Bootstrapping the $(2,0)$ theories in six dimensions

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The $(2, 0)$ theories in six dimensions

The $(2, 0)$ theories are six-dimensional conformal field theories with the maximal amount of supersymmetry (16 $\mathcal{Q}$'s and 16 $\mathcal{S}$'s). They are conventionally described using string- or M-theory.

We do not know of a standard field-theoretic definition, which severely limits our understanding of these theories.

Nevertheless, these theories are the "mother" of many lower dimensional field theories and of tremendous interest for the general study of supersymmetric field theories in $d < 6$.

The $(2,0)$ theories in six dimensions

Some common lore:

- They are local and unitary quantum field theories with $\mathfrak{osp}(8^*|4)$ superconformal invariance.
- The known theories are classified by a simply-laced Lie algebra $\mathfrak{g} \in \{A_n, D_n, E_n\}$. [Witten (1995)]
- They are isolated: there are no marginal deformations that preserve $\mathfrak{osp}(8^*|4)$.
- The large n theories can be described through AdS/CFT.

The $(2,0)$ theories in six dimensions

- They are local and unitary quantum field theories with $\mathfrak{osp}(8^*|4)$ superconformal invariance.

In this sense, they are like any other conformal field theory: we have an infinite set of local operators $\mathcal{O}_i$, transforming in unitary irreducible representations of the (super)conformal algebra. Its correlation functions satisfy the (super)conformal Ward identities, for example

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}} ,$$

and there exists an *operator product expansion* or OPE

$$\mathcal{O}_i(x)\mathcal{O}_j(y) \simeq \sum_k {\lambda_{ij}}^k C[x-y,\partial_y]\mathcal{O}_k(y)$$

which has finite radius of convergence.

Until last year, we did not know of any nontrivial ${\lambda_{ij}}^k$ for finite n.

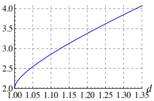
The $(2,0)$ theories in six dimensions

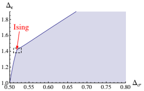
The (quantum numbers of the) $\mathcal{O}_i$ and the OPE coefficients ${\lambda_{ij}}^k$ are subject to the constraints of *crossing symmetry*.

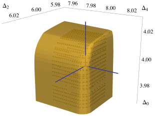
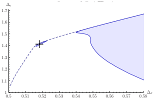
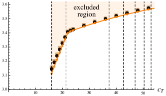
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$

$$= \sum_k \lambda_{12}^{\ \ k}\lambda_{34}^{\ \ k} \quad k \quad = \sum_p \lambda_{13}^{\ \ p}\lambda_{24}^{\ \ p} \quad p$$

The idea of the "bootstrap" is to eploit these constraints with the aspiration that they might completely determine the theory. We have made much progress in putting these constraints to use in higher-dimensional field theories.





[Rattazzi, Rychkov, Tonni, Vichi (2008); many others]

The $(2,0)$ theories in six dimensions

So maybe we can face the problem head-on and ask ourselves

What does crossing symmetry tell us about
six-dimensional $(2,0)$ *theories?*

Would it be possible to:

- Constrain the space of all theories?
- Find the spectrum for the allowed theories?
- Find the OPE coefficients?

The remainder of this talk discusses some initial results in trying to answer these questions.

Outline

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Classification of operators

The local operators $\mathcal{O}_i$ transform in unitary irreducible highest-weight representations of $\mathfrak{osp}(8^*|4)$ with maximal bosonic subalgebra $\mathfrak{so}(6,2) \times \mathfrak{so}(5)_R$.

They are therefore labelled by Δ, $[d_1, d_2, d_3]$, $[b_1, b_2]$.

Examples:

- The free tensor multiplet with $\mathfrak{so}(6) \times \mathfrak{so}(5)_R$ highest weight

5 scalars: $\Phi^{\mathbf{a}}$	$\Delta = 2$	$[0,0,0]$	$[1,0]$
4 fermions: $\Psi_{\alpha\mathbf{A}}$	$\Delta = 5/2$	$[1,0,0]$	$[0,1]$
1 two-tensor: $B_{\mu\nu}$	$\Delta = 3$	$[2,0,0]$	$[0,0]$

- The half-BPS operators

$\mathcal{O}_k^{\{\mathbf{a}_1 \ldots \mathbf{a}_k\}}(x)$	$\Delta = 2k$	$[0,0,0]$	$[k,0]$

  They form a ring with generators that are in one-to-one correspondence with the Casimirs of $\mathfrak{g}$. For example, for the A_n theories we have generators with $k \in \{2, 3, \ldots n+1\}$.

Chiral correlation functions

Consider now a correlation function

$$\langle \mathcal{O}^{I_1}(x_1) \dots \mathcal{O}^{I_n}(x_n) \rangle$$

with the following properties.

1. Consider $\mathbb{Q}$-*chiral* operators satisfying

$$\Delta = d_1/2 + d_2 + 2b_1 \qquad b_2 = d_3 = 0 \qquad b_1 \neq 0$$

   Such operators are the highest weights of a nontrivial $\mathfrak{su}(2) \subset \mathfrak{so}(5)_R$. We add the index I running over the entire $\mathfrak{su}(2)$ multiplet.
2. Take all n points to lie in a plane $\mathbb{R}^2 \subset \mathbb{R}^6$.
3. Contract the $\mathfrak{su}(2)$ indices with *position-dependent* $v_I(\bar{z})$. For example, for a doublet $v(\bar{z}) = (1, \bar{z})$.

Claim: the resulting correlation function is *meromorphic*.

The six-dimensional $(2, 0)$ theories have a chiral algebra!

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$$v_{I_1}(\bar{z}_1) \ldots v_{I_n}(\bar{z}_n) \langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \ldots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle$$

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$$\frac{\partial}{\partial \bar{z}_k} \left(v_{I_1}(\bar{z}_1) \ldots v_{I_n}(\bar{z}_n) \langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \ldots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle \right) = 0$$

with the following properties.

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Chiral correlation functions

Claim:

$$\frac{\partial}{\partial \bar{z}_k} \langle v_{I_1}(\bar{z}_1)\mathcal{O}^{I_1}(z_1,\bar{z}_1)\ldots\ldots v_{I_n}(\bar{z}_n)\mathcal{O}^{I_n}(z_n,\bar{z}_n)\rangle = 0$$

Proof:

- There exists a particular nilpotent supercharge $\mathbb{Q}$ such that

$$[\mathbb{Q}, \mathcal{O}^1(0)\} = 0\,.$$

  for $\mathbb{Q}$-chiral operators. Roughly speaking $\mathbb{Q} = \tilde{\mathcal{Q}} - \mathcal{S}$.
- Holomorphic translations are $\mathbb{Q}$ closed

$$[\mathbb{Q}, P_z] = 0$$

- In the antiholomorphic direction we find that

$$\partial_{\bar{z}} \left(v_I(\bar{z})\mathcal{O}^I(z,\bar{z}) \right) = v_I(\bar{z})[P_{\bar{z}} + \mathcal{R}^-, \mathcal{O}^I(\bar{z})]$$

  and such *twisted* antiholomorphic translations are $\mathbb{Q}$ exact

$$P_{\bar{z}} + \mathcal{R}^- = \{\mathbb{Q}, \ldots\}$$

Meromorphicity then follows from the usual cohomological argument.

Example

Consider the free tensor multiplet where the scalar Φ^+ is $\mathbb{Q}$-chiral.

In this case I is a triplet index, and $v_I(\bar{z}) = (1+\bar{z}^2, 2\bar{z}, i(1-\bar{z}^2))/\sqrt{2}$.

The OPE is

$$\Phi^I(z,\bar{z})\Phi^J(0) \sim \frac{\delta^{IJ}}{(z\bar{z})^2}$$

so

$$v_I(\bar{z})v_J(0)\Phi^I(z,\bar{z})\Phi^J(0) \sim \ldots = \frac{1}{z^2}$$

which is the OPE of a dimension one current. We write

$$[v_I(\bar{z})\Phi^I(z,\bar{z})]_{\mathbb{Q}} \rightsquigarrow j(z)$$

The other $\mathbb{Q}$-chiral operators are normal ordered products and holomorphic derivatives of this basic field.

$\rightarrow$ The complete *chiral algebra* of a free tensor multiplet is the $u(1)$ AKM algebra generated from

$$j(z)j(0) \sim \frac{1}{z^2}$$

Chiral correlation functions

For the interacting theories, we know that:

- The operators in the half-BPS chiral ring are also $\mathbb{Q}$-chiral, and generators of the chiral ring are also generators of the chiral algebra.
- The superconformal index, which counts (with signs) short operators, indicates that there are no further generators.

[Kim^3, Lee (2012)]

We claim that the generators of the chiral algebra are in one-to-one correspondence with the Casimir invariants of $\mathfrak{g}$.

For example, for the A_4 theory we have generators with dimension 2, 3, 4 and 5.

Conjecture: the chiral algebra for the $(2,0)$ theories of type $\mathfrak{g}$ is $W_{\mathfrak{g}}$.

From the twisted OPE of the stress tensor multiplet we find that

$$c_{2d} = c_{6d}$$

in conventions where $c_{6d} = 1$ for a free tensor multiplet.

[Beem, Rastelli, BvR (2014)]

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Three-point functions

OPE coefficients in the chiral algebra determine certain OPE coefficients in six dimensions. Consider for example the three-point functions of the half-BPS operators:

$$\langle \mathcal{O}_{k_1}^{\{\mathbf{a}_1 \ldots \mathbf{a}_{k_1}\}}(x_1) \mathcal{O}_{k_2}^{\{\mathbf{b}_1 \ldots \mathbf{b}_{k_2}\}}(x_1) \mathcal{O}_{k_3}^{\{\mathbf{c}_1 \ldots \mathbf{c}_{k_3}\}}(x_3) \rangle = \frac{\lambda_{\mathfrak{g}}(k_1, k_2, k_3) \mathfrak{C}^{\mathbf{a}_1 \ldots \mathbf{c}_{k_3}}}{x_{12}^{2k_{12}} x_{13}^{2k_{13}} x_{23}^{2k_{23}}}$$

The $\lambda_{\mathfrak{g}}(k_1, k_2, k_3)$ are completely determined by the chiral algebra.

Corollary: $\lambda_{\mathfrak{g}}(k_1, k_2, k_3)$ are three-point functions of the $W_{\mathfrak{g}}$ currents.

This works wonderfully at large n where (for the A_n type theories) we know from AdS/CFT that

$$\lambda_{n \to \infty}(k_1, k_2, k_3) = \frac{2^{2 \sum k_i - 2}}{\pi n^{3/2}} \Gamma\left(\frac{\sum k_i}{2}\right) \frac{\Gamma(\frac{k_{12}+1}{2}) \Gamma(\frac{k_{13}+1}{2}) \Gamma(\frac{k_{23}+1}{2})}{\sqrt{\Gamma(2k_1 - 1) \Gamma(2k_2 - 1) \Gamma(2k_3 - 1)}}$$

with $k_{12} = k_1 + k_2 - k_3$ etc. [Corrado et al; Bastianelli et al (1999)]

This agrees with the large n limit of W_n! (note: $c_{2d} \sim 4n^3$)

[Gaberdiel et al; Campoleoni et al (2011)]

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Outline

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1. Introduction
2. Analytical results
3. Numerical results
4. Conclusions

Numerical results

We have learned a great deal about OPE coefficients of a subset of protected operators. What about other operators? What about the *unprotected operators*? Can we for example constrain their spectrum?

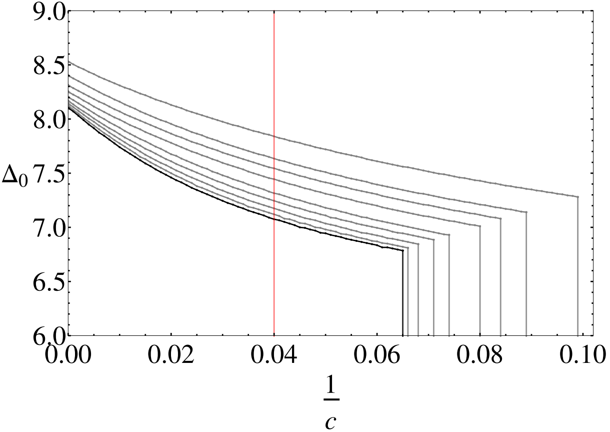
We will resort to numerical methods. I will give a snapshot of initial results, based on the crossing symmetry constraints and the methods of [Rattazzi, Rychkov, Tonni, Vichi (2008)].

We investigated the four point function of stress tensor multiplets.

[Beem, Lemos, Rastelli, BvR (to appear)]

Numerical results

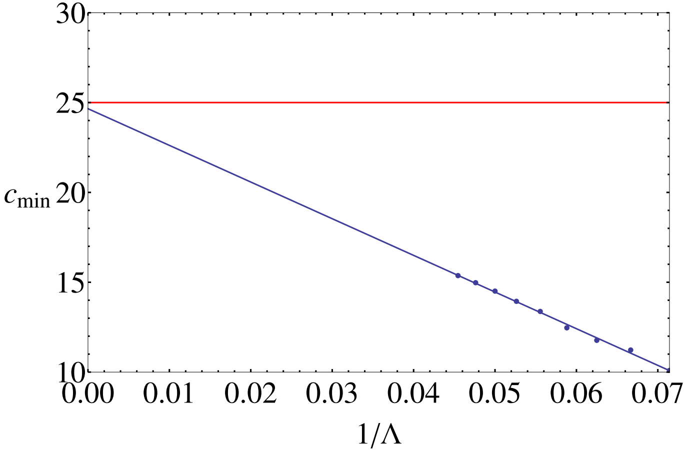
First result: an upper bound Δ_0 for the dimension of the lowest-lying unprotected $\mathfrak{so}(5)_R$ singlet scalar operator.



- The bound is likely to improve with better numerics
- There exists a minimal value of c (no HS currents)
- For very large c we find $\Delta_0 \lesssim 8.1$, in agreement with AdS/CFT.

Numerical results

The lower bound on c as a function of the set of crossing symmetry constraints.



The bound appears to converge to $c \simeq 25$!

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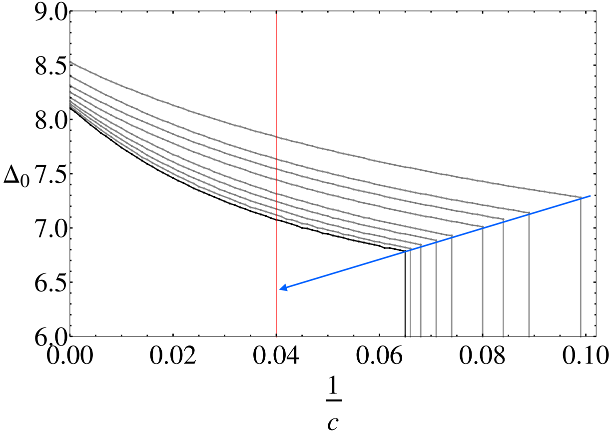

- We claim that the bound is *saturated* by the A_1 theory and that this theory has an unprotected scalar of dimension $\Delta_0 \sim 6.4$.

Conclusions

We gave a snapshot of analytic and numerical results that follow from a study of crossing symmetry for the six-dimensional $(2, 0)$ theories.

- We find exact OPE coefficients
- We find scaling dimensions numerically
- There also exist chiral algebras inside codimension two defects...
- but how about codimension two defect inside the algebra?

Can we really bootstrap the $(2, 0)$ theories? Let's try!