Geometry and Quantum Noise

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CMS

Cambridge
a panorama of previous work...

Peres (1984)
Deutsch (1991)
Srednicki (1994)
Maldacena (2001)
Dyson, Kleban & Susskind (2002)
Barbon & Rabinovici (2003)
Festuccia & Liu (2007)

RECENTLY...
Marolf & Polchinski
Shenker & Stanford
Susskind
Balasubramanian, Berkooz, Ross & Simon
Barbon & Rabinovici
• Introduction- What Does(does not) Geometry capture?

• Geometry Topology and Quantum Noise I - QFT, BH Information

• VERY(!) long time correlations. VERY small.

• Quantum Noise II - מורים ב-Firewalls?

• Geometry and Quantum Noise II

• Discussion
Round I
\( \mathcal{N}=4 \) describes also a theory of a string moving in a background a \( \text{AdS}_5 \times S^5 \) and a black hole in \( \text{AdS}_5 \times S^5 \)

**The AdS/CFT Correspondence**

\[ D=4, \ N=4 \ , \ \text{SUSY Y.M. SU}(N) \]

't Hooft coupling: \( \lambda = Ng_{YM}^2 \) \quad 1/color number: \( \frac{1}{N} \) \quad theta angle: \( \theta_{YM} \)

\( \mathcal{N} = 4 \) SYM was conjectured to be dual to a string theory:

\[ \text{IIB Superstrings on } \text{AdS}_5 \times S^5 \]

\[ \frac{R^2}{\alpha'} = \sqrt{\lambda} \quad \text{coupling: } g_s = \frac{\lambda}{4\pi N} \quad \text{axion: } \langle C \rangle = \theta_{YM} \]
• **AdS$_5$ metric**

\[ ds^2 = - \left(1 + \frac{\rho^2}{R^2}\right) dt^2 + \left(1 + \frac{\rho^2}{R^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2 + R^2 d\Omega_5^2 \]

• Effective temperature

\[ T(r) = \frac{T(0)}{\sqrt{1 + \rho^2/R^2}} \]

• Black Hole in AdS$_5$ metric

\[ ds^2 = - \left(1 + \frac{\rho^2}{R^2} - \frac{M}{C\rho^2}\right) dt^2 + \left(1 + \frac{\rho^2}{R^2} - \frac{M}{C\rho^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2 + R^2 d\Omega_5^2 \]
• For $T < 1/R$
  Only thermal AdS

• For $T \gtrsim 1/R$
  Thermal AdS plus BH in AdS,
  (actually two Black Holes)

• For $T > 1/R$
  BH dominates
Black Hole Information Paradoxes

- BH formation paradox
- Eternal BH paradox (Maldacena)
  Tool for CFT $\rightarrow$ AdS
• In Principle

Initial bulk state $\leftrightarrow$ Initial CFT state

Final bulk state $\leftrightarrow$ Final CFT state

Instead consider slight deviation from thermal equilibrium on the field theory side
Consider

\[ G(t) = Tr \left[ \rho A(t) A(0) \right] \]

For very large time scale
2. Quantum Noise Of Temporal Correlation Functions

In our discussion we will use the term 'quantum noise' to describe the characteristics of correlation functions after a certain time; essentially the time that is required for them to start oscillating around the long time average value they are supposed to attain. To appreciate how this comes about consider first the example of a time self-correlation function for an Hermitian operator \( B \) in an infrared-bounded, unitary quantum system

\[
C(t) = \langle T r[\rho B(t) B(0)] \rangle = \text{Tr}[\rho e^{itH}Be^{-itH}] .
\]

(2.1)

The correlation (2.1) has a spectral representation

\[
C(t) = \sum_{mn} \rho_{m} B_{mn} B_{nm} e^{i(E_{m} - E_{n})t} ,
\]

(2.2)

where we have taken for simplicity a diagonal density matrix in the energy basis. A particular case is provided by the canonical thermal state, \( \rho_{n} = e^{-\beta E_{n}}/\sum_{k} e^{-\beta E_{k}} \), although the present discussion goes through for a general choice of \( \rho_{n} \).

The bounded character of the system is reflected in the discreteness of the energy spectrum. Indeed in this case the correlation is maximal at \( t = 0 \) and decays away for a time period to be estimated. For times small compared to the inverse level separation \( \langle E_{n} - E_{m} \rangle t \ll 1 \) we may approximate the discrete sum in (2.2) by continuous integrals, resulting in either a power law or exponential decay, depending on the detailed energy dependence of the operator matrix elements. However, the discreteness of the spectrum cannot be ignored over very long time scales, since the infinite time average of (2.2) is strictly positive.

\[
C(t) \equiv \lim_{\tau \to \infty} \frac{1}{\tau} \int_{0}^{\tau} dt C(t) = \sum_{m} \rho_{m} |B_{mm}|^{2} .
\]

(2.3)

In fact, the value of the limiting time average is controlled by the diagonal matrix elements of \( B \) in the energy basis. The same diagonal matrix elements control the one-point expectation value \( \text{Tr}[\rho B] \). It will be convenient to work with operators having no expectation values in energy eigenstates. In the following we will arrange for this by dealing with modified operators \( B \) from which the diagonal elements were subtracted. In such a

1. We assume all spectral sums to be sufficiently convergent so that formal manipulations involving commutation of integrals and sums are permitted.

2. 
Aspects of Long Time Scales in Field Theory

Classical                          Quantum

Compact Phase Space $\iff$ Discrete Spectrum

Volume Conservation $\iff$ Unitarity

Then, If

$$G(t_0) = \langle \theta_1(t_0, x_1)|\theta_2(0, x_2) \rangle$$

for any $\epsilon$ there is a $t^p(\epsilon)$ such that

$$|G(t^p(\epsilon)) - G(t^0)| < \epsilon$$

You See It All!
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$$C(t) \equiv \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt C(t) = \sum_m \rho_m |B_{mm}|^2.$$

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In fact, the value of the limiting time average is controlled by the diagonal matrix elements of $B$ in the energy basis. The same diagonal matrix elements control the one-point expectation value $\text{Tr}[\rho B]$. It will be convenient to work with operators having no expectation values in energy eigenstates. In the following we will arrange effort by dealing with modified operators $B$ from which the diagonal elements were subtracted. In such a case, we assume all spectral sums to be sufficiently convergent so that formal manipulations involving commutation of integrals and sums are permitted.
$$L(t) = \exp(-\Gamma t)$$
We begin in Sect. 2 with an operational definition of quantum correlation noise in various types of correlation functions, including EPR systems. In Sect. 3 we discuss the most suitable observables to probe the system once the dynamics is given, with a focus on constraints derived from the theory of quantum chaos. This leads to a choice of operators characterized by the Eigenvalue Thermalization Hypothesis (ETH). In Sect. 4 we turn to estimate the value of the quantum noise and its dependence on the dynamics, the observables and the states. We find a diversity of relations between the value of the noise and the entropy of the system. In Sect. 5 we apply these results to AdS/CFT systems containing both black holes and graviton gases in their dual dynamical description. Finally, we conclude offering some speculations on the interpretation of the EPR=ER conjecture in the light of our results.

**Quantum noise of temporal correlation functions**

In our discussion we will use the term 'quantum noise' to describe the characteristics of correlation functions after a certain time; essentially the time that is required for them to start oscillating around the long time average value they are supposed to attain. To appreciate how this comes about consider first the example of a time self-correlation for an Hermitian operator $B$ in an infrared-bounded, unitary quantum system $C(t) = \text{Tr}[\rho B(t)B(0)] = \text{Tr}[\rho e^{itH}Be^{-itH}]$.

The correlation (2.1) has a spectral representation

$$C(t) = \sum_{mn} \rho_m B_{mn} B_{nm} e^{i(E_m - E_n)t},$$

where we have considered for simplicity a diagonal density matrix in the energy basis. A particular case is provided by the canonical thermal state, $\rho_n = e^{-\beta E_n}/\sum_k e^{-\beta E_k}$, although the presented discussion goes through for a general choice of $\rho_n$.

The bounded character of the system is reflected in the discreteness of the energy spectrum. Indeed in this case the correlation is maximal at $t=0$ and decays away from there for a time period to be estimated.

For times small compared to the inverse level separation $\langle E_n - E_m \rangle t \ll 1$ we may be able to approximate the discrete sums in (2.2) by continuous integral $s$, resulting in either a power law or exponential decay, depending on the detailed energy dependence of the operator matrix elements. However, the discreteness of the spectrum cannot be ignored over very long time scales, since the infinite time average of (2.2) is strictly positive

$$\overline{C(t)} \equiv \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt \ C(t) = \sum_m \rho_m |B_{mm}|^2.$$

Generically positive. For $B$ with no diagonal terms average the square.

An estimate gives a normalisation $\text{Exp}(-S)$ times a number. So the decay must stop, the discrete nature of the spectrum felt and the magnitude is $\text{Exp}(-S)*$
NOISE

The Noise is defined by

\[ |\text{noise}| \equiv \left[ \left| C(t) \right|^2 \right]^{1/2} \]

\[ |C(t)|^2 = \sum_{mnrs} \rho_m \rho_r |B_{mn}|^2 |B_{rs}|^2 e^{i(E_m - E_n + E_s - E_r)t}. \]

B has no diagonal elements so

\[ E_m = E_r \text{ and } E_n = E_s \]
We then define the ratio $t = 0$

When the spectrum is generic, that is when no rational operator. Following usual practice, we can define an effective 'entropy' associated to this Hilbert-space

determined by the smallest energy differences.

This heuristic method for analyzing the properties that result requires some knowledge of the spectral properties of the system,

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any rate, if the long-time decay from the peak is of an.

We are thus left with an average noise

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The Time it takes to reach the average if the decay is exponential is \( t \sim S \)
\[ L(t) = \exp(-\Gamma t) \]
$\Gamma$ is not universal

$$t_H = \frac{1}{\langle w \rangle} \quad <w> = <E_i - E_j>$$

$$<w> \sim \frac{\Gamma}{\Delta n_\Gamma},$$

$\Delta n_\Gamma$ is the number of states in a band of width $\Gamma$.

$$t_H \sim \frac{1}{\Gamma} \exp(S(\beta))$$

$$t^P(\epsilon) \sim \exp(f(\epsilon) \exp S) \quad |G(t^P(\epsilon)) - G(0)| < \epsilon$$
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However, the discreteness of the spectrum cannot be ignored over very long time scales, since the infinite time average of (2.2) is strictly positive,

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt C(t) \equiv \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt \sum_{mn} \rho_m |B_{mn}|^2.$$

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1 We assume all spectral sums to be sufficiently convergent so that formal manipulations involving commutation of integrals and sums are permitted.

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\[ C(t) = \sum_{mn} \rho_m |B_{mn}|^2 e^{i(E_m - E_n)t} \]

\[ C(0) = 1 \]

\[ \rho_m \sim e^{-S} \]

\[ |B_{mn}| \sim e^{-S/2} \]

Page time scale

fast oscillations of amplitude \( e^{-2S} \)
Estimate of Poincare Time

Consider “clocks” $\text{Exp}(iEt)$

\[
t = \frac{1}{v}
\]

\[
v = \left(\frac{\Delta \alpha}{2\pi}\right)\text{Neff}
\]

\[Et \sim \exp \left(\text{Neff} \log\left(\frac{2\pi}{\Delta \alpha}\right)\right) \sim \exp\left(\exp\left(S \log\left(\frac{2\pi}{\Delta \alpha}\right)\right)\right)\]
\[ C(t) = \sum_{mn} \rho_m |B_{mn}|^2 e^{i(E_m - E_n)t} \]

\[ C(0) = 1 \]

\[ \rho_m \sim e^{-S} \]

\[ |B_{mn}| \sim e^{-S/2} \]

Heisenberg time scale
\[ C(t) = \sum_{mn}^{e^{2S}} \rho_m \left| B_{mn} \right|^2 e^{i(E_m - E_n)t} \]

\[ C(0) = 1 \]

\[ \rho_m \sim e^{-S} \]

\[ \left| B_{mn} \right| \sim e^{-S/2} \]

Poincaré time scale
\[ C(t) = \sum_{mn} e^{2S} \rho_m |B_{mn}|^2 e^{i(E_m - E_n)t} \]

\[ C(0) = 1 \]

\[ \rho_m \sim e^{-S} \]

\[ |B_{mn}| \sim e^{-S/2} \]

detailed Poincaré time scale
Some Proportion - Page Times $S$

In units of the Universe’s Life $UL$

Page time for a BH the size of a proton $10^{10}$ ULs

Page time of a BH in $10^9$ sm Quasar 3 km $10^{87}$ ULs

This is just $S$!!!

One reaches for Poincare $10$ to the $10$ five times…
Summary:

Time Scales related by Log

Log S - Scrambling time BH, 1/S boundary (UP, T)

S-Page time, end of decay

Exp(S) - Heisenberg time

Exp(Exp(#S)) - Poincare time
Consider

\[ L(t) = \left| \frac{G(t)}{G(0)} \right|^2 \]
\[ \bar{L} = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \, L(t) \]

The CFT is unitary and has a Gap
\[ \bar{L} \sim \frac{\Delta L}{\Gamma t_H} \sim \exp(-S(\beta)) \]

, 

\[ \bar{L} \sim \exp(-N^2 \ldots) \sim \exp\left(-\frac{1}{G_N} \ldots\right) \]

**Non Perturbative from Gravity Point of View**

For BH background \( \bar{L} \to 0 \), Reason:

**No Gap in the presence of a BH.**
$$\exp \left( 4 \pi \frac{r_*}{\beta} \right)$$

$$r_* = \pi \frac{R}{2}$$
\[
\tilde{L}_{CFT} = \exp(-S) \Rightarrow \tilde{L} = \exp(-S)
\]

But it seems \( \tilde{L}_{Bulk} = 0 \)

Contradiction?

Poincarè Recurrences and Topological Diversity
In a Thermal AdS Background a gap is formed and now

\[ \bar{L}_{Bulk} \approx \exp(-S) > 0 \]
Energy $\gg 1/R$  

$W$  

$U_t(W)_H$  

AdS Black Hole states  

graviton gas states

$(P, Q)$
\[ L(t) = \exp(-N^2t) \]
• $\tilde{L}$ reasonable course grained
• $L(t)$ not reproduced
• Stretched horizon, Brick Wall?
Conclusions

• The Burden of Proof That a Well Defined Information Paradox Exists Shifts to Claimer
• Topological Diversity is Required
• String theory is Quite a Formidable Bastion of Consistency
Geometry Reproduces Average Result

Geometry May Well Miss some $\exp(-S)$ Features.

Question: Is the failure of the thermodynamically dominant contribution to reproduce the average quantum noise accidental?
\[ |\text{TFD}\rangle = \sum_n |n\rangle_A |n\rangle_B e^{-E_n/2T} \]

CFT\textsubscript{A} \hspace{5cm} \text{CFT}\textsubscript{B}

A-side of eternal AdS\textsubscript{bh} \hspace{5cm} B-side of eternal AdS\textsubscript{bh}
\( C(t=0) \sim 1 \)

- Noise \sim \text{Exp}(-S)
\[ |G\rangle = \sum_n |n\rangle_A \; g_{nm} \; |m\rangle_B \]

A-side of ... AdS_{bh}  

B-side of ... AdS_{bh}
\[ |G\rangle = \sum_n |n\rangle_A g_{nm} |m\rangle_B \]

\( CFT_A \) \hspace{2cm} \( CFT_B \)

A-side of ... AdS\(_{bh}\) \hspace{2cm} B-side of ... AdS\(_{bh}\)
More General Case

- Two different operators.
- If the density matrix is diagonal in the energy basis—there is only dependence on time differences.
- Otherwise there is a dependence on both times.
- Even for a diagonal density matrix there is no generic peak at \( t=0 \)

\[
G_{BB'}(t, t') = \text{Tr} \left[ \rho(t) BB'(t') \right] = \sum_{mn} \rho_{mn} B_{nr} B'_{rm} e^{i(E_n - E_r)(t-t')} e^{-i(E_m - E_n)t'}
\]
General Two Sided State leads to a One Sided Density Matrix

\[ |G\rangle = \sum_{mn} g_{mn} |m\rangle_A \otimes |n\rangle_B \]

\[ \rho_{nn'} = \sum_m g_{mn} g_{mn'}^* \]
One Can Consider EPR Correlations on Two and One Sides.

\[ G_{AB}(t_A, t_B) = \langle A(t_A) B(t_B) \rangle_G = \langle G | e^{-it_A H} A e^{it_A H} e^{it_B H} B e^{-it_B H} | G \rangle \]

\[ G_{AB}(t_A, t_B) = \langle G(t_B) | A(t_A - t_B) B(0) | G(t_B) \rangle \]

\[ |G(t)\rangle \equiv \sum_{mn} g_{mn} e^{itH} |m\rangle_A \otimes e^{-itH} |n\rangle_B \]

Can be taken to one side
\[ \langle A(t_A) B(t_B) \rangle_G = \sum_{\alpha\beta} \sqrt{\rho_\alpha \rho_\beta} \left( \Omega_A^\dagger A(t_A) \Omega_A \right)_{\alpha\beta} \left( \Omega_B^\dagger B(t_B) \Omega_B \right)_{\alpha\beta} \]

In the case of doubled EPR states, the \( \rho_\alpha \) measure the degree of entanglement, ranging from zero in the case that only one \( \rho_\alpha \) is non-zero, to maximal entanglement when all of them are equal to one another. The matrices \( \Omega_A \) and \( \Omega_B \) measure the departure from ‘diagonal’ entanglement, by which we refer to the alignment between the Schmidt basis which diagonalizes entanglement and the energy basis which diagonalizes the Hamiltonian. On Bob’s side, ‘alignment’ is equivalent to \( [\rho, H] = 0 \), i.e. stationarity of the Bob-side state.
For enough entanglement one can construct the Alice surrogate

\[ \langle A(t_A) B(t_B) \rangle_G = \text{Tr} \left[ \rho B(t_B) B_A(t_A) \right], \]

\[ (B_A(t_A))_{\alpha\beta} = \sqrt{\rho_\alpha} \left( \Omega^\dagger_A A(t_A) \Omega_A \right)_{\beta\alpha} \frac{1}{\sqrt{\rho_\beta}}, \]

\[ \langle A(t_A) B(t_B) \rangle_{\text{TFD}} = \text{Tr} \left[ \rho_T \tilde{A}(t_A - i\beta/2) B(t_B) \right]. \]
\[ g_{mn} = \sum_{\alpha} (\Omega_A)_{m\alpha} \sqrt{\rho_{\alpha}} (\Omega_B)_{n\alpha} \]

\[ \rho_{nn'} = \sum_{m} g_{mn} g_{mn'}^* \]

\[ \text{Tr} \left[ \rho \, B(t) \, B'(t') \right] = \sum_{\alpha\beta} \rho_{\alpha} \left( \Omega_B^\dagger \, B(t) \, \Omega_B \right)_{\alpha\beta} \left( \Omega_B^\dagger \, B'(t') \, \Omega_B \right)_{\beta\alpha} \]
Representative Dynamics and Observables

- Dynamics- “nearly” Integrable.
- Operators- Fields of Quasi Particles- Sparse- Gravitons in Thermal AdS
Representative Dynamics and Observables

- Dynamics- Chaotic.

- Operators Bs- They do not commute with the Hamiltonian, H, moreover their eigenfunctions are uncorrelated with those of H.

- U is “Pseudo Random”- Black Hole

\[ B_{mn} = (U b U^\dagger)_{mn} = \sum_{\alpha} b_{\alpha} U_{m\alpha} (U_{n\alpha})^* \]
ETH Observable

\[ B_{mn} = \tilde{B}(\tilde{E})\delta_{mn} + b(\tilde{E}, \omega) e^{-S(\tilde{E})/2} R_{mn} \]

\[ \tilde{E} = \frac{1}{2}(E_m + E_n) , \quad \omega = E_m - E_n \]
$$B_{mn} = \tilde{B}(\tilde{E}) \delta_{mn} + b(\tilde{E}, \omega) e^{-S(\tilde{E})/2} R_{mn}$$

$$\sum_{\alpha} |U_{\alpha n}|^2 = 1$$

$$B_{mn} = (U b U^\dagger)_{mn} = \sum_{\alpha} b_{\alpha} U_{m\alpha} (U_{n\alpha})^*$$

U elements are of order Exp(-S/2).

Off diagonal elements are Exp(-S/2), random walk.
Noise Estimates

- Bob’s Noise (one sided)
- EPR Noise (two sides)
- Several Narrow bands Noise.
- Thermal Gas Noise
Bob’s Noise

ETH, one “narrow” band with thermal width $T$

Constant functions in the band.

\[ \text{ETH} \ast \text{ETH} = \text{ETH} \]

First term is “smooth” in $m,n$

Second term gives the leading answers

\[ (R^B)_{mn} (R^{B'})_{rs} = (D_{BB'})_{mn} \delta_{ms} \delta_{nr} + (\text{erratic})_{mnr} \]

To see this, we pick the first term in (3.2) to find the “smooth” component of the correlation as

\[ (D^s)_{BB'} (t) = \sum_{mn} \rho_m (BB')_{mn} e^{i(E_m - E_n)t} \]
Noise from the Smooth Part.

\[ G_{BB'}^{(s)}(t) \sim |b b'| e^{-S} \sum_{\alpha} \rho_{\alpha} \sum_{mn} (\Omega_B^\dagger)_{\alpha m} e^{i(E_m - E_n)t} (\Omega_B)_{m\alpha} \]
\[
|\text{noise}^{(s)}|_{\text{pure diag}} \sim |b \ b'| \ e^{-S/2}
\]

\[
|\text{noise}^{(s)}|_{\text{pure non-diag}} \sim |b \ b'| \ e^{-S}
\]

\[
|\text{noise}^{(s)}|_{\text{mixed diag}} \sim |b \ b'| \ e^{-S}
\]

\[
|\text{noise}^{(s)}|_{\text{mixed non-diag}} \sim |b \ b'| \ e^{-3S/2}
\]
Noise From the Erratic Component-Dominates.

\[ G_{BB'}^{(e)}(t) \sim |b b'| e^{-S/2} \sum_\alpha \rho_\alpha \left( \Omega_B^\dagger R_{BB'} \Omega_B \right)_{\alpha\alpha} \sim |b b'| e^{-S/2} \sum_\alpha \rho_\alpha (R_{BB'}^\dagger \Omega)_{\alpha\alpha} \]

\[ |\text{noise}|_{\text{mixed}} \sim |b b'| e^{-S} , \quad |\text{noise}|_{\text{pure}} \sim |b b'| e^{-S/2} \]

Does NOT depend on the alignment of B!
EPR Noise

\[ G_{AB}(t) = \sum_{\alpha\beta} \sqrt{\rho_\alpha \rho_\beta} \left( \Omega_A^\dagger A(t) \Omega_A \right)_{\alpha\beta} \left( \Omega_B^\dagger B(0) \Omega_B \right)_{\alpha\beta} \]
EPR Noise
For the Diagonal Term
\[ |TFD> = \sum_n |n>_A |n>_B e^{-E_n/2T} \]

A-side of eternal AdS\(_{bh}\)

B-side of eternal AdS\(_{bh}\)
EPR Noise

For the Diagonal State

Value of the Correlator at $t=0$

$O(1)$ Peak as a “Geometry”

The Noise is

$ab\exp(-S)$
$| G \rangle = \sum_n | n \rangle_A \ g_{nm} \ | m \rangle_B$

\[ CFT_A \quad \text{----} \quad CFT_B \]

A-side of $\ldots \ AdS_{bh}$

B-side of $\ldots \ AdS_{bh}$
EPR Noise

For a non diagonal state

Value of the correlator at $t=0$ \( O(ab\exp(-S)) \) peak “NOT” as a “Geometry”

The noise is HOWEVER again

\[ ab\exp(-S) \]

This does NOT depend on the amount of Entanglement.
To ensure $O(1)$ amplitude for the noise at time, $t$
One needs a state dependent condition.

$$(R^A)_{mn} (R^B)_{rs} \sim (\Omega_A \Omega_B^T)_{mr} (\Omega_A^* \Omega_B^\dagger)_{ns} + \text{(erratic)}_{mnrs}$$
Several Bands

Large gap, large difference in entropy

- The high-energy entropy scales as the central charge, of order $N^2$, dominated by states described as black holes in the bulk picture (plasma of glue in the CFT picture).
- There is a band of low-energy states resembling graviton-gas states in the bulk (glueballs in the CFT) with entropy of $O(1)$ in the large $N$ limit.
- The low energy band extends from the spectral mass gap up to the energies of $O(N^2)$ where the 'black hole' states start to dominate the density of states.

- The natural dynamical assumption is that of quantum chaos for the high-energy (black hole) band and thermal gas for the low-energy (graviton) band.
- Interactions among gravitons are suppressed by powers of $1/N^2$.
- Hence, at moderate values of $N$ we may consider also a chaotic model for an interacting graviton gas. In this case the main difference between the high and low energy bands would be just the jump in density of states.

- With the AdS/CFT application in mind, special simplifications occur when restricting attention to regularized operators whose fixed energy width $\Gamma_B$ is small compared to the overall energy range of interest.
- For example, we may consider a model with two bands (high and low) with $E_h - E_l \gg \Gamma_B$. In this case we can approximate the $B_{mn}$ matrix by a block-diagonal form as in the figure, in the discrete uniform density representation of matrix entries. If the upper band is chaotic and the lower band has perturbative quasiparticle-like states, a single-particle operator would have the ETH form in the upper band and the sparse quasiparticle form in the lower band.
- Fine details of the spectrum may include additional intermediate bands with different types of black holes and/or Hagedorn transients.
The Lower(est) Band Dominates.

\[ \rho = \sum_b p_b \rho_b = \sum_b p_b e^{-S_b} 1_b, \quad \sum_b p_b = 1 \]

\[ C(t) \approx \sum_b p_b C_b(t) \quad C_b(t) \sim |\text{noise}|_b f_b(t) \]

\[ C(t) \sim \sum_b |bb'|_b p_b e^{-S_b} f_b(t) \]

\[ p_b \bigg|_{\text{canonical}} = \frac{e^{-I_b(\beta)}}{Z(\beta)} \]
The Lowest Energy Band Dominates

\[ Z(\beta) \equiv \sum_b e^{-I_b(\beta)} \quad \text{and} \quad I_b(\beta) = \beta E_b - S_b \]

\[ e^{-I_b(\beta)} e^{-S_b} \]

\[ \exp(-\beta E_b) \]

* For bands which are all quasi integrable, the noise is determined by the thermodynamical dominant.
Thermal Gas Noise

\[ B_1 = \frac{1}{L \frac{d-1}{2}} \sum_s \left( b_s a_s + b_s^* a_s^\dagger \right) \]

\[ \langle B_1(t)B_1(0) \rangle_{\text{gas}} = \frac{1}{L^{d-1}} \sum_s \left[ (1 + f(\omega_s)) |b_s|^2 e^{-i\omega_s t} + f(\omega_s) |b_s|^2 e^{i\omega_s t} \right] + \text{inter} \]

\[ f(\omega_s) = \left( e^{\beta \omega_s} - 1 \right)^{-1} \]
Thermal Gas Noise-Large

\[
\frac{|\langle B_1(t)B_1(0)\rangle_{\text{gas}}|^2}{\text{noise}} \sim \frac{1}{L^{2d-2}} \sum_{\omega_s < T} (1 + 2f(\omega_s) + 2f(\omega_s)^2) |b_s|^4 \sim L^{2-2d} (LT)^{d-2}
\]

\[
\frac{|\text{noise}|}{|\text{peak}|} \sim \frac{1}{(LT)^{d/2}} \sim \frac{1}{\sqrt{S_{\text{gas}}}}
\]

The peak scales as

\[
\langle B_1^2 \rangle \sim L^{1-d} (LT)^{d-1}.
\]
• Geometry reproduces correctly the average property.

• Geometry reproduced a VERY SMALL non perturbative result.

• Geometry does not reproduce even finer details of the non perturbative behaviour of the time dependent correlations.
This will have consequences in AdS CFT
Listen to the AdS Noise
Several Bands
Large gap, large difference in entropy properties. In particular, the high-energy entropy scales as the central charge, of order $N^2$, dominate by states which describe as black hole states in the bulk (glue in the CFT picture.) In addition, there is a band of low-energy states looking like graviton-gas states in the bulk (glueballs in the CFT) with entropy of $O(1)$ in the large $N$ limit. The low energy band extends from the spectral mass gap up to the energies of $O(N^2)$ where the 'black hole' states start to dominate the density of states.

The natural dynamical assumption is that of quantum chaos for the high-energy (black hole) band and thermal gas for the low-energy (graviton) band. Interactions among gravitons are suppressed by powers of $1/N^2$. Hence, at moderate values of $N$ we may consider also a chaotic model for an interacting graviton gas. In this case the main difference between the high and low energy bands would be just the jump in density of states.

With the AdS/CFT application in mind, simplifications occur when restricting attention to regularized operators whose fixed energy width $\Gamma_B$ is small compared to the overall energy range of interest. For example, we may consider a model with two bands (high and low) with $E_h \gg E_l \gg \Gamma_B$. In this case we can approximate the $B_{mn}$ matrix by a block-diagonal form as in the figure, in the discrete uniform density representation of matrix entries. If the upper band is chaotic and the lower band has perturbative quasiparticle-like states, a single-particle operator would have the ETH form in the upper band and the sparse quasiparticle form in the lower band.

Fine details of the spectrum may include additional intermediate bands with different types of black holes and/or Hagedorn transients.

Figure 3: If the relevant part of the spectrum is approximated by two narrow bands with very different average energies, $E_h \gg E_l$, and dimensionalities, $e^{S_h} \gg e^{S_l}$, the ETH ansatz for regularized operators can be simplified by writing a block-diagonal form as in the figure, in the discrete uniform density representation of matrix entries. If the upper band is chaotic and the lower band has perturbative quasiparticle-like states, a single-particle operator would have the ETH form in the upper band and the sparse quasiparticle form in the lower band.
The Lowest Energy Band Dominates
This is NOT an accident

\[ Z(\beta) \equiv \sum_b e^{-I_b(\beta)} \text{ and } I_b(\beta) = \beta E_b - S_b \]

\[ e^{-I_b(\beta)} e^{-S_b} \]

\[ \exp(-\beta E_b) \]
ETH For BHs and Strings.

For the Gas:

\[ B \sim \frac{1}{N} \text{Tr} \ F^n \]
For $T$ small relative to the critical $T$ 
$S$ and $I$ are $O(1)$ in 

$$|\text{noise}|_{\text{canonical}} \sim |b|^2 \frac{e^{-|\Delta I|}}{\sqrt{S_{\text{gas}}}}$$
For $T \gg T_c$ critical

\[
|\text{noise}|_{T \gg T_c} \sim |b|^2 e^{I_{bh}} \left[ \frac{1}{(RT)^{9/2}} + O \left( e^{-c_{\text{Hag}} \lambda^{5/2}} \right) \right]
\]

\[
+ O \left( e^{-c_{\text{sh}} N^2 / \lambda^{7/4}} \right) + O \left( e^{-c_{\text{bh}} N^2} \right)
\]
The Average Noise is determined by the lowest band, the fast O(1) variations are determined by it as well. But the height and the long time variations are determined by the thermodynamical dominant configuration.
Slogans:

1. Diversity Counts.

2. Geometry can capture non perturbative average observables.

3. Geometry may well miss some parts.

4. For ETH the lowest energy band dominates the value of the average noise. Not the thermodynamical leading one.