The Adiabatic Path To the Eightfold Way of Hydrodynamic Dissipation

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[1502.00636], [1412.1090], [1312.0610]

Wilsonian effective field theories

- The Wilsonian paradigm provides a powerful way to think about QFTs.
- Typically, we tend to consider the Wilsonian effective action for vacuum (or near-vacuum) dynamics, and therefore focus on low energy physics in pure states.
- Low energy dynamics in mixed states is a fascinating question for a QFT and shows up in a wide variety of circumstances:
  - non-equilibrium dynamics of QFTs.
  - semi-classical gravity in the presence of horizons etc.
- Question: How does one upgrade the Wilsonian paradigm to deal with mixed states?

Density matrices and doubling

$$\langle\psi| \to e^{iHt}\langle\psi|$$

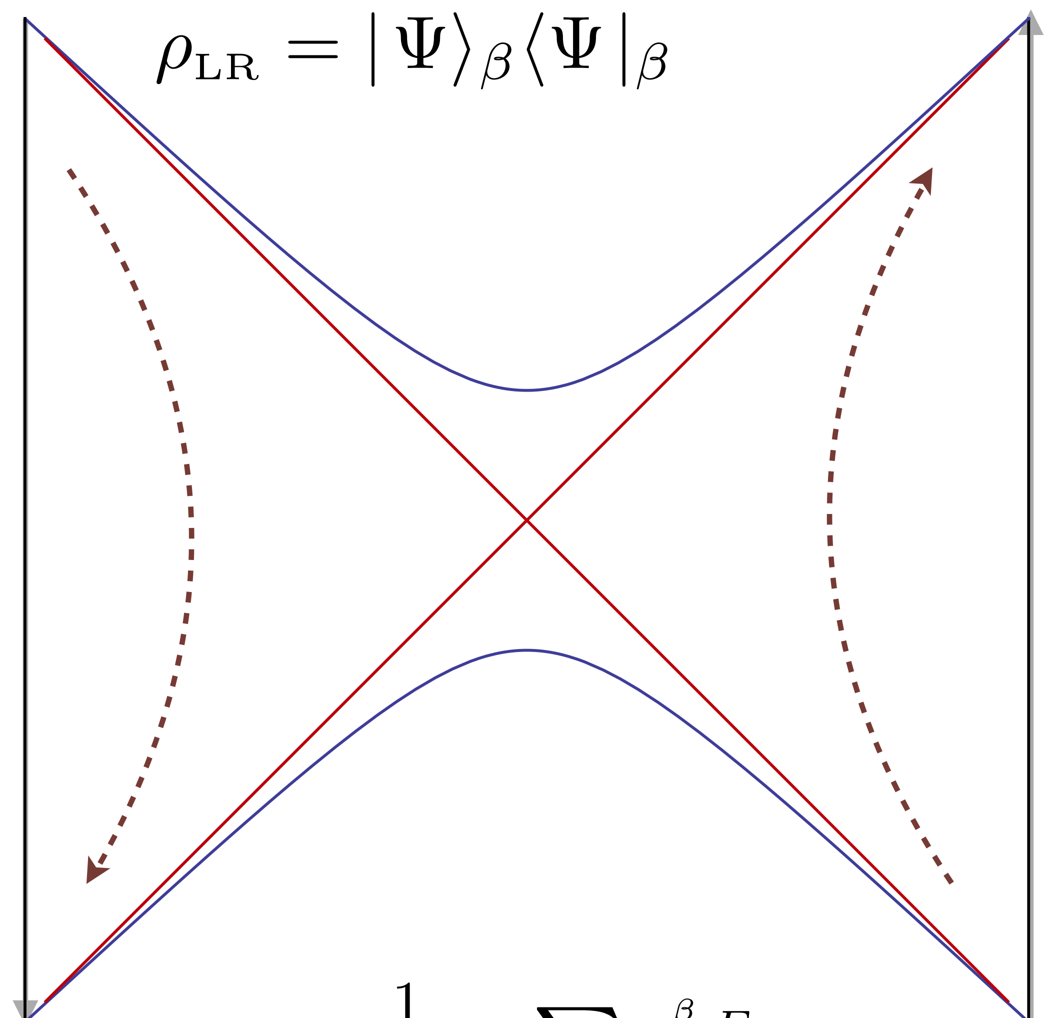
$$|\psi\rangle \to e^{-iHt}\,|\psi\rangle$$

$$\rho = |\psi\rangle\langle\psi| \to e^{-iHt}\,|\psi\rangle\langle\psi|\,e^{iHt}$$

Density matrices and doubling

$$\rho_\beta = \frac{1}{Z}\, e^{-\beta H} = \mathrm{Tr}_{\mathrm{L}}\left(\rho_{\mathrm{LR}}\right)$$

$$\rho_{\mathrm{LR}} = |\,\Psi\rangle_\beta \langle \Psi\,|_\beta$$



$$|\,\Psi\rangle_\beta = \frac{1}{\sqrt{Z(\beta)}} \sum e^{\frac{\beta}{2}\,E_a}\; |\,\mathfrak{r}_a\,\mathfrak{l}_a\rangle$$

Density matrices & equilibrium dynamics

- Equilibrium QFT is well understood in this thermofield double, or Schwinger-Keldysh construction
- This is useful for computing real time correlation functions with microscopic unitarity constraints being imposed via the KMS condition (periodicity in Euclidean time).
- The gravitational analog for equilibrium dynamics is the eternal black hole spacetime which constructs the Hartle-Hawking thermofield state (cf., ER=EPR).

Israel '76; Maldacena '01

- Similarly, we understand how to compute real time correlation functions in equilibrium from holography.

Son, Starinets '02; Herzog, Son '02; Skenderis, van Rees '08

Out of equilibrium: Here be dragons

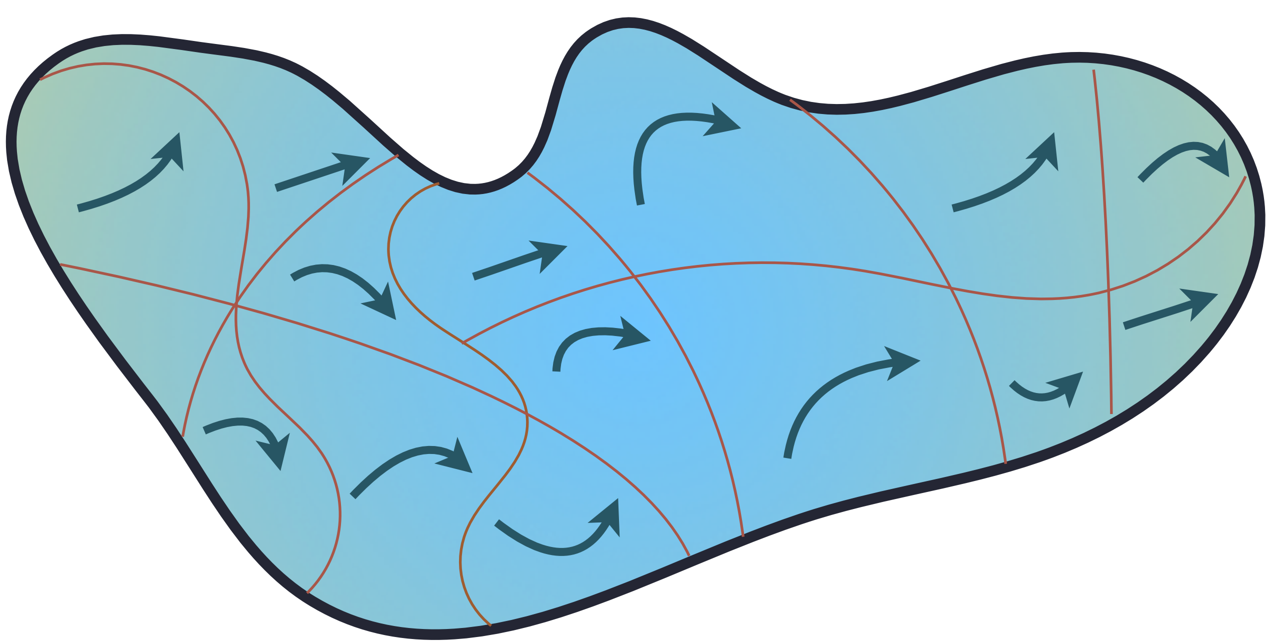
- However, away from equilibrium the story becomes a lot more murky.
- Integrating out degrees of freedom in the mixed state induces interactions between the two parts of the Schwinger-Keldysh contour – these are Feynman-Vernon influence functionals.
- Arbitrary Feynman-Vernon functionals are problematic and need constraining (tension with microscopic unitarity constraints).

✦ The Hydrodynamic Path

- Hydrodynamics accords a perfect opportunity to formulate a Wilsonian construction in background density matrices.
- **Goal:** Construct an effective action for hydrodynamic dissipation.

The hydrodynamic effective field theory

- Relativistic fluid dynamics is best thought of as an effective field theory for quantum systems in local, but not global, thermal equilibrium.
- The description in terms of fluid dynamics is valid when departures from equilibrium are on scales that are large compared to the characteristic mean free path of the underlying quantum dynamics.



$$\ell_{\rm mfp} \ll L\,, \qquad t_{\rm mfp} \ll t$$

- Local domains of equilibrated fluid can be characterized by the local temperature/energy density and conserved charges.
- Energy/charge flux exchanged across the domains: velocity field.

Axioms of Hydrodynamics I: Fields

- Hydrodynamics describes low-energy, near-equilibrium fluctuations of an equilibrium Gibbsian density matrix on scales large compared to the characteristic mean free path.
- The macroscopic description involves currents which capture energy-momentum and charge transport $T^{\mu\nu},\ J^{\mu}$ (and entropy current J_S^{μ}).
- The currents are functionals of the hydrodynamic fields, which are the intensive variables characterizing the density matrix and background sources.
  - temperature and chemical potential and a flux vector (fluid velocity) $T, \mu, u^{\mu}, \qquad u^{\mu}\, u_{\mu} = -1$
  - background metric and electromagnetic potential $g_{\mu\nu}, A_{\mu}$

Axioms of Hydrodynamics II: Data

* Repackage the dynamical degrees of freedom in a vector an scalar

thermal vector $\boldsymbol{\beta}^\mu \equiv \frac{u^\mu}{T}\,, \qquad \Lambda_{\boldsymbol{\beta}} \equiv \frac{\mu}{T} - \frac{u^\sigma}{T} A_\sigma$ thermal twist

* The currents of hydrodynamics are expressed as functionals of the hydrodynamical fields and the background sources.

- currents $T^{\mu\nu}, J^\mu, J^\mu_S$
- fields $\boldsymbol{\Psi} \equiv \{g_{\mu\nu}, A_\mu, \boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$
- constitutive relations

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}\left[\boldsymbol{\Psi}\right] = T^{\mu\nu}\left[g_{\alpha\beta}, A_\alpha, \boldsymbol{\beta}^\alpha, \Lambda_{\boldsymbol{\beta}}\right] \\ J^\mu &= J^\mu\left[\boldsymbol{\Psi}\right] = J^\mu\left[g_{\alpha\beta}, A_\alpha, \boldsymbol{\beta}^\alpha, \Lambda_{\boldsymbol{\beta}}\right] \\ J^\mu_S &= J^\mu_S\left[\boldsymbol{\Psi}\right] = J^\mu_S\left[g_{\alpha\beta}, A_\alpha, \boldsymbol{\beta}^\alpha, \Lambda_{\boldsymbol{\beta}}\right]\,. \end{aligned}$$

Axioms of Hydrodynamics III: Dynamics

- The dynamical content of hydrodynamics is the statement of conservation, modulo work done by sources and anomalies:

$$\nabla_\nu T^{\mu\nu} = J_\nu \cdot F^{\mu\nu} + \mathrm{T}^{\mu\perp}_H \qquad D_\nu J^\nu = \mathrm{J}^{\perp}_H$$

work term

covariant anomalies

- These are effectively *Ward identities* for the one-point functions of the conserved currents in the fluctuating Gibbs density matrix.

- The task of a hydrodynamicist is to specify the currents as a functional of the hydrodynamic fields, consistent with the dynamics, constructing a current algebra of sorts, but…

Axioms of Hydrodynamics IV: Constraints

- From a macroscopic, statistical viewpoint, one has to demand that a local form of the second law of thermodynamics is upheld.

$$\exists \;\; J^{\mu}_{S}[\mathbf{\Psi}]: \;\; \forall \;\; \mathbf{\Psi}_{\text{on-shell}} \;\; \nabla_{\mu} J^{\mu}_{S}[\mathbf{\Psi}] \geq 0$$

- This is required to be upheld on-shell, and complicates the analysis of hydrodynamics, for without it the current algebra can be analyzed purely in terms of representation theory.
- Note that usually one only requires the existence of *some* entropy current.
- From a microscopic viewpoint the entropy current is rather mysterious; it is not associated with any underlying symmetry per se.

- **Opportunity:** *Understand a Wilsonian hydrodynamic theory consistent with second law.*

Neutral fluids

- A neutral fluid is characterized by its energy-momentum stress tensor

$$T^{\mu\nu} = \epsilon(T)\, u^\mu\, u^\nu + p(T)\, P^{\mu\nu} - \eta(T)\, \sigma^{\mu\nu} - \zeta(T)\, \Theta\, P^{\mu\nu} + \cdots$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu$$

spatial metric

$$\nabla_\mu u_\nu = \sigma_{(\mu\nu)} + \omega_{[\mu\nu]} + \Theta\, P_{\mu\nu} - u_\mu\, \mathfrak{a}_\nu$$

shear vorticity acceleration

expansion

- The second law forces some of the transport data to satisfy some inequalities, e.g., the viscosities are non-negative definite (friction)

$$J^\mu_S = s\, u^\mu + \cdots$$

$$\nabla_\mu J^\mu_S = \eta\, \sigma^2 + \zeta\, \Theta^2 + \cdots \qquad \eta, \zeta \geq 0$$

Wilsonian formulation of hydrodynamics

- A satisfactory Wilsonian framework should satisfy 3 primary criteria:
  - Consistency with current algebra formulation.
  - Dynamics ≈ current conservation.
  - Rationalize the entropy current & second law.

- Based on what we have seen so far, we can expect:
  - Doubling of fields.
  - Constraints on Feynman-Vernon terms.

- We also get something new and unexpected: an emergent gauge symmetry!

Benchmarking hydrodynamics

- To set the stage for our Wilsonian framework, we need to understand hydrodynamic constitutive relations compatible with second law.
- Ideally, this should data should be given to us off-shell, since we are aiming to construct effective action.

STRATEGY

- Take the entropy current constraint off-shell.
- Classify all off-shell physical constitutive relations.
- Derive the resulting constitutive relation from an effective action.

Off-shell entropy production

- ✦ Take the statement of the second law off-shell Lagrange multipliers

$$\nabla_\mu J_S^\mu + \boldsymbol{\beta}_\mu \left(\nabla_\nu T^{\mu\nu} - J_\nu \cdot F^{\mu\nu} - \mathrm{T}_H^{\mu\perp}\right) + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\lambda A_\lambda) \cdot \left(D_\nu J^\nu - \mathrm{J}_H^\perp\right) = \Delta \geq 0$$

$$\boldsymbol{\beta}^\mu \equiv \frac{u^\mu}{T}\,, \qquad \Lambda_{\boldsymbol{\beta}} \equiv \frac{\mu}{T} - \frac{u^\sigma}{T} A_\sigma$$

entropy production by dissipation

- ✦ The Lagrange multipliers are fixed to be the hydrodynamic fields exploiting field redefinition freedom.

- ✦ This off-shell formalism motivates separation of transport into:

  - ✴ dissipative (Class D)

  - ✴ adiabatic

Free energy current

- Package the information in terms of a Gibbs free energy current, switching from a microcanonical to grand-canonical language:

$$
\begin{aligned}
J_S^\sigma &= -\left[\boldsymbol{\beta}_\nu T^{\nu\sigma} + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\nu A_\nu)\cdot J^\sigma + \frac{\mathcal{G}^\sigma}{T}\right] \\
&\equiv (J_S^\sigma)_{can} - \frac{\mathcal{G}^\sigma}{T}\,.
\end{aligned}
$$

- The off-shell second law statement can be phrased now as

$$
\boxed{-\left[\nabla_\sigma\left(\frac{\mathcal{G}^\sigma}{T}\right) - \frac{\mathcal{G}_H^\perp}{T}\right] = \frac{1}{2}T^{\mu\nu}\delta_{\mathfrak{B}}\, g_{\mu\nu} + J^\mu\cdot\delta_{\mathfrak{B}} A_\mu + \Delta}
$$

$$
\begin{aligned}
\delta_{\mathfrak{B}}\, g_{\mu\nu} &\equiv \pounds_{\boldsymbol{\beta}} g_{\mu\nu} = \nabla_\mu \boldsymbol{\beta}_\nu + \nabla_\nu \boldsymbol{\beta}_\mu\,, \\
\delta_{\mathfrak{B}} A_\mu &\equiv \pounds_{\boldsymbol{\beta}} A_\mu + \partial_\mu \Lambda_{\boldsymbol{\beta}} + [A_\mu, \Lambda_{\boldsymbol{\beta}}]
\end{aligned}
$$

diffeomorphism

flavour gauge transformation

Hydrodynamic taxonomy

✦ The off-shell formalism is quite powerful. One can classify hydrodynamic constitutive relations into eight distinct classes:

- ✳ Class D: dissipative class
- ✳ Class B: Berry-like transport
- ✳ Class A: anomaly induced transport
- ✳ Class C: conserved entropy

$$\mathcal{G}^{\mu} = \mathfrak{S}\,\boldsymbol{\beta}^{\mu} + \mathfrak{V}^{\mu}\,, \qquad \mathfrak{V}^{\mu}\,\boldsymbol{\beta}_{\mu} = 0$$

longitudinal vector

transverse vector (conserved)

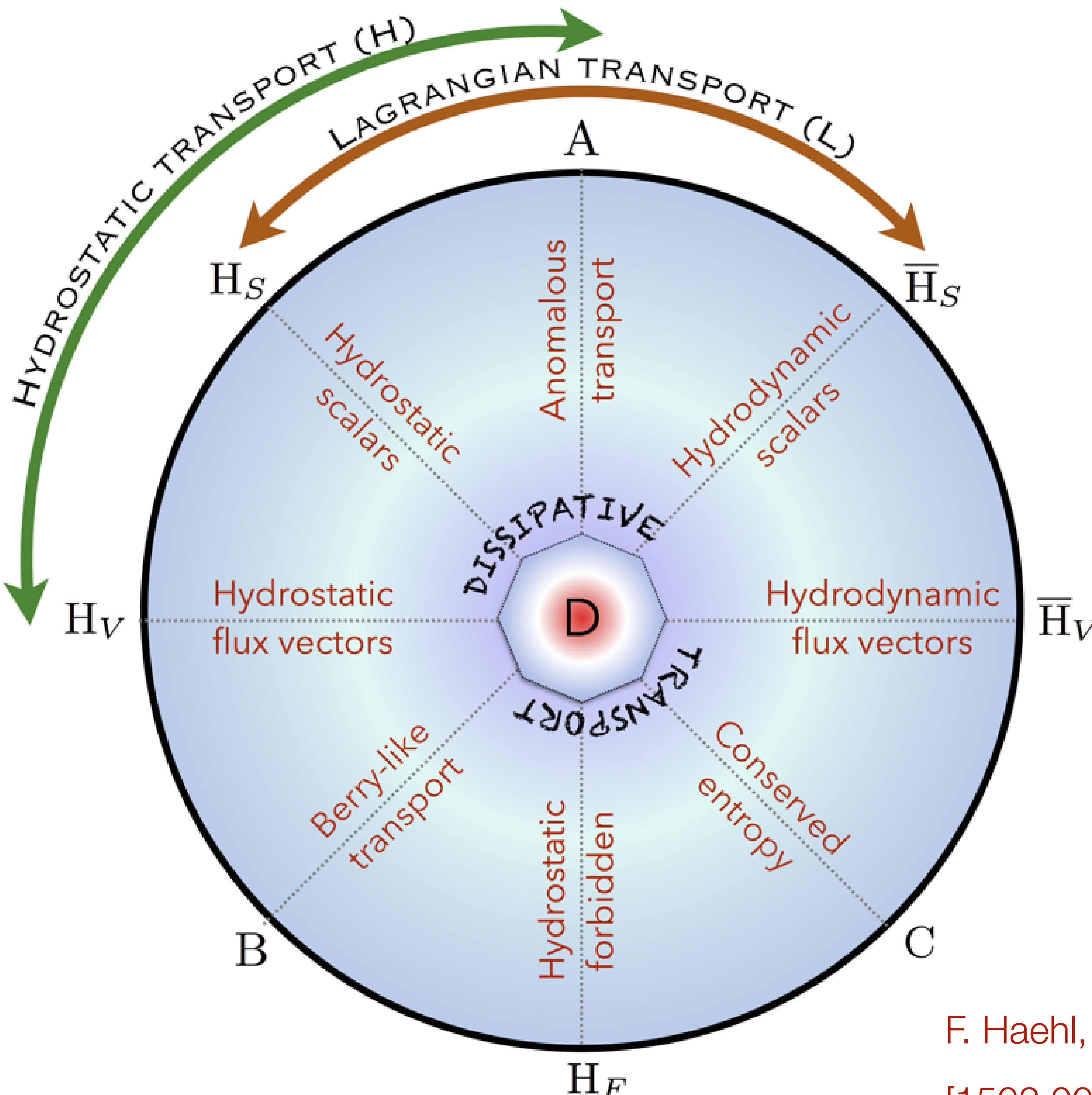
Free energy scalars

- ✳ Class H_S: Hydrostatic scalars
- ✳ Class $\overline{H}_S$: Landau-Ginzburg scalars

Free energy vectors

- ✳ Class H_V: Hydrostatic vectors
- ✳ Class $\overline{H}_V$: Gibbsian vectors

Eightfold classification of hydrodynamic transport



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Class H: Hydrostatics ($H_S \cup H_V$)

- ✦ Hydrodynamic transport can be classified into two categories
  - ✳ Hydrostatic or thermodynamic response: fixed by equilibrium
  - ✳ Genuine hydrodynamic transport
- ✦ Hydrostatic data can be understood by time-independent configurations of the fluid in the presence of non-trivial (spatially varying) background sources.
- ✦ Can equivalently be encoded in a generating function, the equilibrium partition function which is a functional of stationary background sources.

$$\mathfrak{K} \equiv \{K^\mu, \Lambda_K\}\,, \quad g_{\mu\nu}\, K^\mu\, K^\nu \leq 0 \longrightarrow \delta_{_\mathfrak{K}} g_{\mu\nu} = \delta_{_\mathfrak{K}} A_\mu = 0$$

Banerjee et. al. '12 Jensen et. al. '12

Class H: Hydrostatics

- The hydrostatic partition function is the integral of the (consistent) free energy current over the Wick rotated Euclidean manifold.

$$W_{\text{Hydrostatic}} = -\left[\int_{\Sigma_E} \left(\frac{\mathcal{G}^\mu_{cons}}{T}\right)\, d^{d-1}S_\mu\right]_{\text{Hydrostatic}}$$

spatial integral!

- Since the free energy current is a vector field, it decomposes into

$$\mathcal{G}^\mu = \mathfrak{S}\,\boldsymbol{\beta}^\mu + \mathfrak{V}^\mu\,, \qquad \mathfrak{V}^\mu\,\boldsymbol{\beta}_\mu = 0\,.$$

longitudinal vector

transverse vector (conserved)

$$\mathrm{H} = \mathrm{H}_S \cup \mathrm{H}_V$$

partition fn scalars

partition fn vectors

Entropy constraint: Hydrostatic forbidden (H_F)

- The scalars and vectors which do not vanish in equilibrium parameterize the free energy current and in turn generate the currents after varying with respect to the sources.

$$\delta W_{\text{Hydrostatic}} = \left[\int_{\Sigma_E} \left(\frac{1}{2} T^{\mu\nu}_{cons}\, \delta g_{\mu\nu} + J^{\mu}_{cons} \cdot \delta A_{\mu} \right) \boldsymbol{\beta}^{\alpha} d^{d-1} S_{\alpha} \right]_{\text{Hydrostatic}}$$

- At any given derivative order however, there are fewer scalars than the tensor structures in the currents.
- *Hydrostatics implies that certain constitutive relations are forbidden.*
- Intuitively think of hydrostatics as time-independent configurations; turning on time dependence one should find no linear term, for it can produce entropy of either sign.

Bhattacharyya ('13, '14)

Example: Ideal fluids

- For an ideal fluid, the hydrostatic partition function is generated by the pressure *p(T,μ)*.

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + (p - \zeta\,\Theta)\, P^{\mu\nu} - \eta\, \sigma^{\mu\nu}\,, \qquad J^\mu_S = s\, u^\mu$$

- Adiabaticity (or sign-definiteness of Δ) implies that all the zeroth order transport is determined by *p(T)*.

$$\epsilon + p - T\, s = 0$$ ← Gibbs-Duhem relation

$$\frac{d\epsilon}{dT} - T\,\frac{ds}{dT} = 0$$ ← Clausius relation

- At second order: 5 constraints for the neutral fluid.

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu\,, \qquad \Theta = \nabla_\alpha u^\alpha\,, \qquad \sigma_{\mu\nu} = \nabla_{<\mu} u_{\nu>}$$

$$\mathcal{A}_{<\alpha\beta>} = \left(P_{\alpha\mu} P_{\beta\nu} - \frac{1}{d-1}\, P_{\alpha\beta}\, g_{\mu\nu}\right) \mathcal{A}^{\mu\nu}$$

Class D: Dissipation

- ✦ Focus on positivity of Δ order by order in the gradient expansion. Deviations from equilibrium: $\delta_{\mathscr{B}} g \quad \delta_{\mathscr{B}} A$

  - viscous dissipative terms $\quad \eta\, \sigma^{\mu\nu} + \zeta\, \Theta\, P^{\mu\nu} \Longrightarrow \Delta = \eta\, \sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta\, \Theta^2 \sim (\delta_{\mathcal{B}} g)^2$
  - descendant operators $\quad \delta_{\mathscr{B}} g\, D\mathcal{O}_{k-2}$
  - product composites $\quad (\delta_{\mathscr{B}} g)^k \quad (\delta_{\mathscr{B}} A)^k$

  → sub-dissipative

- ✦ Sub-dissipative terms can be subsumed under viscous dissipative terms.

- ✦ **Theorem**: Entropy constraints operate only at leading order in the gradient expansion!

Bhattacharyya ('11, '13, '14)

- ✦ Useful restatement of the argument using tensor valued differential operators acting on $\delta_{\mathscr{B}} g \quad \delta_{\mathscr{B}} A$

Set Δ =0 henceforth.

Class A: Anomalies

$$\nabla_\sigma \left(\frac{\mathcal{G}^\sigma}{T} \right) - \frac{\mathcal{G}^\perp}{T} = -\frac{1}{2} T^{\mu\nu} \delta_{\mathcal{B}} g_{\mu\nu} - J^\mu \cdot \delta_{\mathcal{B}} A_\mu$$

Adiabaticity Equation

- The anomalous constitutive relations are particular constitutive relations, which can be determined once and for all and thence we can focus on the anomaly-free part of adiabaticity equation (AE). Loganayagam '11

Jensen, Loganayagam, Yarom '13

- It should be noted that the anomalous constitutive relations are a finite class, owing to the topological origins of the associated contributions.

- These constitutive relations can be obtained from an effective action which is the integral of a particular transgression form built from the anomaly polynomial.

Haehl, Loganayagam, MR '13-'15

Class B: Berry-like transport

- This class of constitutive relations solves adiabaticity trivially. Non-equilibrium, non-dissipative data!

$$(T^{\mu\nu})_{\rm B} \equiv -\frac{1}{4}\left(\mathcal{N}^{(\mu\nu)(\alpha\beta)} - \mathcal{N}^{(\alpha\beta)(\mu\nu)}\right)\, \delta_{_\mathscr{B}} g_{\alpha\beta} + \mathcal{X}^{(\mu\nu)\alpha}\cdot \delta_{_\mathscr{B}} A_\alpha$$
$$(J^\alpha)_{\rm B} \equiv -\frac{1}{2}\mathcal{X}^{(\mu\nu)\alpha}\delta_{_\mathscr{B}} g_{\mu\nu} - \mathcal{S}^{[\alpha\beta]}\cdot \delta_{_\mathscr{B}} A_\beta$$

- The entropy current is canonical (given just by projections of energy-momentum and charge currents)

Hall Transport in 3 dimensions

$$(T^{\mu\nu})_B = -\tilde{\eta}_{_H}\, u_\rho\, \left(\varepsilon^{\rho\mu\alpha}\, \sigma^\nu_\alpha + \varepsilon^{\rho\nu\alpha}\, \sigma^\mu_\alpha\right)$$
$$(J^\alpha)_B = \tilde{\sigma}_{_H}\cdot u_\rho\, \varepsilon^{\rho\alpha\beta}\left[E_\beta - T\, D_\beta\left(\frac{\mu}{T}\right)\right]$$

Neutral fluids in arbitrary dimensions

$$(T^{\mu\nu})_B = -\lambda_\sigma\left(\Theta\,\sigma^{\mu\nu} - \sigma^2\, P^{\mu\nu}\right) - \lambda_\omega\left(\omega^{\mu\alpha}\sigma^\nu_\alpha + \omega^{\nu\alpha}\sigma^\mu_\alpha\right)$$

Class C: Conserved entropy

- AE can be solved by considering an exactly conserved entropy current.

$$(J_S^\mu)_{\rm C} = \mathsf{J}^\mu\,, \qquad (T^{\mu\nu})_{\rm C} = 0\,, \qquad (J^\mu)_{\rm C} = 0$$

- Currents must be cohomologically non-trivial (non-Komar terms) for them to be physically interesting.

- Eg., Wen-Zee current in 3 spacetime dimensions (more generally Euler currents in odd spacetime dimensions.

$$\mathsf{J}^\sigma_{\rm Euler} = \frac{1}{2} c_{\rm Euler}\ \varepsilon^{\sigma\alpha\beta}\ \varepsilon^{\mu\nu\lambda}\ u_\mu \left(\nabla_\alpha u_\nu \nabla_\beta u_\lambda - \frac{1}{2} R_{\nu\lambda\alpha\beta}\right)$$

- These currents count the degeneracy of topological states in the thermal density matrix and can be realized holographically (eg., Gauss-Bonnet contribution to black hole entropy in ABJM like theories).

Class $\overline{\mathrm{H}}$v: Gibbsian vectors

- Just as hydrostatic vectors entered into parameterization of the free energy current, there are non-trivial hydrodynamic vectors which lead to adiabatic constitutive relations.
- These are parameterized by tensor valued differential operators

$$
\begin{aligned}
(T^{\mu\nu})_{\overline{\mathrm{H}}_V} &\equiv \frac{1}{2}\left[D_\rho \mathfrak{C}_{\mathcal{N}}^{\rho(\mu\nu)(\alpha\beta)}\, \delta_{\mathbb{B}} g_{\alpha\beta} + 2\, \mathfrak{C}_{\mathcal{N}}^{\rho(\mu\nu)(\alpha\beta)}\, D_\rho \delta_{\mathbb{B}} g_{\alpha\beta}\right] \\
&\qquad + D_\rho \mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha} \cdot \delta_{\mathbb{B}} A_\alpha + 2\, \mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha} \cdot\, D_\rho \delta_{\mathbb{B}} A_\alpha \\
(J^\alpha)_{\overline{\mathrm{H}}_V} &\equiv \frac{1}{2}\left[D_\rho \mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha} \delta_{\mathbb{B}} g_{\mu\nu} + 2\, \mathfrak{C}_{\mathcal{X}}^{\rho(\mu\nu)\alpha}\, D_\rho \delta_{\mathbb{B}} g_{\mu\nu}\right] \\
&\qquad + D_\rho \mathfrak{C}_{\mathcal{S}}^{\rho(\alpha\beta)} \cdot \delta_{\mathbb{B}} A_\beta + 2\, \mathfrak{C}_{\mathcal{S}}^{\rho(\alpha\beta)} \cdot\, D_\rho \delta_{\mathbb{B}} A_\beta
\end{aligned}
$$

- No explicit data on such transport, but they do appear in charged fluids at second order in gradients.

Class L = $H_S \cup \overline{H}_S$

- Consider diffeomorphism and gauge invariant scalar Lagrangian densities which are functionals of hydrodynamic fields $\boldsymbol{\Psi} \equiv \{g_{\mu\nu}, A_\mu, \boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$

$$S_{\text{hydro}} = \int d^d x \sqrt{-g}\; \mathcal{L}\left[\boldsymbol{\Psi}\right]$$

- The basic variational principle of this theory defines currents:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\delta\left(\sqrt{-g}\;\mathcal{L}\right) - \nabla_\mu(\not{\delta}\Theta_{_{\text{PS}}})^\mu \\ = \frac{1}{2}\,T^{\mu\nu}\;\delta g_{\mu\nu} + J^\mu\cdot\delta A_\mu + T\,V_\sigma\;\delta\boldsymbol{\beta}^\sigma + T\,\zeta\cdot\left(\delta\Lambda_{\boldsymbol{\beta}} + A_\sigma\,\delta\boldsymbol{\beta}^\sigma\right)\end{aligned}$$

- Entropy density is defined as in thermodynamics

$$s \equiv \left(\frac{1}{\sqrt{-g}}\,\frac{\delta}{\delta T}\int\sqrt{-g}\;\mathcal{L}\left[\boldsymbol{\Psi}\right]\right)\Bigg|_{\{u^\sigma,\,\mu,\,g_{\alpha\beta},\,A_\alpha\}=\text{fixed}} \qquad\qquad J^\mu_S = s\,u^\mu$$

Class L adiabaticity

Now diffeomorphism and flavour gauge symmetries of the Lagrangian imply a set of Bianchi identities:

$$\nabla_\nu T^{\mu\nu} = J_\nu \cdot F^{\mu\nu} + \frac{g^{\mu\nu}}{\sqrt{-g}}\delta_{\mathscr{B}}\left(\sqrt{-g}\; T V_\nu\right) + g^{\mu\nu}\, T\,\zeta \cdot \delta_{\mathscr{B}} A_\nu$$

$$D_\sigma J^\sigma = \frac{1}{\sqrt{-g}}\delta_{\mathscr{B}}\left(\sqrt{-g}\; T\zeta\right)$$

Together with the identity and an off-shell Euler relation

$$\nabla_\sigma J^\sigma_S = \nabla_\sigma (T\, s\, \boldsymbol{\beta}^\sigma) = \frac{1}{\sqrt{-g}}\delta_{\mathscr{B}}\left(\sqrt{-g}\; T s\right)$$

$$T\, s + \mu \cdot \zeta + u^\sigma\, V_\sigma = 0$$

one ends up with the non-anomalous AE

$$\nabla_\mu J^\mu_S + \boldsymbol{\beta}_\mu \left(\nabla_\nu T^{\mu\nu} - J_\nu \cdot F^{\mu\nu}\right) + \left(\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\lambda A_\lambda\right) \cdot D_\nu J^\nu = 0$$

Dynamics in Class L

- The dynamics in Class L is supposed to reduce to the conservation of energy-momentum and charge currents.
- Naive variation with respect to $\{\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$ does not respect this requirement, since it would lead to vanishing of the adiabatic heat/charge currents.
- Constrained variational principle: vary the hydrodynamic fields along a family related by Lie transport.

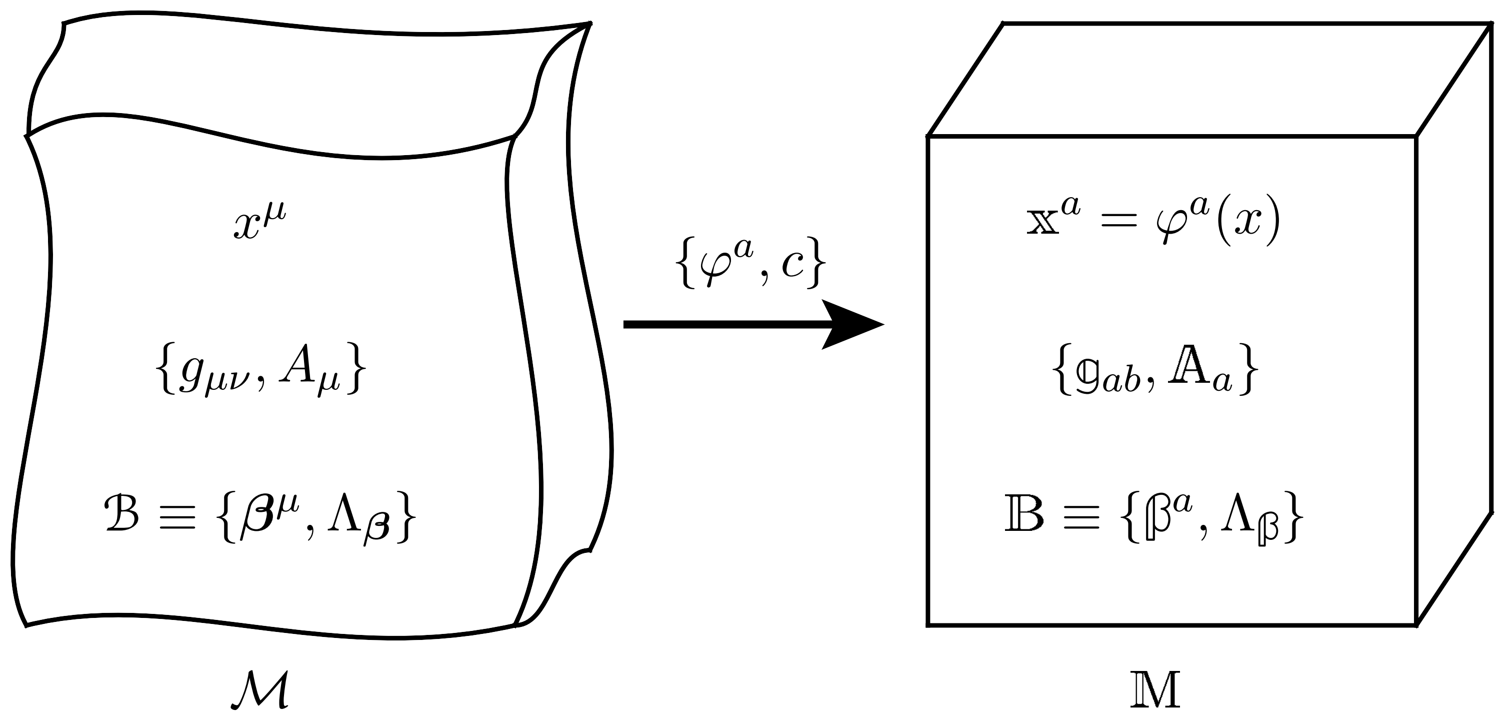
$$\mathbb{\delta} : \quad \mathbb{\delta}\boldsymbol{\beta}^\mu = \delta_\chi \boldsymbol{\beta}^\mu \,, \qquad \mathbb{\delta}\Lambda_{\boldsymbol{\beta}} = \delta_\chi \Lambda_{\boldsymbol{\beta}} \,, \qquad \mathbb{\delta} g_{\mu\nu} = \mathbb{\delta} A_\mu = 0$$

- This variation leads to equations of motion which when combined with the Bianchi identities leads to conservation

$$\frac{1}{\sqrt{-g}}\delta_{\mathscr{B}}\left(\sqrt{-g}\; T\, V_\mu\right) + T\,\zeta\cdot\delta_{\mathscr{B}} A_\mu \simeq 0$$
$$\frac{1}{\sqrt{-g}}\delta_{\mathscr{B}}\left(\sqrt{-g}\; T\zeta\right) \simeq 0$$

$$\xRightarrow{+\text{ Bianchi}} \qquad \nabla_\nu T^{\mu\nu} \simeq 0 \,, \qquad D_\nu J^\nu \simeq 0$$

Reference fields for Class L



The constrained variational principle can be alternately phrased as fixing a reference configuration and varying along the pull-back maps by diffeos and gauge transformations.

$$e_a^\mu \, \partial_\nu \varphi^a = \delta_\nu^\mu \ , \qquad e_a^\mu \, \partial_\mu \varphi^b = \delta_b^a \ .$$

$$\boldsymbol{\beta}^\mu = e_a^\mu(x) \ \mathbb{\beta}^a[\varphi(x)]$$

$$\Lambda_{\boldsymbol{\beta}} = c(x) \ \Lambda_{\mathbb{\beta}}[\varphi(x)] \ c^{-1}(x) + \boldsymbol{\beta}^\sigma(x) \, \partial_\sigma c(x) \ c^{-1}(x)$$

Eightfold effective action?

✦ We have distilled the essence of the second law and have our benchmarks.

✦ Prognosis for an effective action respecting this classification scheme?

✦ With a single set of hydrodynamic dof we do rather poorly (2/8).

* Class H_S: Hydrostatic scalars ✔
* Class $\bar{H}_S$: Landau-Ginzburg scalars ✔
* Class A: anomalous transport ✓̸
* Class B: Berry-like transport ✓̸
* Class C: conserved entropy ?
* Class D: dissipative class ???
* Class H_V: Hydrostatic vectors ?
* Class $\bar{H}_V$: Gibbsian vectors ?

A: Dubovsky, Nicolis, Hui '12; Haehl, Loganayagam, MR '13

B: Nicolis, Son '11; Haehl, MR '13; Geracie, Son '14

Symmetry from the eightfold way

- The eightfold classification includes constitutive relations which do not admit a simple Lagrangian description (6/8 classes).
- However, there exists a framework which has an enhanced symmetry and captures all of the adiabatic transport in a single Lagrangian density. (for the 7 classes).

• the background sources	$\{g_{\mu\nu}, A_\mu\}$	
• the fluid fields	$\{\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$	
• partners for the sources	$\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$	Schwinger-Keldysh like
• KMS gauge field	$A^{(T)}{}_\mu$	$U(1)_T$ invariant ensures adiabaticity

The Eightfold Lagrangian

✦ The adiabatic constitutive relations can be derived in one swoop from a Lagrangian density that is invariant under diffeomorphisms, flavour gauge transformations and the KMS $U(1)_T$ symmetry.

$$\mathcal{L}_T = \frac{1}{2} T^{\mu\nu}\, \tilde{g}_{\mu\nu} + J^{\mu} \cdot \tilde{A}_{\mu} + \left(J_S^{\sigma} + \boldsymbol{\beta}_{\nu} T^{\nu\sigma} + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^{\nu} A_{\nu}) \cdot J^{\sigma}\right) \mathsf{A}^{(\mathsf{T})}{}_{\sigma}$$

✦ The $U(1)_T$ symmetry ensures that the influence functionals which are allowed in the Schwinger-Keldysh construction respect the second law.

✦ A complete map between the Schwinger-Keldysh construction and the picture involving the partner sources and KMS photon is being developed, but there is a rather suggestive heuristic….

Wherefrom KMS gauge field?

✦ The non-canonical part of the entropy current is a Noether current.

$$-\frac{\mathcal{G}^\sigma}{T} = \boldsymbol{\beta}^\sigma\, \mathcal{L} - \left(\oint_{\mathbb{B}} \Theta_{\text{PS}}\right)^\sigma$$

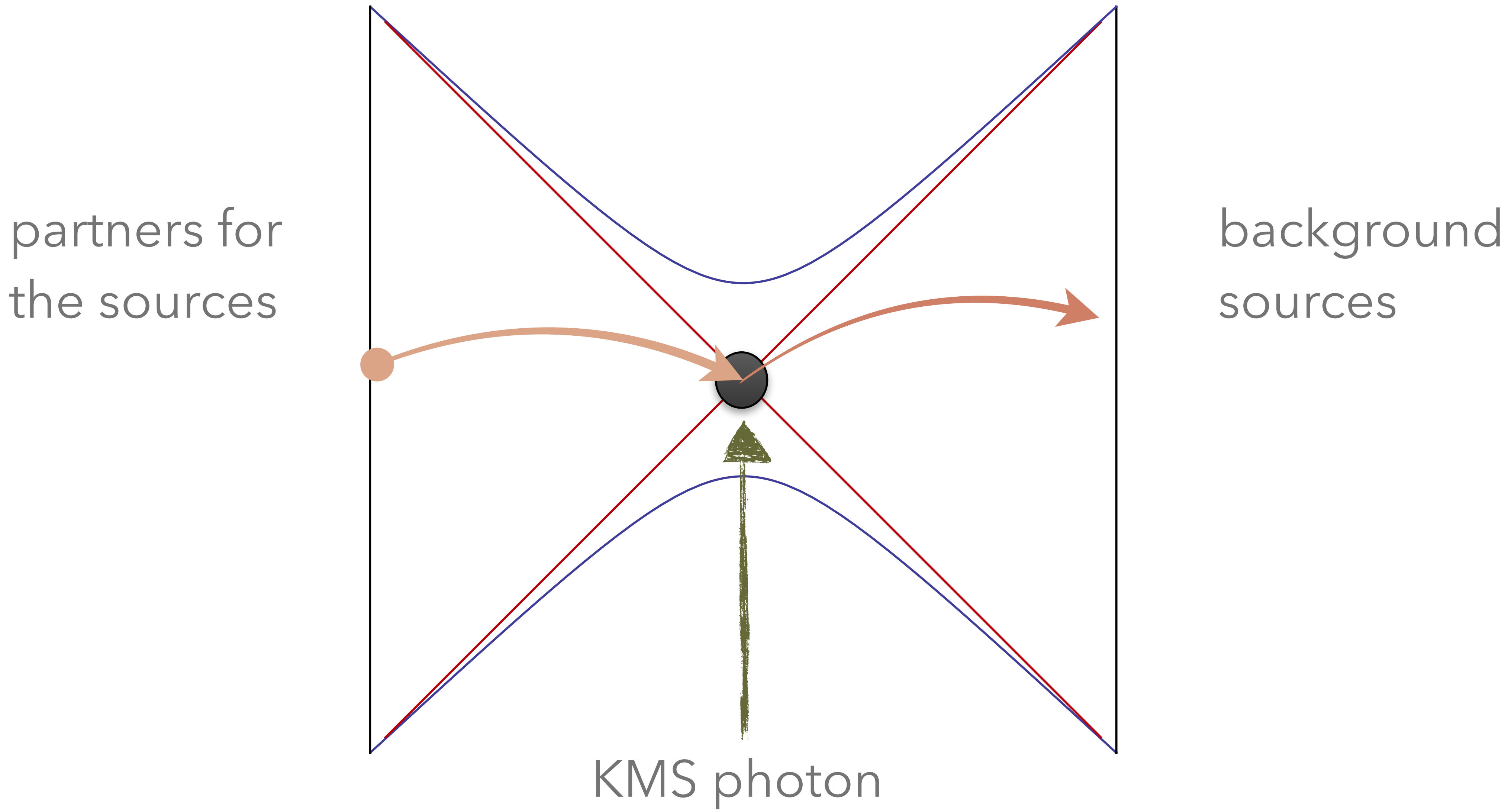
Iyer, Wald '94

✦ We claim that this is in fact the Noether current of an Abelian gauge field, whose conservation equation is indeed the Adiabaticity equation!

✦ Empirically, we have determined the $U(1)_T$ transformations of various fields and sources and shown that the diffeomorphism + flavour + $U(1)_T$ algebra closes.

✦ **Claim:** Gauging and Higgsing KMS gauge symmetry with the partner sources treated as Goldstone modes should allow incorporation of dissipation. Stay tuned…

A gravitational heuristic for KMS gauge invariance



Schwinger-Keldysh like construction, with KMS photon ensuring consistency with second law (macroscopic manifestation of KMS conditions).

Haehl, Loganayagam, MR (wip)

Classification of Weyl invariant fluids

✦ Weyl invariant neutral (and to some extent charged) fluids have been well studied from both

✳ kinetic theory (weak coupling) York, Moore '08

✳ holography via fluid/gravity (strong coupling) Baier et. al.; Bhattacharyya et. al., '07

✦ Given the data at hand we can ask whether this class of hydrodynamic systems is cognizant of the adiabatic eightfold way.

✦ The answer turns out to be in the affirmative indicating that these systems are aware of the classification scheme we propose.

Classification of Weyl invariant fluids

- The stress tensor for a conformal holographic fluid can be expressed in the eightfold basis as:

$$
\begin{aligned}
T^{\mu\nu} = {} & p\,(d\,u^\mu u^\nu + g^{\mu\nu}) - \eta\,\sigma^{\mu\nu} \\
& + (\lambda_1 - \kappa)\,\sigma^{<\mu\alpha}\sigma_\alpha^{\ \nu>} + (\lambda_2 + 2\,\tau - 2\kappa)\ \sigma^{<\mu\alpha}\omega_\alpha^{\ \nu>} \\
& + \tau\left(u^\alpha \mathcal{D}^{\mathcal{W}}_\alpha \sigma^{\mu\nu} - 2\,\sigma^{<\mu\alpha}\omega_\alpha^{\ \nu>}\right) + \lambda_3\,\omega^{<\mu\alpha}\omega_\alpha^{\ \nu>} \\
& + \kappa\ \left(C^{\mu\alpha\nu\beta}\,u_\alpha\,u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha^{\ \nu>} + 2\,\sigma^{<\mu\alpha}\omega_\alpha^{\ \nu>}\right).
\end{aligned}
$$

Hs

D

B

$\overline{\mathrm{H}}$s

- While the shear viscosity takes on the universal value, the second order transport satisfies two interesting relations

Haack, Yarom '08

$$\lambda_1 = \kappa\,, \qquad \lambda_2 = 2\,(\kappa - \tau)$$

Holographic fluids

- Adiabatic Transport coefficients for holographic fluids up to second order in the gradient expansion can be derived from a simple effective action:

$$\mathcal{L}^{\mathcal{W}} = c_{\text{eff}} \left(\frac{4\pi T}{d}\right)^{d} - c_{\text{eff}} \left(\frac{4\pi T}{d}\right)^{d-2} \left[\frac{{}^{\mathcal{W}}R}{(d-2)} + \frac{1}{2}\,\omega^2 + \frac{1}{d}\,\text{Harmonic}\left(\frac{2}{d}-1\right)\,\sigma^2\right]$$

- This formula is derived empirically; it would be great to give a first principles derivation from gravitational dynamics.

- Minimum dissipation conjecture: Holographic fluids not only attain the minimum allowed value of shear viscosity, but also ensure that the entropy production in any fluid flow is minimized.

Status Quo

✓ Classification of hydrodynamic transport.

✓ Effective action reproducing this classification scheme.

  * 7 of 8 classes work.

✓ Correct dynamics: constrained variational principle

➡ Relation to Schwinger-Keldysh?

➡ Connections to horizon dynamics?

✦ Hints that we are on the right track provided by existing analyses of hydrodynamic transport in holography and kinetic theory.

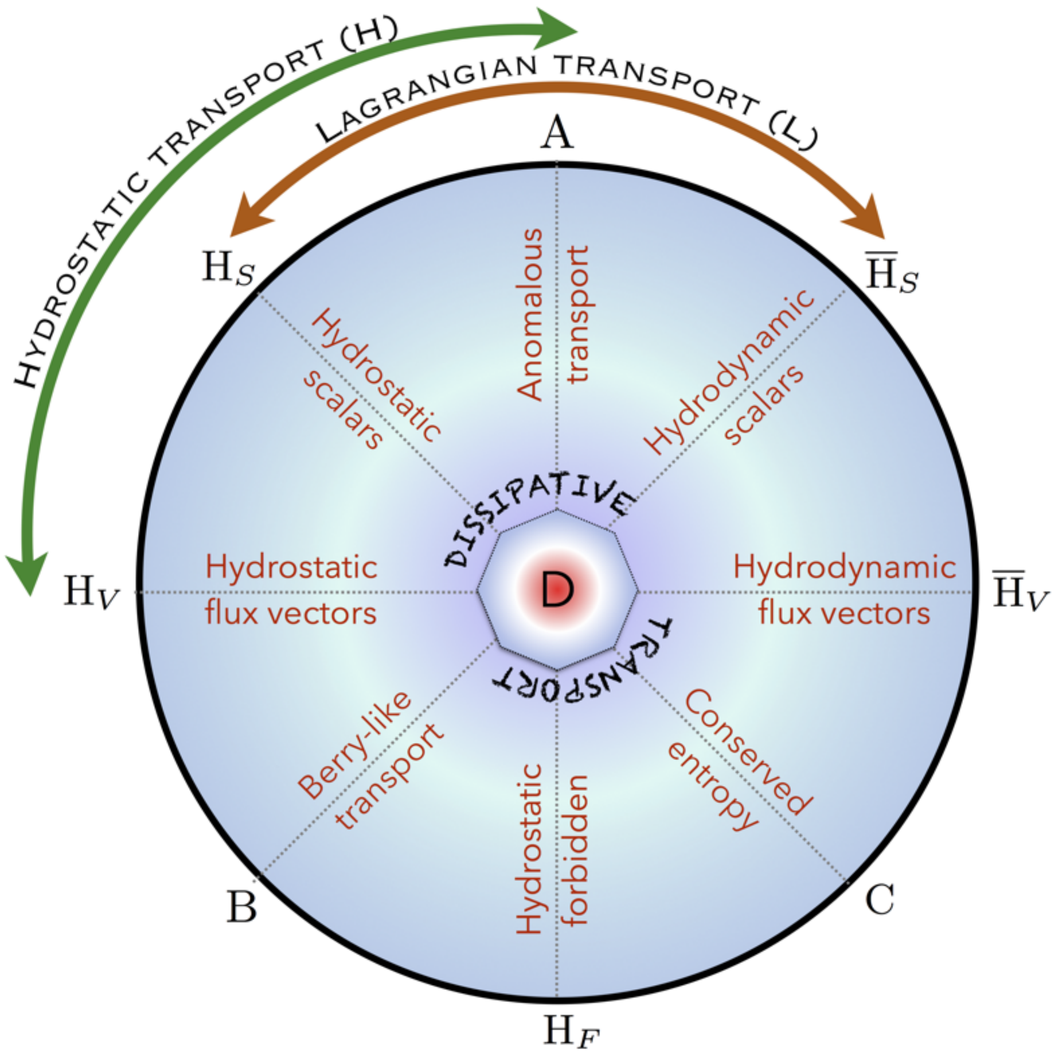
Summary

- There is a complete classification of hydrodynamic transport, at all orders in the gradient expansion.
- The key concept that facilitates this analysis is adiabaticity equation, which permits an off-shell analysis of the second law constraint.
- Various physical fluid systems that have been independently analyzed are cognizant of the adiabatic eightfold classification.
- The classification scheme not simply useful for structure purposes, but more pragmatically should allow simplifications of various computations.
- We see hints of an new symmetry principle that suggests a deep structure of non-equilibrium QFTs.

Open Questions

- Understand the microscopic origins of KMS flavour invariance.
- Determine the constraints on influence functionals in non-equilibrium dynamics arising from this underlying symmetry (expect it to be Higgsed in the non-equilibrium phase).
- Relation to fluctuation-dissipation relations?
- Derive the holographic fluid Lagrangian from the dynamics of gravity in asymptotically AdS spacetimes.

More Qs: Section 19 of 1502.00636


HYDROSTATIC TRANSPORT (H)
LAGRANGIAN TRANSPORT (L)
A
H_S
$\overline{H}_S$
H_V
$\overline{H}_V$
B
C
H_F
Anomalous transport
Hydrostatic scalars
Hydrodynamic scalars
Hydrostatic flux vectors
Hydrodynamic flux vectors
Berry-like transport
Conserved entropy
Hydrostatic forbidden
DISSIPATIVE
TRANSPORT
D

Thank you!