Magnetic Ground States in AdS/CFT

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Started Phd in 1987

- String theory and in particular the heterotic string dominated the scene.
- Understanding the structure of d=2 CFTs as the cornerstone of the subject was a major focus

- D=11 supergravity was at best a curiosity
- The D=11supermembrane of Bergshoeff, Sezgin, Townsend was a bit of a pariah....

GREAT!

Partially Broken Rigid Supersymmetry

- Aim: can we develop a better understanding of the local fermionic kappa (Siegel) symmetry that exists for the Green-Schwarz type actions for strings and branes [Hughes,Liu,Polchinski][Bergshoeff,Sezgin,Townsend] [Achucarro,Evans,Townsend,Wiltshire]

- After gauge-fixing the kappa symmetry, the spacetime supersymmetry gets transformed into supersymmetry on the world-volume. Some of the supersymmetry is linearly realised and some non-linearly realised.

- Naively violates a No-Go Theorem.

First paper with Paul in 1988:

Supersymmetry "on the brane": world volume supersymmetry from space-time supersymmetry of the four-dimensional supermembrane

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7389 papers on SPIRES with "brane" in title.
Some of Paul's contributions:

- "P-brane democracy"
- "Are two-branes better than one?"
- "Brane surgery"
- "PhreMology: Calibrating M-branes"

- One key point is that supersymmetry algebra contains topological charges. For example, for the supermembrane

$$\{Q_\alpha, Q_\beta\} = P_\mu(\Gamma^\mu)_{\alpha\beta} + Z_{\mu\nu}(\Gamma^{\mu\nu})_{\alpha\beta}$$

Topological Extensions of the Supersymmetry Algebra for Extended Objects
de Azcarraga, JPG, Izquierdo, Townsend

Directly lead to my interest in BPS states, Montonen-Olive duality, susy black holes....

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- 1990 - 100+ job applications - no offers!
- "Effortless transition from heresy to orthodoxy" - PKT

Applied AdS/CFT

- Can we use the AdS/CFT correspondence to study strongly coupled systems that arise in condensed matter physics or QCD?
- One focus of activity: analyse the behaviour of CFTs when held at finite temperature and charge density and/or in a uniform magnetic field
- What type of ground states can we have?
- Interesting to study this question in a top-down context

- Study a family of magnetic $AdS_3 \times \mathbb{R}^2$ solutions of SO(6) gauged SUGRA in D=5 ("magneto-vacs") [Romans][Almuhairi, Polchinski] (c.f. [Cowdall, Townsend])
- These can be uplifted on S^5 and hence are relevant to N=4 SYM.
- Some solutions can also be uplifted to D=10 and D=11 SUGRA in other ways and are relevant for infinite classes of N=2 SCFTs and N=1 SCFTs
- Are any of the AdS_3 solutions ground states for these d=4 SCFTs when placed in a uniform magnetic field plus deformations,vevs?

- There is an interesting sub-family of $AdS_3 \times \mathbb{R}^2$ solutions that are supersymmetric and hence stable. To show that they are good ground states of the SCFTs we constructed susy domain wall solutions that flow from AdS_5 in the UV to $AdS_3 \times \mathbb{R}^2$ in the IR

- For the non-susy solutions we need to investigate stability - are there modes that violate the AdS_3 BF bound?

  We will find rich classes of instabilities including spatially modulated instabilities

- Instability implies they are not good ground states.
  However, they indicate interesting physics.
  Take a domain wall that flows from AdS5 in the UV to $AdS_3 \times \mathbb{R}^2$ in the IR. It is unstable. If we heat it up, the black hole will also be unstable up to a critical temperature at which a new branch of black hole exists, corresponding to a new phase. The nature of the instability indicates what kind of phase this will be.

D=5 SO(6) Gauged SUGRA

- D=5 SO(6) gauge SUGRA

  metric

  15 SO(6) gauge fields

  12 two-forms

  42 scalars - parametrise coset $E_{6(6)}/USp(8)$

- Any solution of D=5 SO(6) gauged supergravity can be uplifted on S5 to obtain an exact solution of type IIB supergravity.

- AdS5 vacuum uplifts to AdS5xS5 which is dual to N=4 SYM in d=4

Additional truncation

- $U(1)^3 \subset SO(6)$ truncation

metric	$g_{\mu\nu}$	
$U(1)^3$ gauge fields	A^i_μ	$i = 1, 2, 3$
2 neutral scalar fields	ϕ_a	$a = 1, 2$

The 2 scalars can be packaged into three constrained scalars X_i

$$X_1 = e^{-\frac{1}{\sqrt{6}}\phi_1 - \frac{1}{\sqrt{2}}\phi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}}\phi_1}$$

Magnetic AdS3 Solutions in D=5

$$\begin{aligned} ds^2 &= L^2\, ds^2\,(AdS_3) + dx_1^2 + dx_2^2 \\ F^i &= 2q^i\, dx_1 \wedge dx_2, \qquad i = 1,2,3 \\ \phi_1 &= f_1, \quad \phi_2 = f_2 \end{aligned}$$

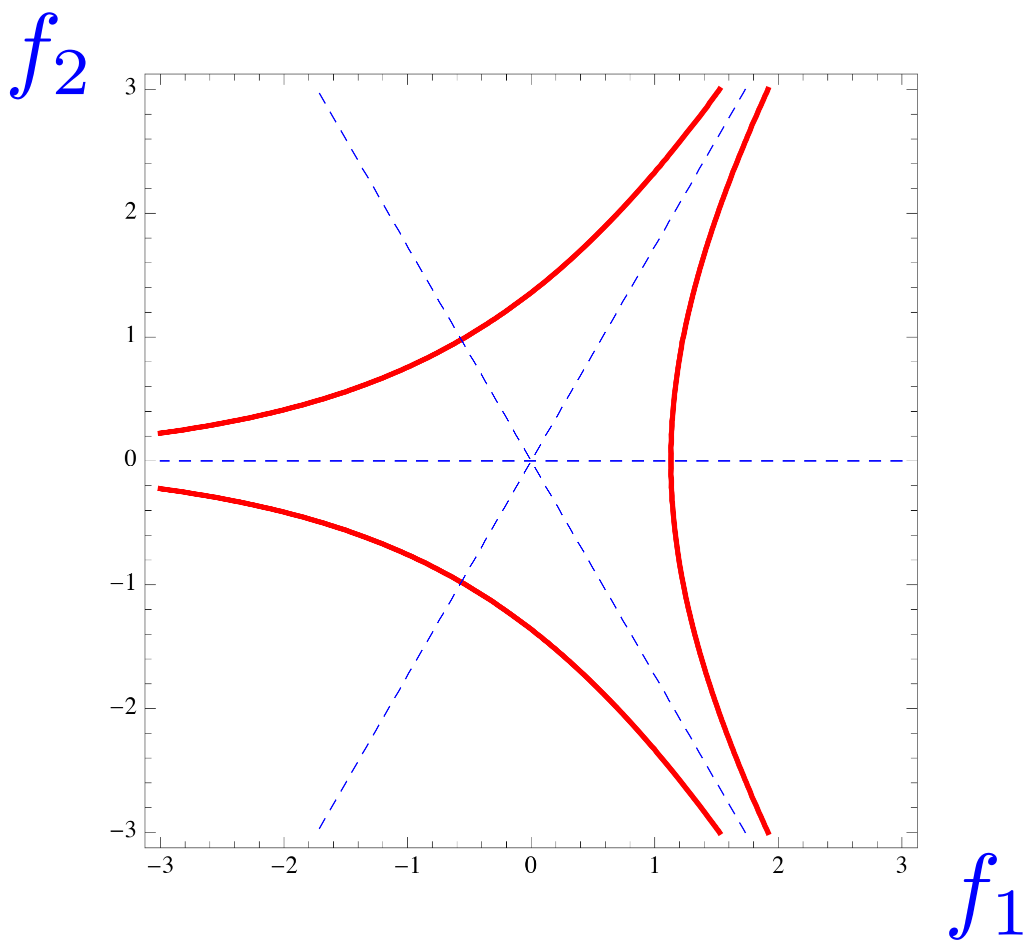
$$L^{-2} = \sum_{I=1}^{3} (\bar{X}_i)^{-1}, \qquad (q^i)^2 = \bar{X}_i\,,$$

$$\begin{aligned} \bar{X}_1 &= e^{-\frac{1}{\sqrt{6}} f_1 - \frac{1}{\sqrt{2}} f_2} \\ \bar{X}_2 &= e^{-\frac{1}{\sqrt{6}} f_1 + \frac{1}{\sqrt{2}} f_2} \\ \bar{X}_3 &= e^{\frac{2}{\sqrt{6}} f_1} \end{aligned}$$

- There is a locus of supersymmetric solutions -- dual to (0,2) supersymmetry in d=2

$$q_1 + q_2 + q_3 = 0 \qquad q_1 + q_2 - q_3 = 0$$

$$q_1 - q_2 + q_3 = 0 \qquad -q_1 + q_2 + q_3 = 0$$

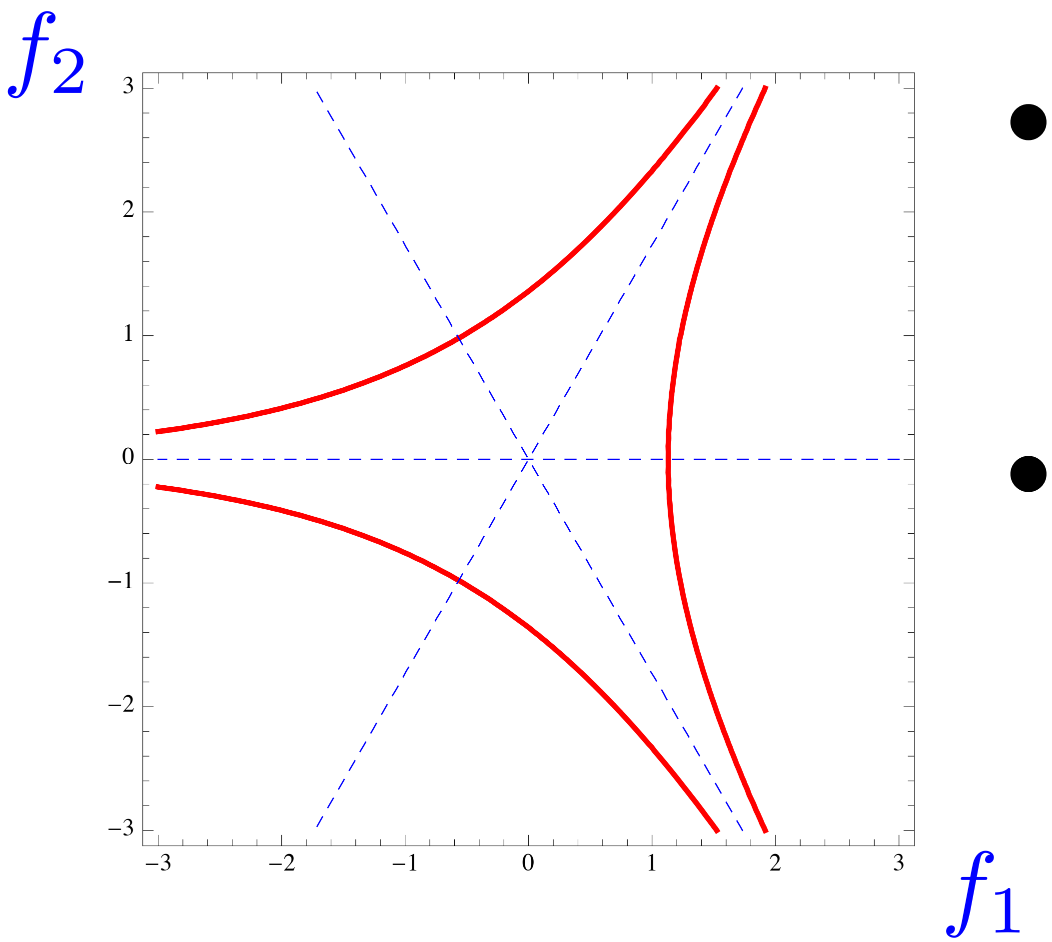


- The supersymmetric solutions are stable. Are they ground states of N=4 SYM? Yes!

Supersymmetric Domain walls

(c.f. [JPG, Tong, Townsend])

- We constructed supersymmetric domain wall solutions that interpolate between AdS_5 in the UV, with magnetic field deformation, and the susy $AdS_3 \times \mathbb{R}^2$ solutions in the IR
- The susy domain walls describe the ground states of N=4 SYM in a magnetic field and that there is an emergent d=2 (0,2) SCFT in the far IR



- Central charge comparison using free limit of N=4 SYM and the strong coupling value do not agree! [Almuhairi, Polchinski]
- Why? The weakly coupled does not approach the free theory because there is a relevant interaction.....

Instabilities

1. Spatially modulated instabilities

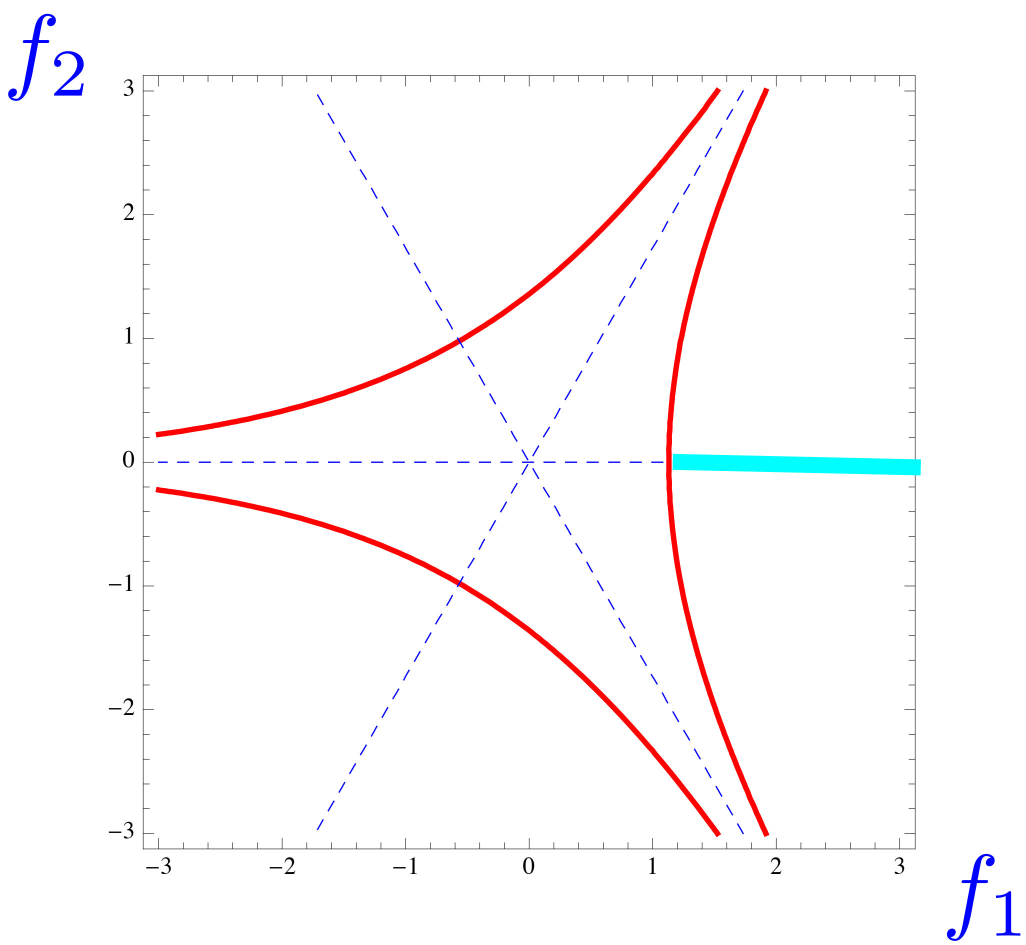
Inside the $U(1)^3$ truncation:

$$\delta A^1 = a(y)\sin(k\,x_1)dx_2, \qquad \delta A^2 = -a(y)\sin(k\,x_1)dx_2\,,$$
$$\delta\phi_2 = w(y)\cos(k\,x_1)$$

$$\mathbf{v} = (a, w)$$

$$(\Box_{AdS_3} - L^2 M^2)\mathbf{v} = 0$$

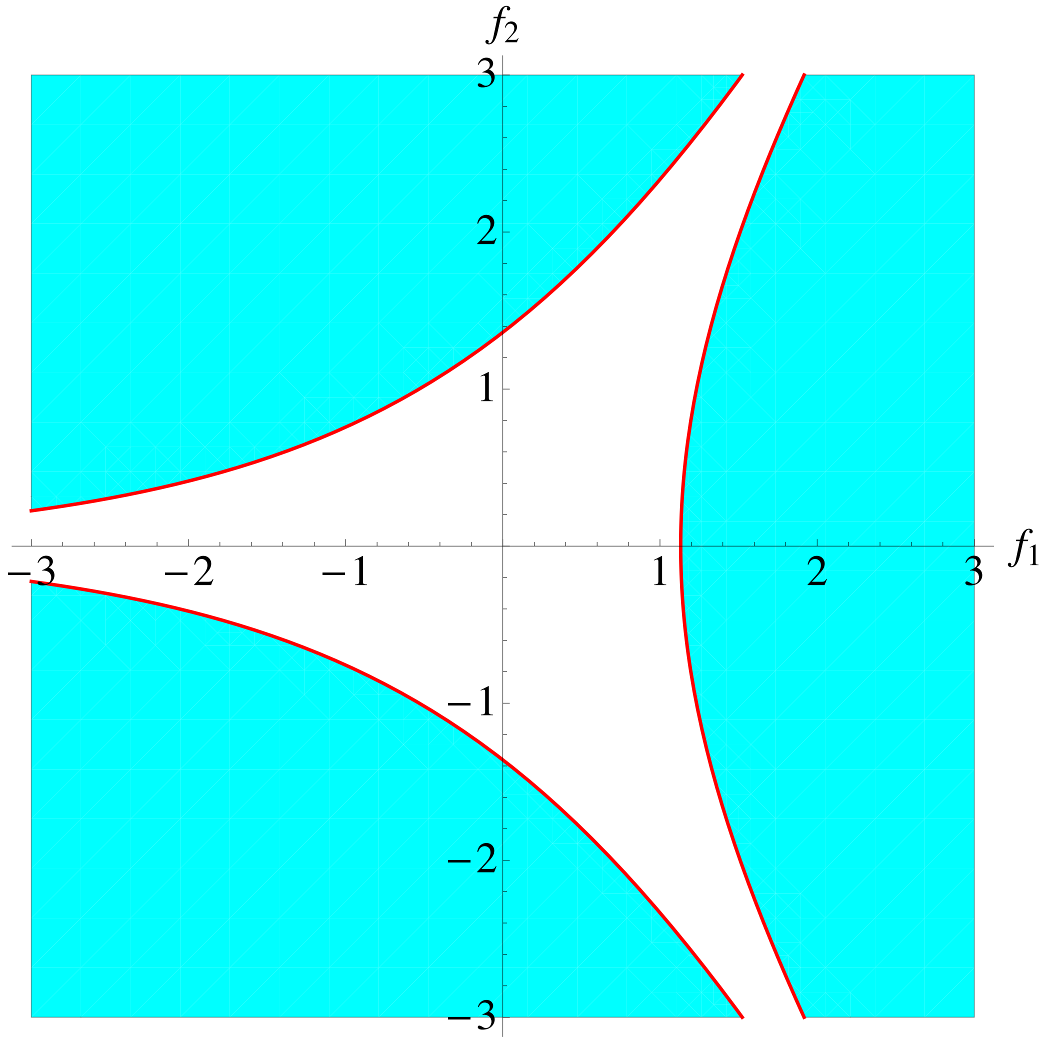
$$M^2 = \begin{pmatrix} k^2 & 2\sqrt{2}q^1\,k \\ 4\sqrt{2}q^1(\bar{X}_1)^{-2}\,k & 4(\bar{X}_1)^{-1} + k^2 \end{pmatrix}$$



Minimum for $k \neq 0$

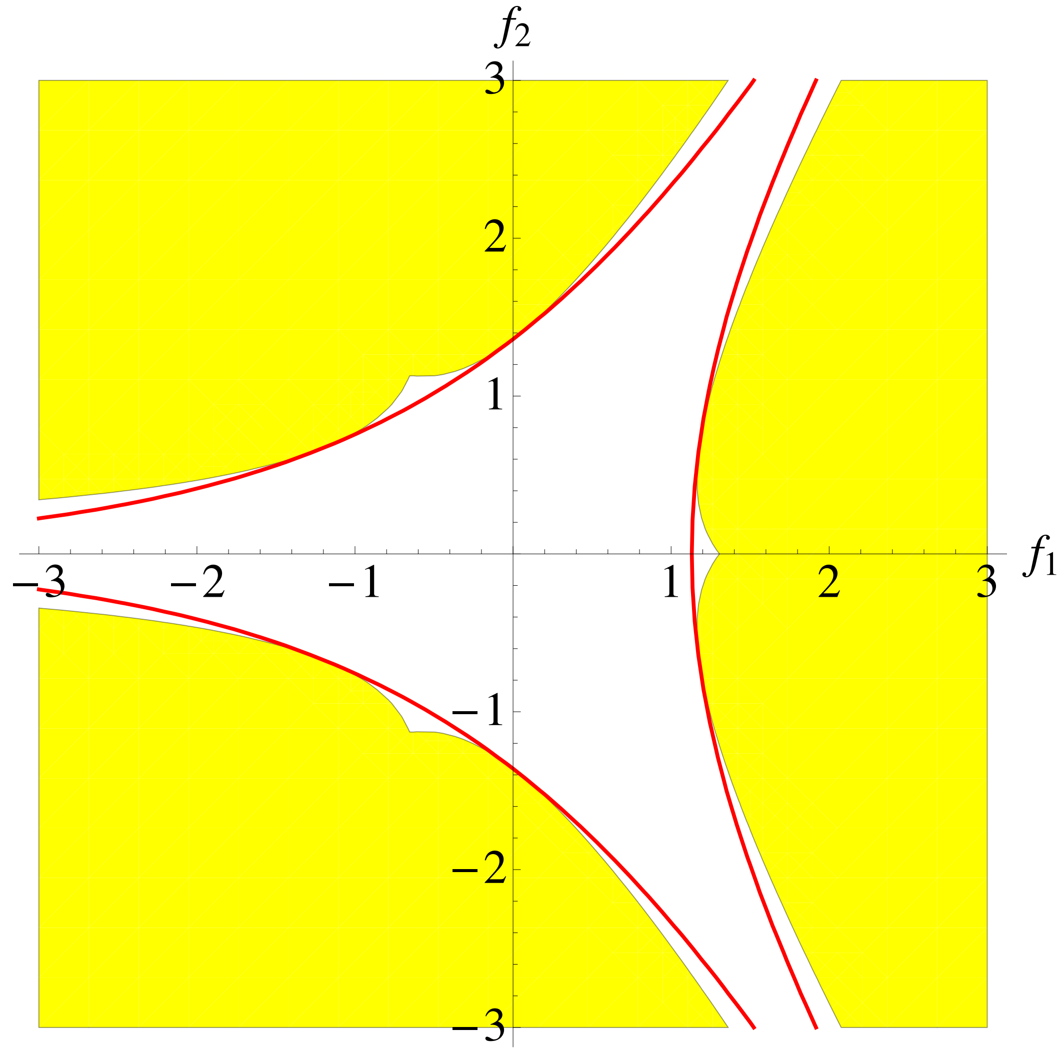
and violates BF bound on blue line

More general analysis (includes mixing with metric modes)



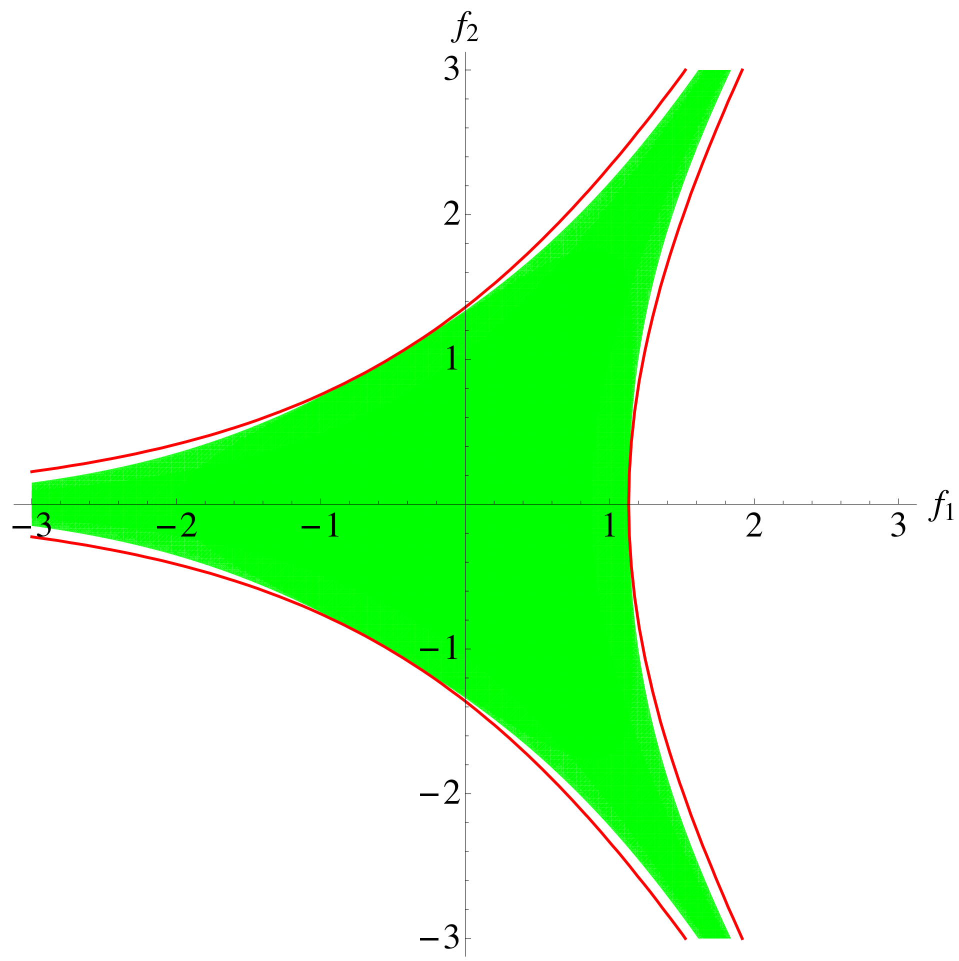
- For the supersymmetric solutions: the spatially modulated modes saturate the BF bound and preserve supersymmetry

II. Spatially modulated instabilities of charged scalar modes



- Since the modes are charged, these instabilities are associated with s-wave superfluidity (cf reentrant superconductivity)
- The BF staurating modes intersect the susy locus at six places. Again for these solutions the BF saturating modes preserve supersymmetry

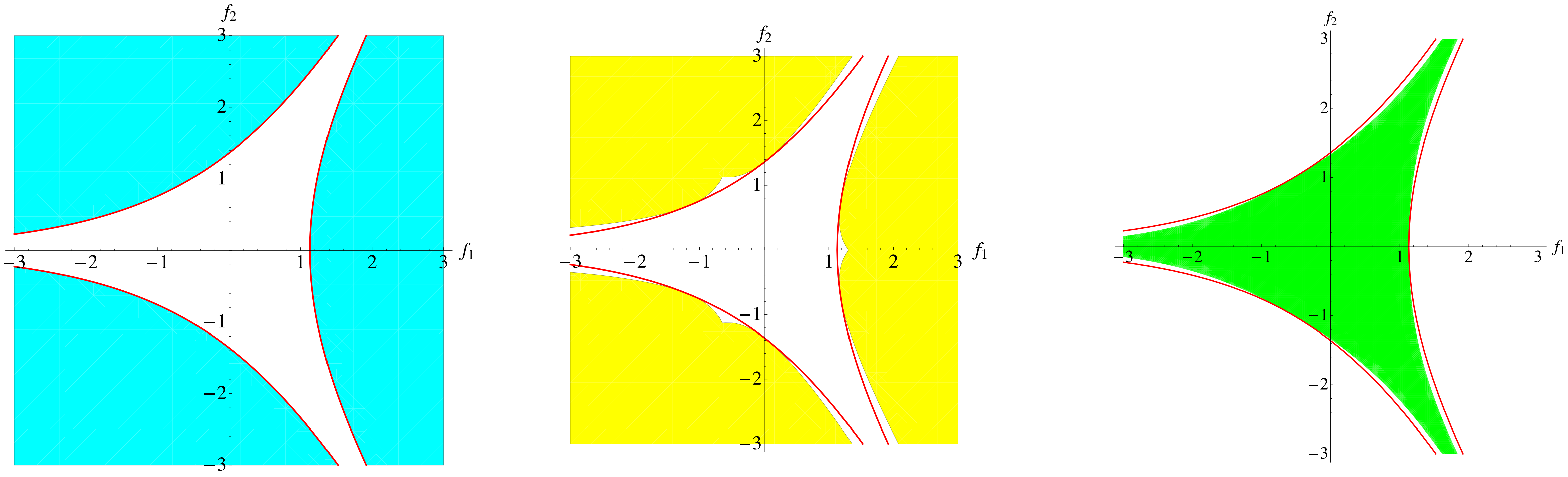
- III. Instabilities involving charged vector fields



- These are associated with p-wave superfluids

- Analogous to W-boson condensation in magnetic field [Ambjorn, Nielsen] and rho-meson condensation in QCD [Chernodub][Ammon et al]

- Summary of stability for SO(6) gauged SUGRA



- These instabilities show that the shaded $AdS_3 \times \mathbb{R}^2$ solutions cannot be the IR ground states of N=4 SYM in a magnetic field

- They also indicate the type of finite temperature instabilities one expects and the modes involved. The fact that there are various instabilities suggests that there will be rich story of competing phases

- Probably many more supersymmetric ground states nearby

Final Comments

- What is the nature of N=4 SYM in a magnetic field?
  The supersymmetric story is solid, but it would be good to understand the central charge better.
  Also there is clearly a very rich set of competing phases that need to be analysed further.

- Similar comments apply to infinite classes of N=2 SCFTs in d=4

- There is a very similar story for solutions of D=4 SO(8) gauged supergravity corresponding to d=3 SCFTs in magnetic fields.
  [Donos,JPG,Pantelidou]
  Stable domain wall solutions interpolating between AdS_4 in the UV and $AdS_2 \times \mathbb{R}^2$ in IR. Ground states with finite entropy.

HAPPY BIRTHDAY PAUL!