exp(-S(\beta))$$

,

$$\bar{L} \sim \exp(-N^2 \ldots) \sim \exp\left(-\frac{1}{G_N} \ldots\right)$$

Non Perturbative from Gravity Point of View

For BH background $\bar{L} \to 0$, Reason:
No Gap in the presence of a BH.



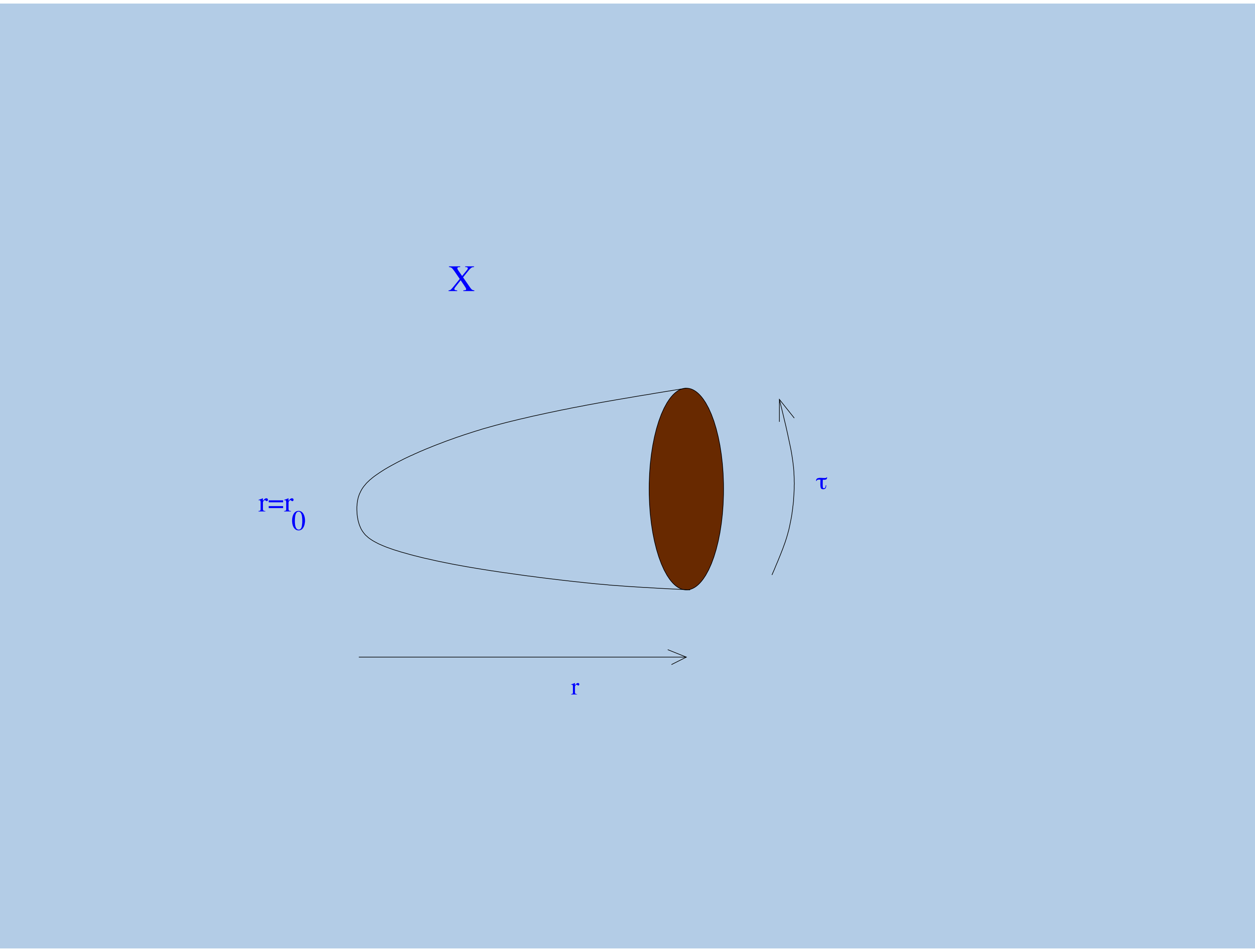
V_{eff}
exp (4 π r_* / β)
r_*
$-\infty \leftarrow r_*$
$r_* = \pi R / 2$

$$\bar{L}_{CFT} = \exp(-S) \Rightarrow \bar{L} = \exp(-S)$$

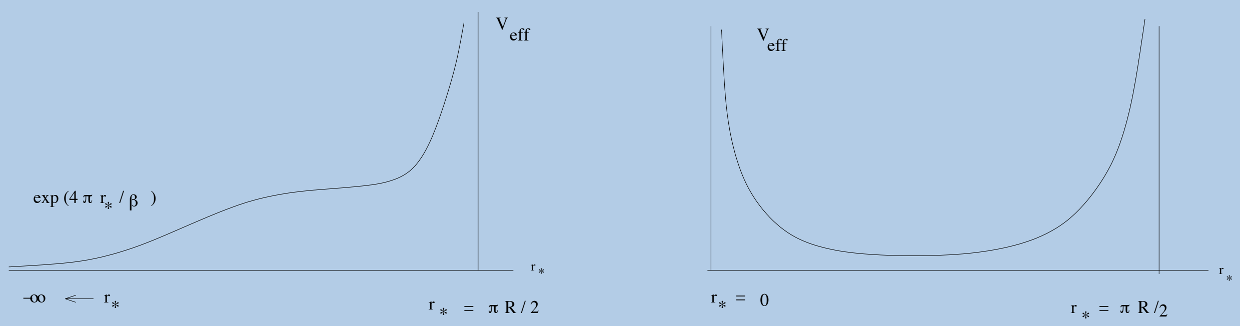
But it seems $\bar{L}_{Bulk} = 0$

Contradiction ?

Poincarè Recurrences and Topological Diversity

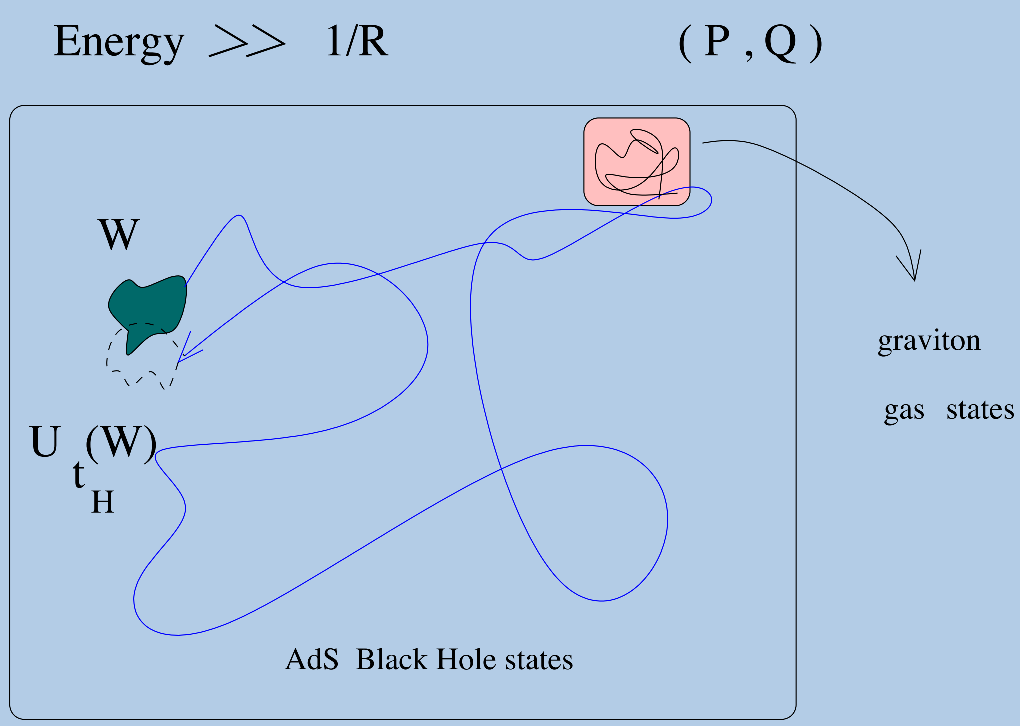


X
r=r0
τ
r

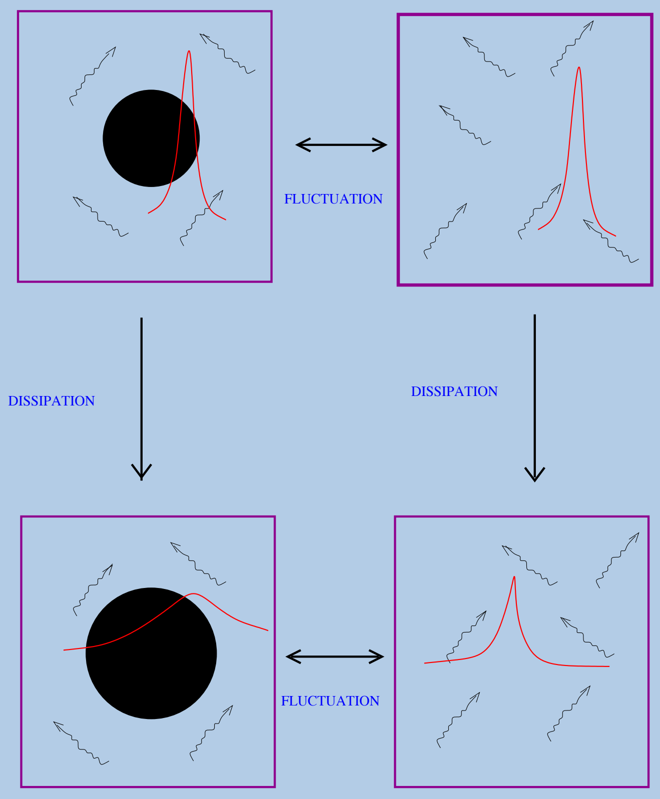


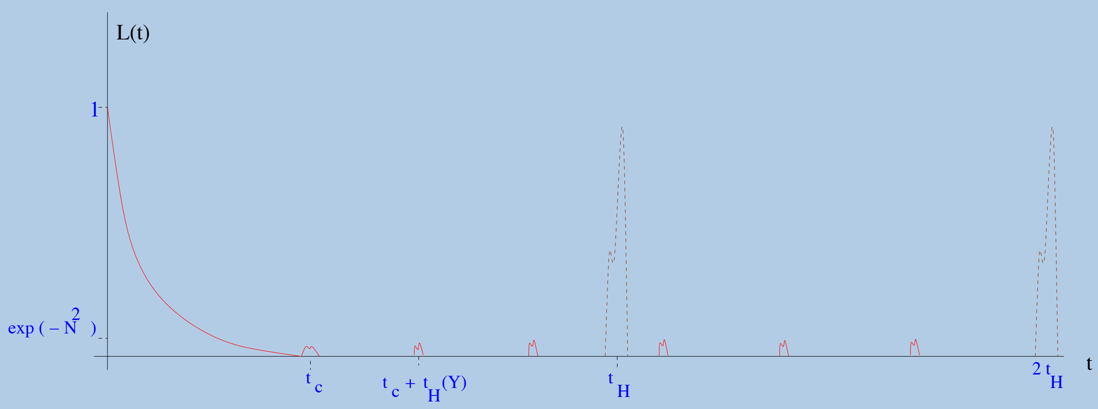
In a Thermal AdS Background a gap is formed and now

$$\bar{L}_{Bulk} \approx \exp(-S) > 0$$



Energy >> 1/R
(P , Q)
W
U t_H (W)
graviton
gas states
AdS Black Hole states


FLUCTUATION
DISSIPATION
DISSIPATION
FLUCTUATION



L(t)
1
exp (− N
2
)
t
c
t
c
+ t
H
(Y)
t
H
2 t
H
t

- $\bar{L}$ **reasonable course grained**
- $L(t)$ **not reproduced**
- **Stretched horizon, Brick Wall?**

Conclusions

- **The Burden of Proof That a Well Defined Information Paradox Exists Shifts to Claimer**
- **Topological Diversity is Required**
- **String theory is Quite a Formidable Bastion of Consistency**

Geometry Reproduces Average Result

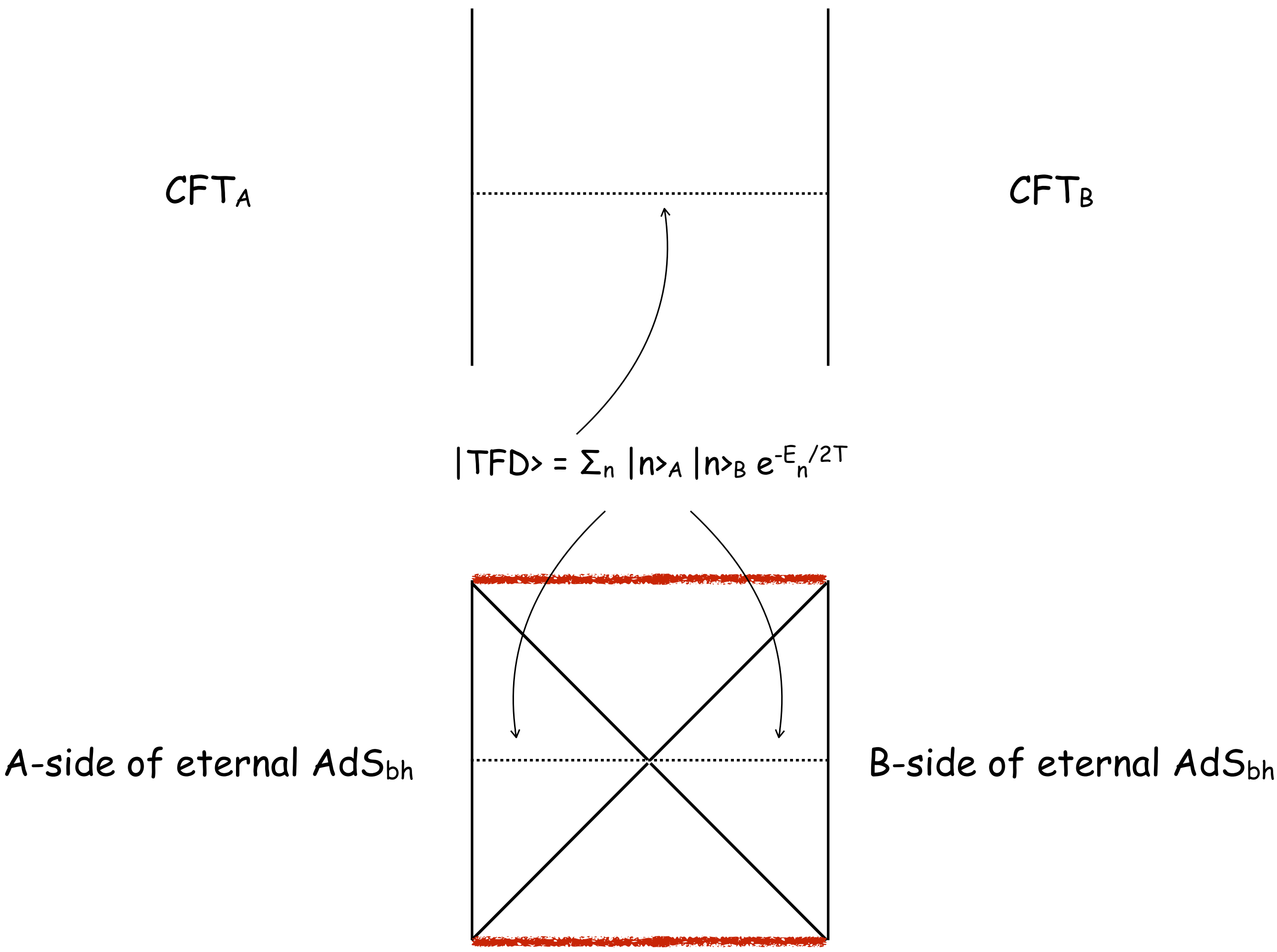
Geometry May Well Miss some Exp(-S) Features.

Question: Is the failure of the thermodynamically dominant contribution to reproduce the average quantum noise accidental?

Firewall

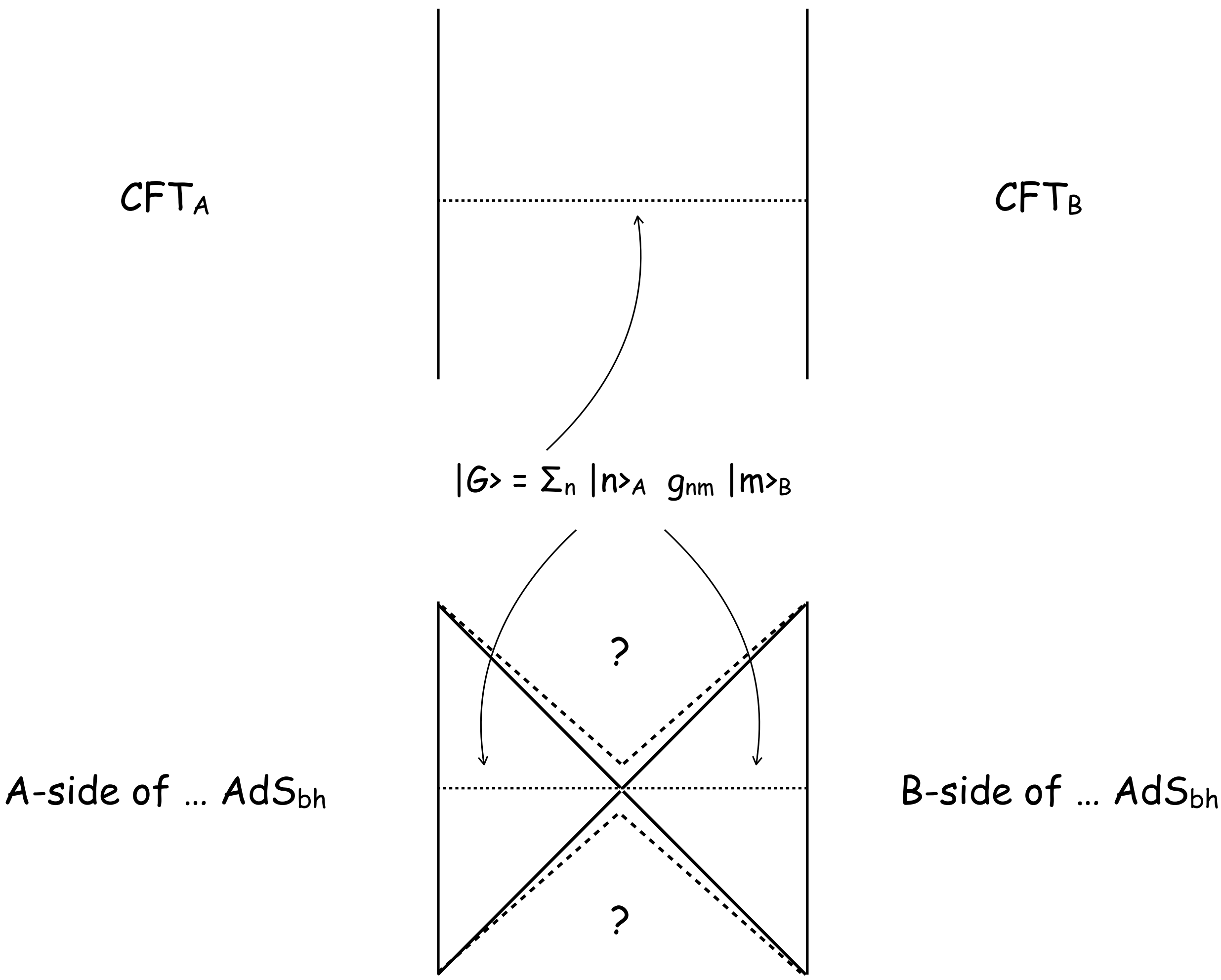
ER= EPR

???

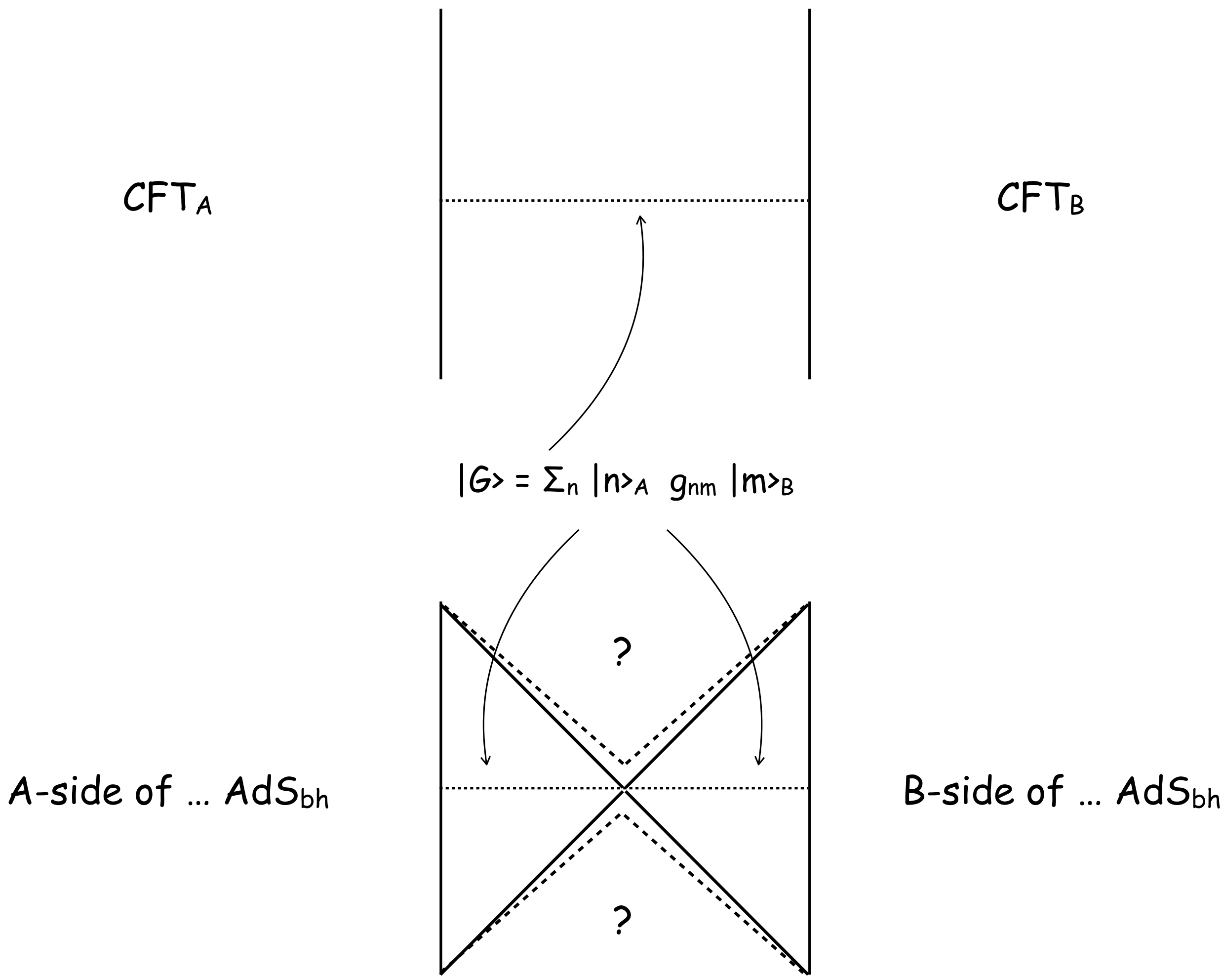

CFT_A
CFT_B
|TFD> = Σ_n |n>_A |n>_B e^{-E_n/2T}
A-side of eternal AdS_bh
B-side of eternal AdS_bh

$$C(t=0)\sim 1$$

- Noise~ $Exp(-S)$



CFT_A
CFT_B
|G> = Σ_n |n>_A g_nm |m>_B
?
?
A-side of … AdS_bh
B-side of … AdS_bh


CFT_A
CFT_B
|G> = Σ_n |n>_A g_nm |m>_B
?
?
A-side of … AdS_bh
B-side of … AdS_bh

More General Case

- **Two different operators.**
- **If the density matrix is diagonal in the energy basis-there is only dependence on time differences.**
- **Otherwise there is a dependence on both times.**
- **Even for a diagonal density matrix there is no generic peak at t=0**

$$G_{BB'}(t,t') = \mathrm{Tr}\left[\,\rho\, B(t)\, B'(t')\,\right] = \sum_{mnr} \rho_{mn}\, B_{nr}\, B'_{rm}\, e^{i(E_n - E_r)(t-t')}\, e^{-i(E_m - E_n)t'}$$

General Two Sided State leads to a One Sided Density Matrix

$$|G\rangle = \sum_{mn} g_{mn}\,|m\rangle_A \otimes |n\rangle_B$$

$$\rho_{nn'} = \sum_{m} g_{mn}\, g^{*}_{mn'}$$

One Can Consider EPR Correlations on Two and One Sides.

$$G_{AB}(t_A, t_B) = \langle A(t_A)\, B(t_B) \rangle_G = \langle G| e^{-it_A H}\, A\, e^{it_A H}\, e^{it_B H}\, B\, e^{-it_B H} |G\rangle$$

$$G_{AB}(t_A, t_B) = \langle G(t_B) \,|\, A(t_A - t_B)\, B(0) \,|\, G(t_B) \rangle$$

$$|G(t)\rangle \equiv \sum_{mn} g_{mn}\, e^{itH} |m\rangle_A \otimes e^{-itH} |n\rangle_B$$

Can be taken to one side

$$\langle A(t_A)\, B(t_B)\rangle_G = \sum_{\alpha\beta} \sqrt{\rho_\alpha \rho_\beta}\, \left(\Omega_A^\dagger\, A(t_A)\, \Omega_A\right)_{\alpha\beta} \left(\Omega_B^\dagger\, B(t_B)\, \Omega_B\right)_{\alpha\beta}$$

In the case of doubled EPR states, the ρ_α measure the degree of entanglement, ranging from zero in the case that only one ρ_α is non-zero, to maximal entanglement when all of them are equal to one another. The matrices Ω_A and Ω_B measure the departure from 'diagonal' entanglement, by which we refer to the alignment between the Schmidt basis which diagonalizes entanglement and the energy basis which diagonalizes the Hamiltonian. On Bob's side, 'alignment' is equivalent to $[\rho, H] = 0$, i.e. stationarity of the Bob-side state.

For enough entanglement one can construct the Alice surrogate

$$\langle\, A(t_A)\, B(t_B)\, \rangle_G = \mathrm{Tr}\, \left[\rho\, B(t_B)\, B_{A(t_A)} \right] \ ,$$

$$\left(B_{A(t_A)}\right)_{\alpha\beta} = \sqrt{\rho_\alpha}\, \left(\Omega_A^\dagger\, A(t_A)\, \Omega_A\right)_{\beta\alpha} \frac{1}{\sqrt{\rho_\beta}}$$

$$\langle A(t_A) B(t_B)\rangle_{\mathrm{TFD}} = \mathrm{Tr}\, \left[\rho_T\, \widetilde{A}(t_A - i\beta/2) B(t_B)\right]$$

$$g_{mn} = \sum_{\alpha} (\Omega_A)_{m\alpha} \sqrt{\rho_\alpha} \, (\Omega_B)_{n\alpha}$$

$$\rho_{nn'} = \sum_{m} g_{mn} \, g^*_{mn'}$$

$$\mathrm{Tr} \, [\rho \, B(t) \, B'(t')] = \sum_{\alpha\beta} \rho_\alpha \left(\Omega_B^\dagger \, B(t) \, \Omega_B \right)_{\alpha\beta} \left(\Omega_B^\dagger \, B'(t') \, \Omega_B \right)_{\beta\alpha}$$

Representative Dynamics and Observables

- Dynamics- “nearly” Integrable.
- Operators- Fields of Quasi Particles- Sparse- Gravitons in Thermal AdS

Representative Dynamics and Observables

- **Dynamics- Chaotic.**
- **Operators Bs- They do not commute with the Hamiltonian, H, moreover their eigenfunctions are uncorrelated with those of H.**
- **U is “Pseudo Random”- Black Hole**

$$B_{mn} = (U\, b\, U^{\dagger})_{mn} = \sum_{\alpha} b_{\alpha}\, U_{m\alpha}\, (U_{n\alpha})^{*}$$

ETH Observable

$$B_{mn} = \bar{B}(\bar{E})\delta_{mn} + b(\bar{E}, \omega)\, e^{-S(\bar{E})/2}\, R_{mn}$$

$$\bar{E} = \frac{1}{2}(E_m + E_n)\ , \quad \omega = E_m - E_n$$

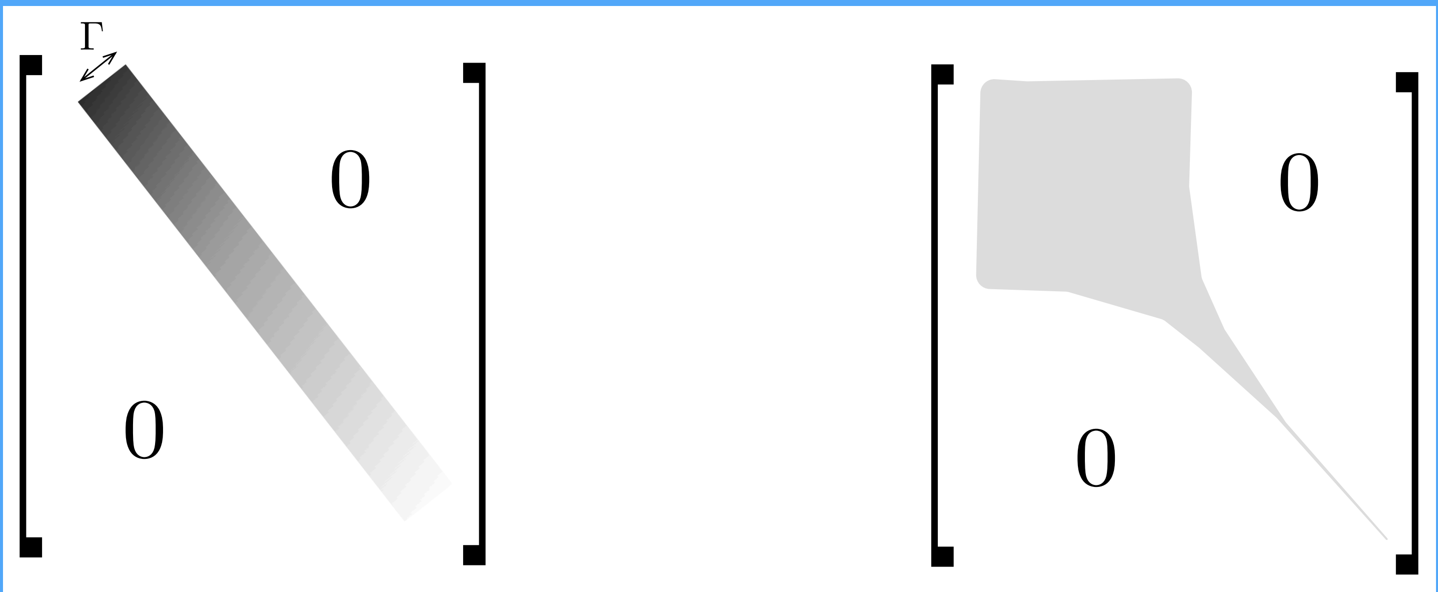
$$B_{mn} = \bar{B}(\bar{E})\delta_{mn} + b(\bar{E}, \omega)\, e^{-S(\bar{E})/2}\, R_{mn}$$

$$\sum_\alpha |U_{\alpha n}|^2 = 1$$

$$B_{mn} = (U\, b\, U^\dagger)_{mn} = \sum_\alpha b_\alpha\, U_{m\alpha}\, (U_{n\alpha})^*$$

U elements are of order Exp(-S/2).

Off diagonal elements are Exp(-S/2), random walk.



Γ
0
0
0
0

Noise Estimates

- Bob’s Noise(one sided)
- EPR Noise(two sides)
- Several Narrow bands Noise.
- Thermal Gas Noise

Bob's Noise

ETH, one "narrow" band with thermal width T

Constant functions in the band.

ETH*ETH=ETH

$$(R^B)_{mn}\,(R^{B'})_{rs} = (\mathcal{D}_{BB'})_{mn}\,\delta_{ms}\,\delta_{nr} + (\text{erratic})_{mnrs}$$

First term is "smooth" in m,n

Second term gives the leading answers

Noise from the Smooth Part.

$$G^{(s)}_{BB'}(t) \sim |b\, b'\,|\, e^{-S} \sum_{\alpha} \rho_{\alpha} \sum_{mn} (\Omega^{\dagger}_{B})_{\alpha m}\, e^{i(E_m - E_n)t}\, (\Omega_B)_{m\alpha}$$

$$|\mathrm{noise}^{(s)}|_{\mathrm{pure\ diag}} \sim |b\, b'|\, e^{-S/2}$$

$$|\mathrm{noise}^{(s)}|_{\mathrm{pure\ non-diag}} \sim |b\, b'|\, e^{-S}$$

$$|\mathrm{noise}^{(s)}|_{\mathrm{mixed\ diag}} \sim |b\, b'|\, e^{-S}$$

$$|\mathrm{noise}^{(s)}|_{\mathrm{mixed\ non-diag}} \sim |b\, b'|\, e^{-3S/2}$$

Noise From the Erratic Component-Dominates.

$$G^{(e)}_{BB'}(t) \sim |b\,b'|\, e^{-S/2} \sum_\alpha \rho_\alpha \left(\Omega_B^\dagger\, R_{BB'}\, \Omega_B\right)_{\alpha\alpha} \sim |b\,b'|\, e^{-S/2} \sum_\alpha \rho_\alpha \left(R_{\Omega^\dagger BB'\Omega}\right)_{\alpha\alpha}$$

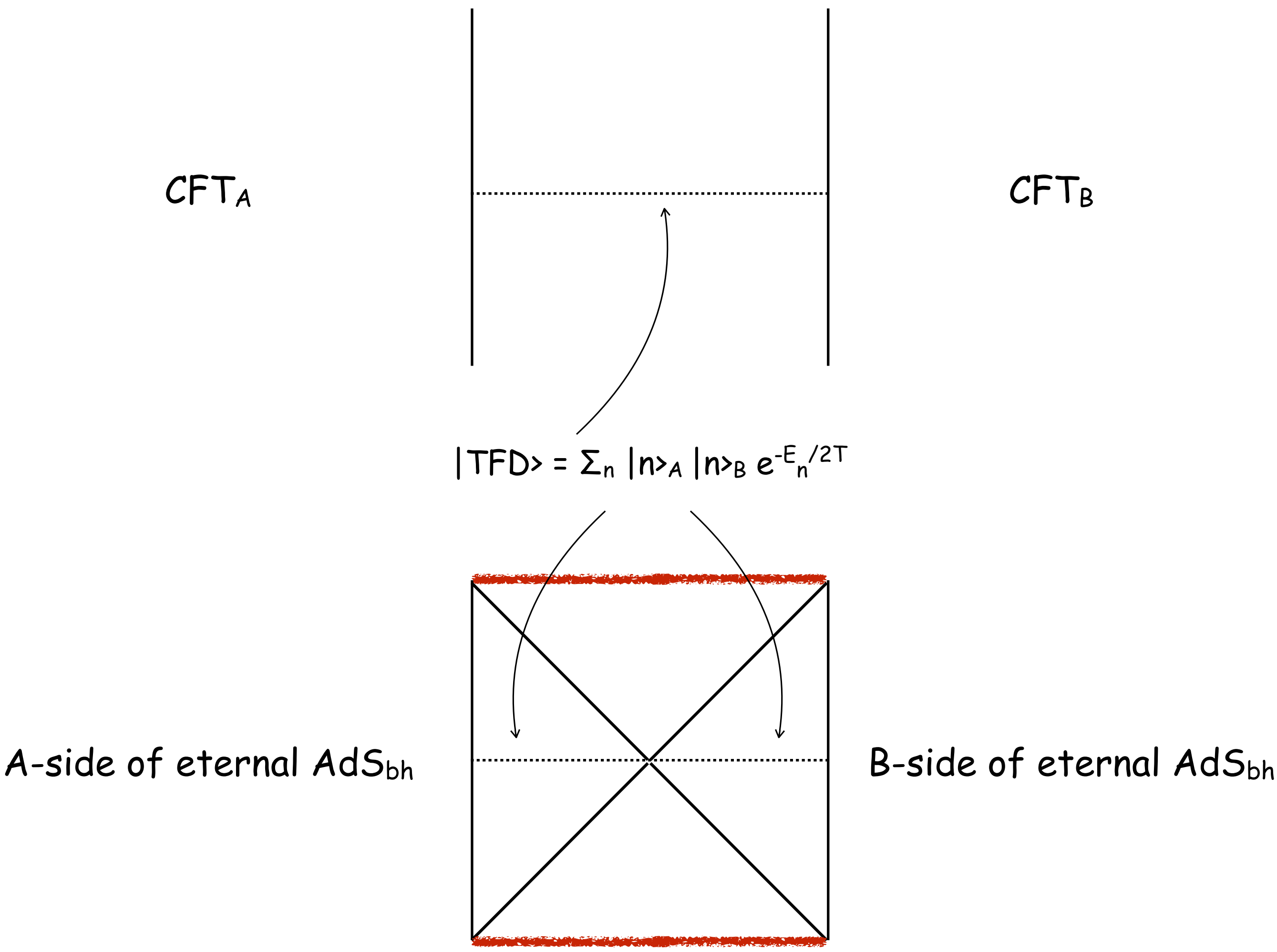
$$|\mathrm{noise}|_{\mathrm{mixed}} \sim |b\,b'|\, e^{-S} \ , \qquad |\mathrm{noise}|_{\mathrm{pure}} \sim |b\,b'|\, e^{-S/2}$$

Does NOT depend on the alignment of B!

EPR Noise

$$G_{AB}(t) = \sum_{\alpha\beta} \sqrt{\rho_\alpha \rho_\beta} \left(\Omega_A^\dagger \, A(t) \, \Omega_A \right)_{\alpha\beta} \left(\Omega_B^\dagger \, B(0) \, \Omega_B \right)_{\alpha\beta}$$

EPR Noise
For the Diagonal Term


CFT$_A$
CFT$_B$
|TFD> = Σ$_n$ |n>$_A$ |n>$_B$ e$^{-E_n/2T}$
A-side of eternal AdS$_{bh}$
B-side of eternal AdS$_{bh}$

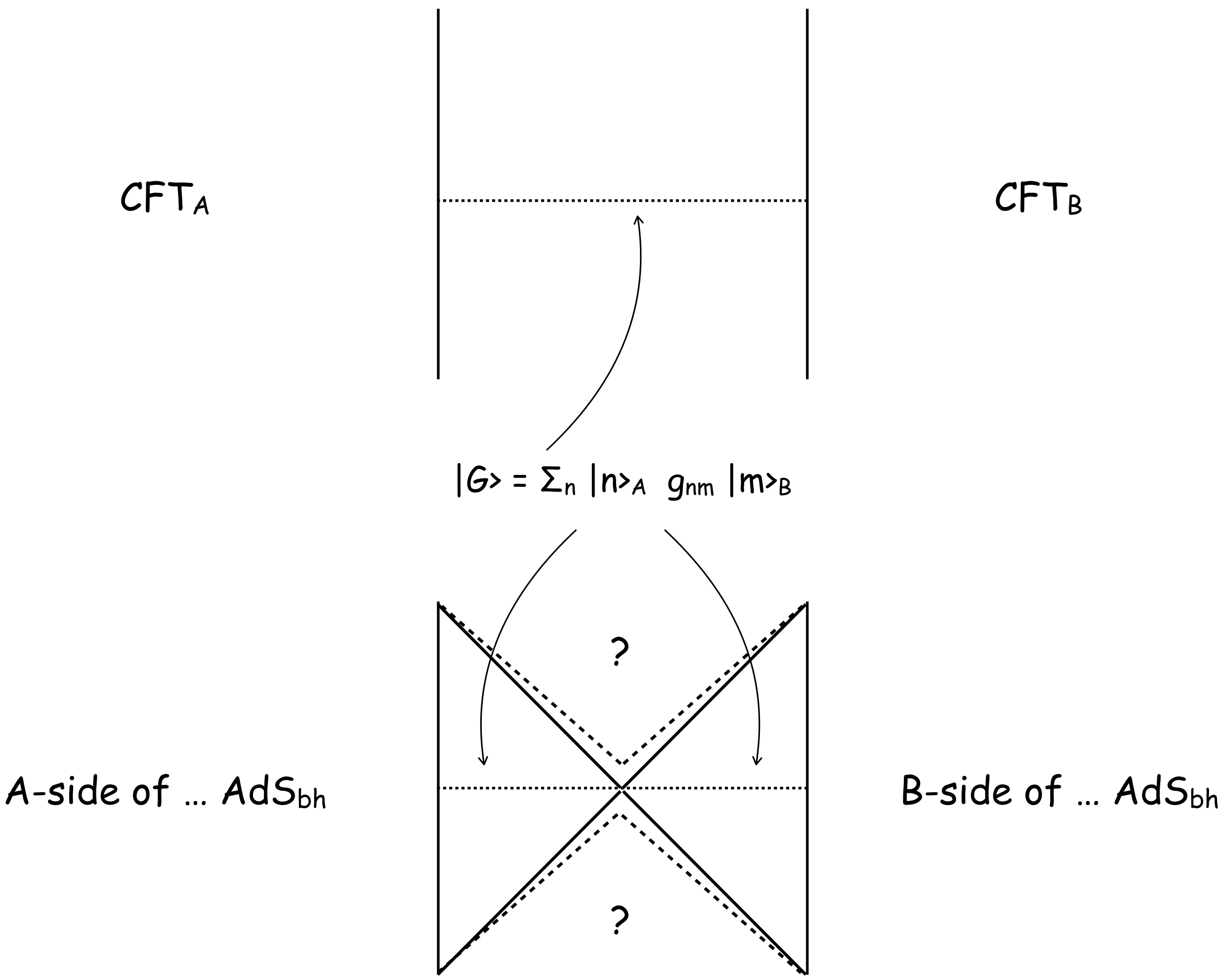
EPR Noise

For the Diagonal State

Value of the Correlator at t=o
O(1) Peak as a “Geometry”

The Noise is

abExp(-S)



CFT_A
CFT_B
$|G\rangle = \Sigma_n |n\rangle_A \; g_{nm} \; |m\rangle_B$
?
A-side of … AdS_{bh}
B-side of … AdS_{bh}
?

EPR Noise

For a non diagonal state

Value of the correlator at t=o
O(abExp(-S)) peak "NOT" as a "Geometry"

The noise is HOWEVER again

abExp(-S)

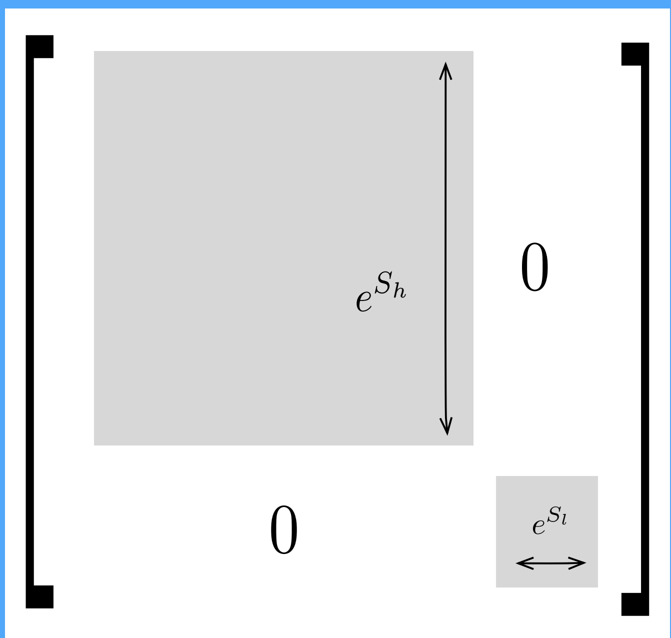
This does NOT depend on the amount of

Entanglement.

To ensure O(1) amplitude for the noise at time, t
One needs a state dependent condition.

$$(R^A)_{mn}\,(R^B)_{rs} \sim (\Omega_A\,\Omega_B^T)_{mr}\,(\Omega_A^*\,\Omega_B^\dagger)_{ns} + (\mathrm{erratic})_{mnrs}$$

Several Bands
Large gap, large difference in entropy



The Lower(est) Band Dominates.

$$\rho = \sum_b p_b\, \rho_b = \sum_b p_b\, e^{-S_b} \mathbf{1}_b\ , \qquad \sum_b p_b = 1$$

$$C(t) \approx \sum_b p_b\, C_b(t) \qquad C_b(t) \sim |\text{noise}|_b\, f_b(t)$$

$$C(t) \sim \sum_b |bb'|_b\, p_b\, e^{-S_b}\ f_b(t)$$

$$p_b\Big|_{\text{canonical}} = \frac{e^{-I_b(\beta)}}{Z(\beta)}$$

The Lowest Energy Band Dominates

$$Z(\beta) \equiv \sum_b e^{-I_b(\beta)} \text{ and } I_b(\beta) = \beta E_b - S_b$$

$$e^{-I_b(\beta)} e^{-S_b}$$

$$\exp(-\beta E_b)$$

*** For bands which are all quasi integrable, the noise is determined by the thermodynamical dominant.**

Thermal Gas Noise

$$B_1 = \frac{1}{L^{\frac{d-1}{2}}} \sum_s \left(b_s\, a_s + b_s^*\, a_s^\dagger\right)$$

$$\langle B_1(t) B_1(0)\rangle_{\rm gas} = \frac{1}{L^{d-1}} \sum_s \left[(1 + f(\omega_s))|b_s|^2\, e^{-i\omega_s t} + f(\omega_s)|b_s|^2\, e^{i\omega_s t}\right] \ + \text{inter}$$

$$f(\omega_s) = (e^{\beta\omega_s} - 1)^{-1}$$

Thermal Gas Noise-Large

$$\overline{|\langle B_1(t)B_1(0)\rangle_{\text{gas}}|^2} \sim \frac{1}{L^{2d-2}}\sum_{\omega_s<T}\left(1+2f(\omega_s)+2f(\omega_s)^2\right)|b_s|^4 \sim L^{2-2d}\,(LT)^{d-2}$$

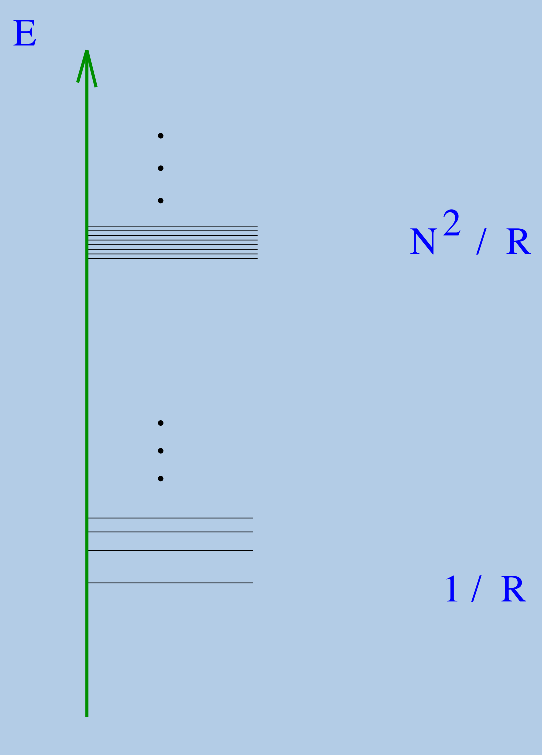
$$\frac{|\text{noise}|}{|\text{peak}|} \sim \frac{1}{(LT)^{d/2}} \sim \frac{1}{\sqrt{S_{\text{gas}}}}$$

The peak scales as $\langle B_1^2\rangle \sim L^{1-d}(LT)^{d-1}$.

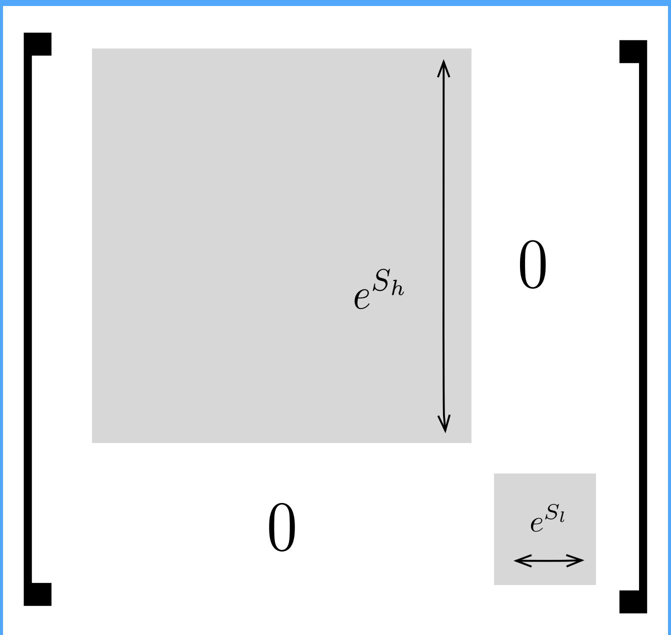
- **Geometry reproduces correctly the average property.**
- **Geometry reproduced a VERY SMALL non perturbative result.**
- **Geometry does not reproduce even finer details of the non perturbative behaviour of the time dependent correlations.**

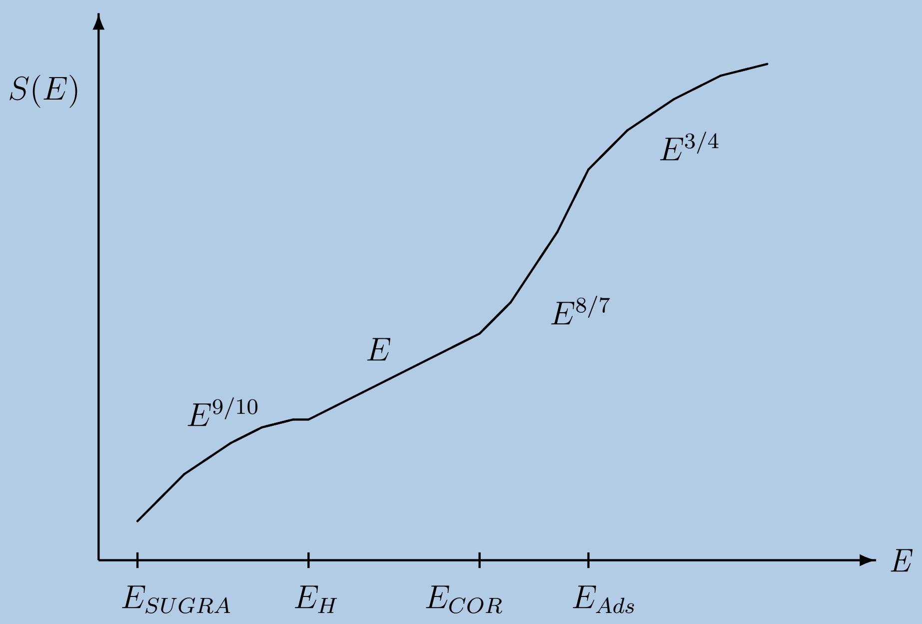
This will have consequences in AdS CFT

Listen to the AdS Noise


E
N^2 / R
1 / R

Several Bands
Large gap, large difference in entropy





$S(E)$
$E^{3/4}$
$E^{8/7}$
E
$E^{9/10}$
E
E_{SUGRA}
E_H
E_{COR}
E_{Ads}

The Lowest Energy Band Dominates
This is NOT an accident

$$Z(\beta) \equiv \sum_b e^{-I_b(\beta)} \text{ and } I_b(\beta) = \beta E_b - S_b$$

$$e^{-I_b(\beta)} e^{-S_b}$$

$$\exp(-\beta E_b)$$

ETH For BHs and Strings.

For the Gas:

$$B \sim \frac{1}{N} \mathrm{Tr}\, F^{\,n}$$

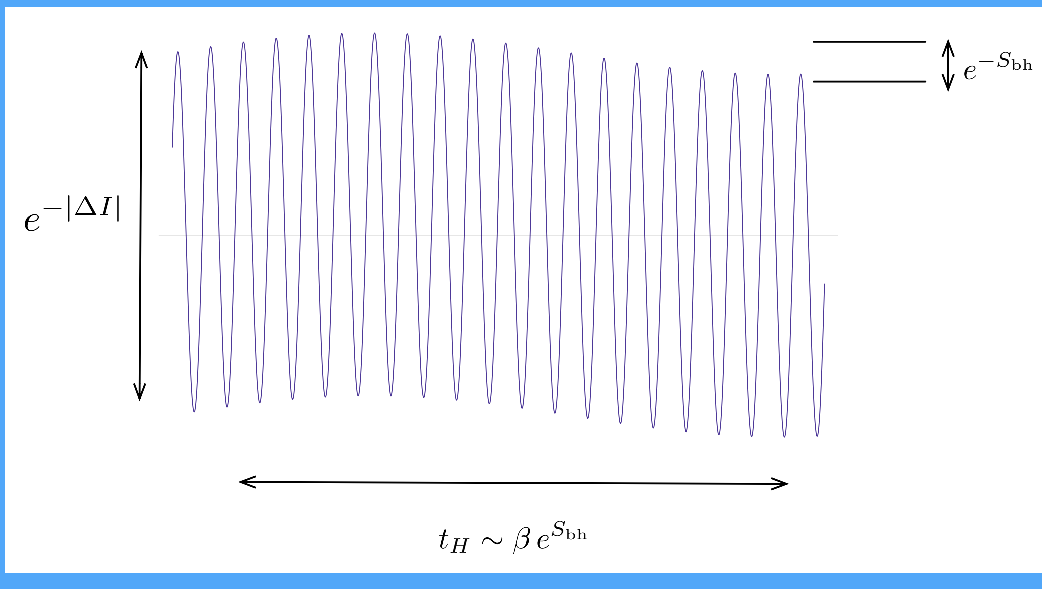
For T small relative to the critical T S and I are O(1) in

$$|\mathrm{noise}|_{\mathrm{canonical}} \sim |b|^2 \, \frac{e^{-|\Delta I|}}{\sqrt{S_{\mathrm{gas}}}}$$

For T>>T critical

$$|\text{noise}|_{T \gg T_c} \sim |b|^2 \, e^{I_{\text{bh}}} \left[\frac{1}{(RT)^{9/2}} + O\left(e^{-c_{\text{Hag}} \lambda^{5/2}}\right) + O\left(e^{-c_{\text{sh}} N^2/\lambda^{7/4}}\right) + O\left(e^{-c_{\text{bh}} N^2}\right) \right]$$

The Average Noise is determined by the lowest band , the fast O(1) variations are determined by it as well. But the hight and the long time variations are determined by the thermodynamical dominant configuration.



Slogans:

1. Diversity Counts.

2. Geometry can capture non perturbative average observables.

3. Geometry may well miss some parts.

4. For ETH the lowest energy band dominates the value

of the average noise. Not the thermodynamical leading one.