Axial couplings in HHχPT at NLO

with Will Detmold and Stefan Meinel

arXiv:1108.4544, Phys.Rev.D84, 094502 (2011)
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Motivation

- LHCb phenomenology, b baryon physics.
- First step: better control of chiral extrapolations in lattice calculations.
- Heavy hadron decay widths.

Outline*

- Introduction: HHχPT in finite volume.
- The axial currents and axial couplings at NLO.
- Impact on (recent) numerical calculations.
- Conclusion.

* For simplicy, I only present SU(2) "full QCD" results in this talk.

Single-HQ hadron states in HHχPT

- Heavy mesons,

$$H_i^{(\bar{b})} = \left(B^*_{i,\mu}\gamma^\mu - B_i\gamma_5\right)\frac{1-\not{v}}{2}.$$

- Heavy baryons with $s_l = 0$ ($s = 1/2$),

$$T = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \Lambda_b \\ -\Lambda_b & 0 \end{pmatrix}.$$

- Heavy baryons with $s_l = 1$,

$$S^\mu_{ij} = \sqrt{\frac{1}{3}}\left(v^\mu + \gamma^\mu\right)\gamma_5\mathcal{B}_{ij} + \mathcal{B}^{*\mu}_{ij} \quad \left(\mathcal{B}_{ij} : s = 1/2,\ \mathcal{B}^*_{ij} : s = 3/2\right)$$

$$\mathcal{B} = \begin{pmatrix} \Sigma_b^{+1} & \frac{1}{\sqrt{2}}\Sigma_b^0 \\ \frac{1}{\sqrt{2}}\Sigma_b^0 & \Sigma_b^{-1} \end{pmatrix},\ \mathcal{B}^* = \begin{pmatrix} \Sigma_b^{+*} & \frac{1}{\sqrt{2}}\Sigma_b^{0*} \\ \frac{1}{\sqrt{2}}\Sigma_b^{0*} & \Sigma_b^{-*} \end{pmatrix}.$$

Symmetries in HHχPT

- Heavy-quark spin, S_h : $H_i^{(\bar{b})} \to H_i^{(\bar{b})} S_h^{-1}$, and similarly for T and S^μ.
- Chiral, $L \times R$.
- Unbroken light-flavour, $U(x)$:

$$H_i^{(\bar{b})}(x) \to U_i{}^j(x) H_j^{(\bar{b})}(x), \; T_{ij} \to U_i{}^k(x) U_j{}^l(x) T_{kl}, \; S_{ij}^\mu \to U_i{}^k(x) U_j{}^l(x) S_{kl}^\mu,$$

$\Rightarrow$ Use the "ξ-basis" for the Goldstone fields.

- $\xi \equiv \exp(i\Phi/f) = \sqrt{\Sigma}$.
- $\xi(x) \to L\; \xi(x)\; U^\dagger(x) = U(x)\; \xi(x)\; R^\dagger$.
- Vector and axial fields transform involving only $U(x)$,

$$V^\mu = \frac{1}{2}\left(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger\right), \; A^\mu = \frac{i}{2}\left(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger\right).$$

- Vector field transforms like "gauge field".

HHχPT Lagrangian

G.Burdman and J.Donoghue; P.Cho; M.B.Wise; T.M.Yan *et al.*; *circa* 1991.

$$\mathcal{L}_{\mathsf{HH}\chi\mathsf{PT}} = \mathcal{L}_{\mathsf{HH}} + \mathcal{L}_{\mathsf{pure-Goldstone}},$$

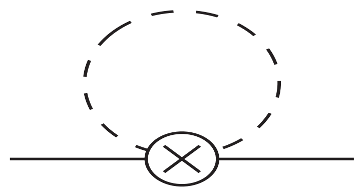
$$\mathcal{L}_{\mathsf{HH}}^{(\mathsf{LO})} = -i\,\mathrm{tr}_{\mathsf{D}}\left[\bar{H}^{(\bar{b})i} v_\mu \mathcal{D}^\mu H_i^{(\bar{b})}\right] + i\left(\bar{T} v_\mu \mathcal{D}^\mu T\right)_{\mathsf{f}} - i\left(\bar{S}^\nu v_\mu \mathcal{D}^\mu S_\nu\right)_{\mathsf{f}} + \Delta^{(B)}\left(\bar{S}^\nu S_\nu\right)_{\mathsf{f}}$$
$$+ g_1\,\mathrm{tr}_{\mathsf{D}}\left[\bar{H}_i^{(\bar{b})}\gamma_\mu\gamma_5 H_j^{(\bar{b})} A^{ij}\right] + i g_2\,\epsilon_{\mu\nu\sigma\rho}\left(\bar{S}^\mu v^\nu A^\sigma S^\rho\right)_{\mathsf{f}} + \sqrt{2}\,g_3\left[\left(\bar{T} A^\mu S_\mu\right)_{\mathsf{f}} + \left(\bar{S}_\mu A^\mu T\right)_{\mathsf{f}}\right].$$

- HH's are (almost) onshell, with fixed velocities.
- Chiral covariant derivatives involve the vector field, V^μ.
- $\Delta^{(B)}$ does not vanish in the chiral and HQ limits.
- Three LEC's in $\mathcal{L}_{\mathsf{HH}}^{(\mathsf{LO})}$, not well determined.
- More mass splittings from higher-order terms in the χ and HQ expansions.

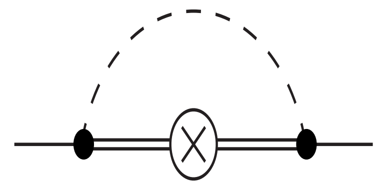
Generic one-loop structure

self-energy

(wavefunction)



Tadpole



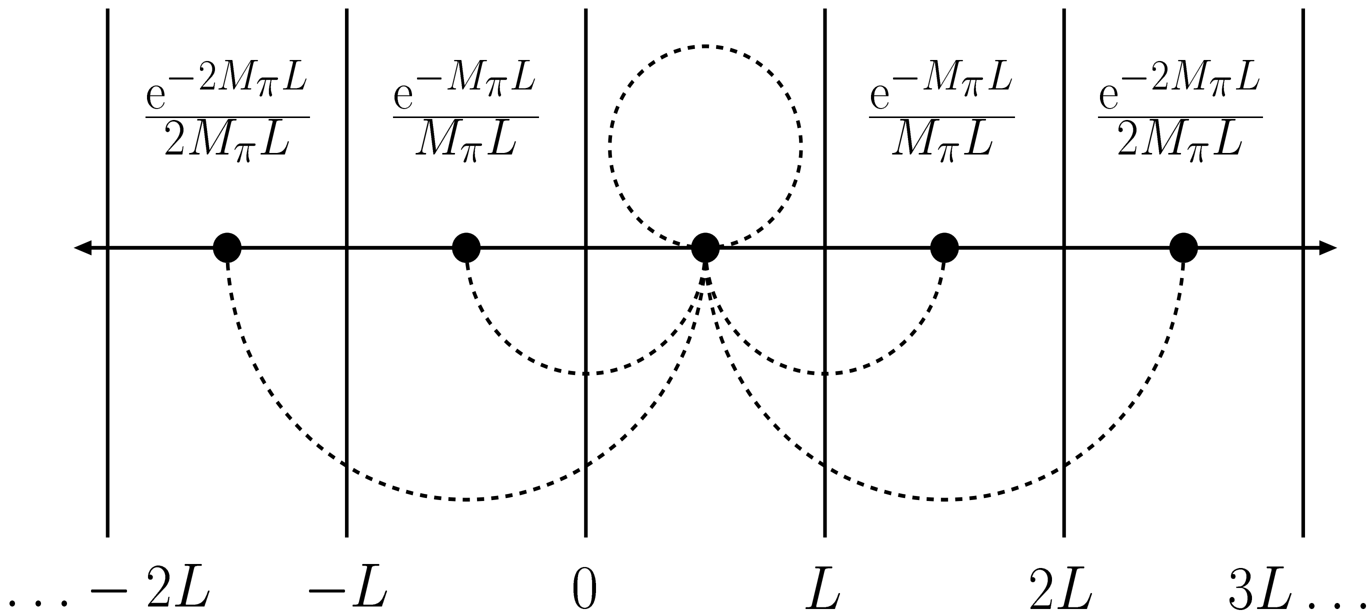
Sunset

- Self-energy and sunset are $O(g^2_{1,2,3})$ higer compared to tadpole.
- Generic NLO formula

$$\mathcal{A} \;=\; \mathcal{A}_{\text{LO}}\left(1 + g^2 L + g'^2 L' + L''\right) + \mathcal{A}_{\text{NLO-analytic}}.$$

One-loop HHχPT in finite volume

D.Arndt and C.-J.D.L., 2004.



- Pions wrapping around the spatial volume.

  $\Rightarrow$ replacing integrals by sums in loops.

- Effects reduced in self-energy and sunset due to virtuality.

Axial currents

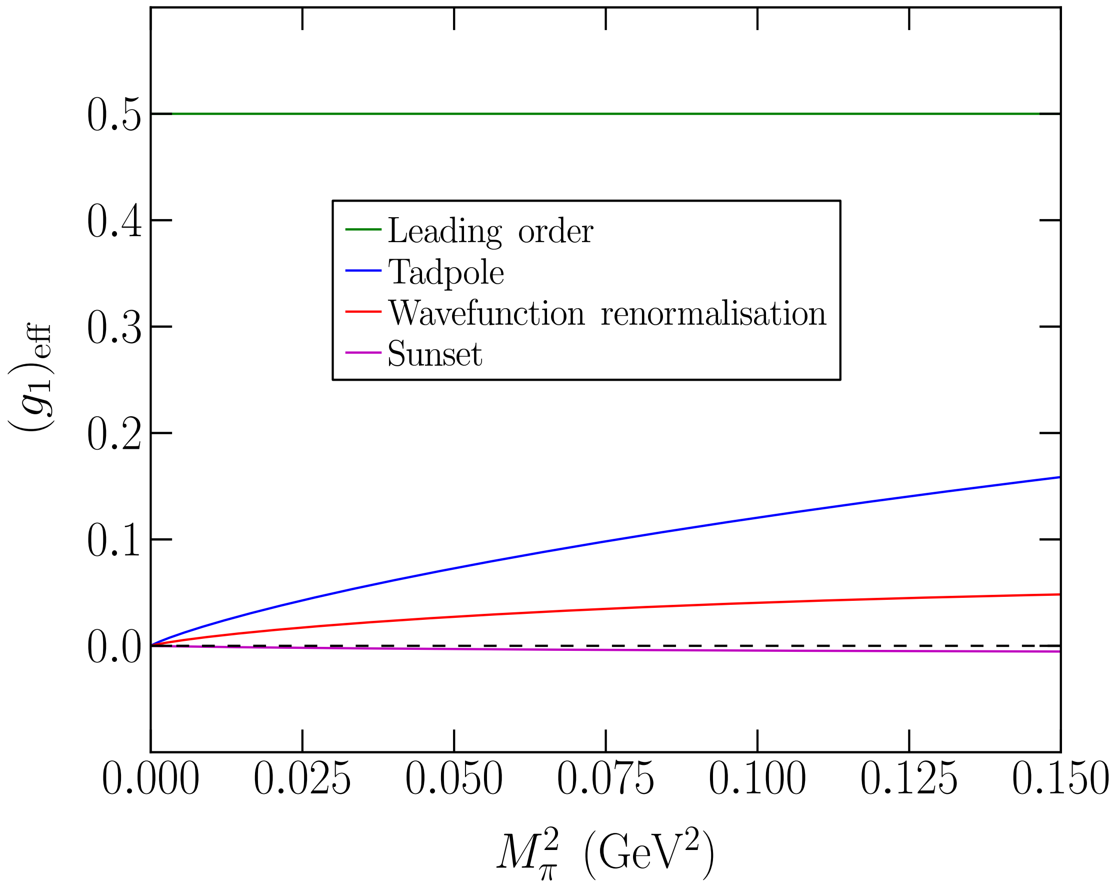
$$J^A_{ij,\mu} = g_1 \,\mathrm{tr_D}\left[\bar{H}_k^{(\bar{b})} H_l^{(\bar{b})} \left(\tau^{(+)}_{ij,\xi}\right)^{kl} \gamma_\mu\gamma_5\right] + i g_2\, \epsilon_{\mu\nu\sigma\rho}\left(\bar{S}^\nu v^\sigma \tau^{(+)}_{ij,\xi} S^\rho\right)_\mathrm{f}$$

$$+\sqrt{2}\, g_3 \left[\left(\bar{S}_\mu \tau^{(+)}_{ij,\xi} T\right)_\mathrm{f} + \left(\bar{T}\tau^{(+)}_{ij,\xi} S_\mu\right)_\mathrm{f}\right] + \text{higher order.}$$

- $\tau^{(+)}_{ij,\xi} = \left(\xi^\dagger \tau_{ij}\xi + \xi\tau_{ij}\xi^\dagger\right)/2$, where $(\tau_{ij})_{kl} = \delta_{il}\delta_{jk}$.
- Obtained using the Noether theorem.
- Matrix elements,

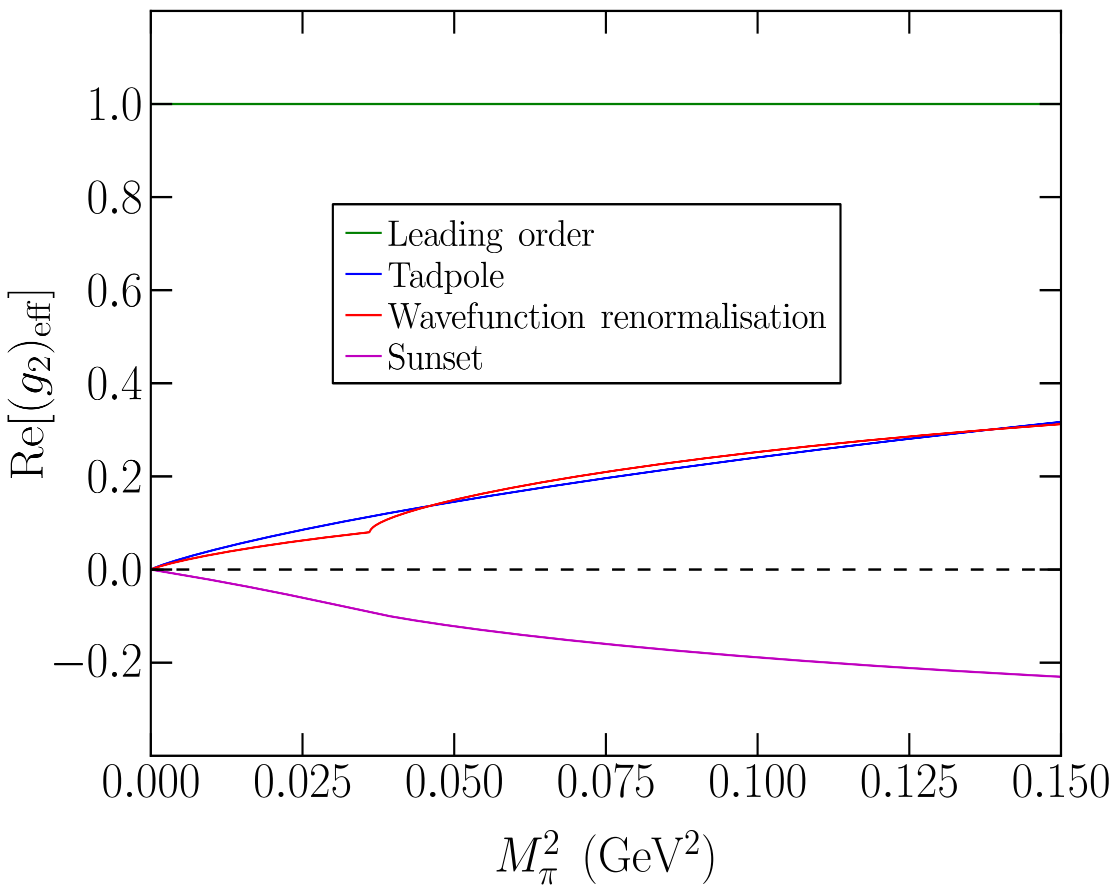
$$\begin{aligned}
\langle B_j^*|J^A_{ij,\mu}|B_i\rangle &= -2\,(g_1)_\mathrm{eff}\,\epsilon^*_\mu, \\
\langle S_{kj}|J^A_{ij,\mu}|S_{ki}\rangle &= -\frac{i}{\sqrt{2}}(g_2)_\mathrm{eff}\, v^\sigma \epsilon_{\sigma\mu\nu\rho}\bar{U}^\nu\ U^\rho, \\
\langle S_{kj}|J^A_{ij,\mu}|T_{ki}\rangle &= -\,(g_3)_\mathrm{eff}\,\bar{U}_\mu\ \mathcal{U}.
\end{aligned}$$

Individual loop contributions: $(g_1)_{\rm eff}$



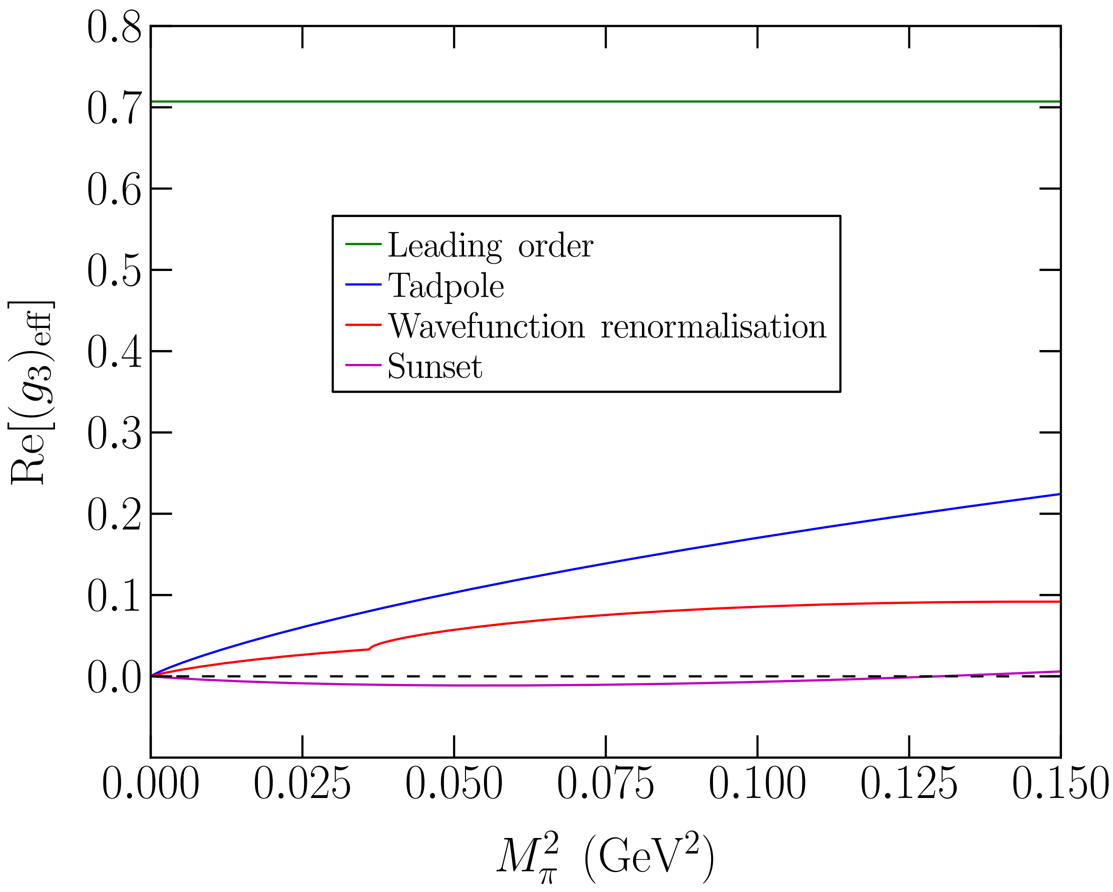
$g_1 = 0.5, \;\; \mu = 4\pi f$

Individual loop contributions: Re$[(g_2)_{\text{eff}}]$



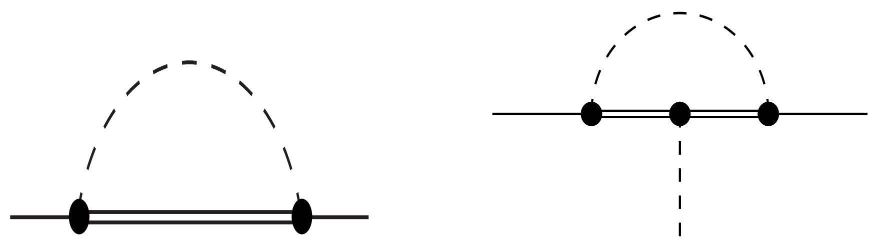
$g_2 = 1, \ , g_3 = \sqrt{2}/2, \ \mu = 4\pi f$

Individual loop contributions: Re[$(g_3)_{\text{eff}}$]



$g_2 = 1, \ , g_3 = \sqrt{2}/2, \ \mu = 4\pi f$

Comparison with $\langle H_1|H_2\pi\rangle$ result



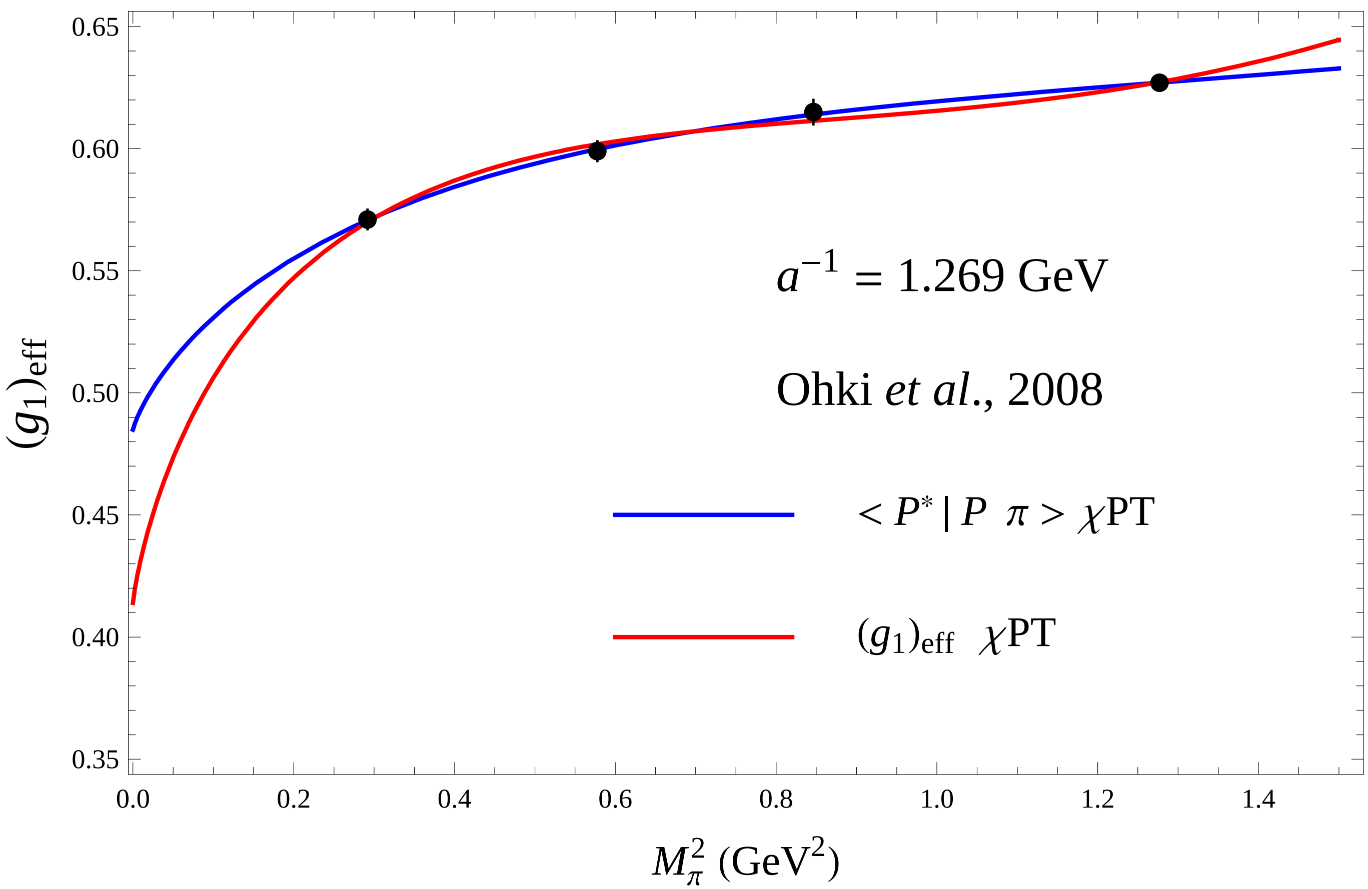
- Tadpole in $\langle H_1|H_2\pi\rangle$ is 1/3 of that in $(g_i)_{\text{eff}}$.

  $\Rightarrow$ It cancels with pion wavefunction renormalisation.

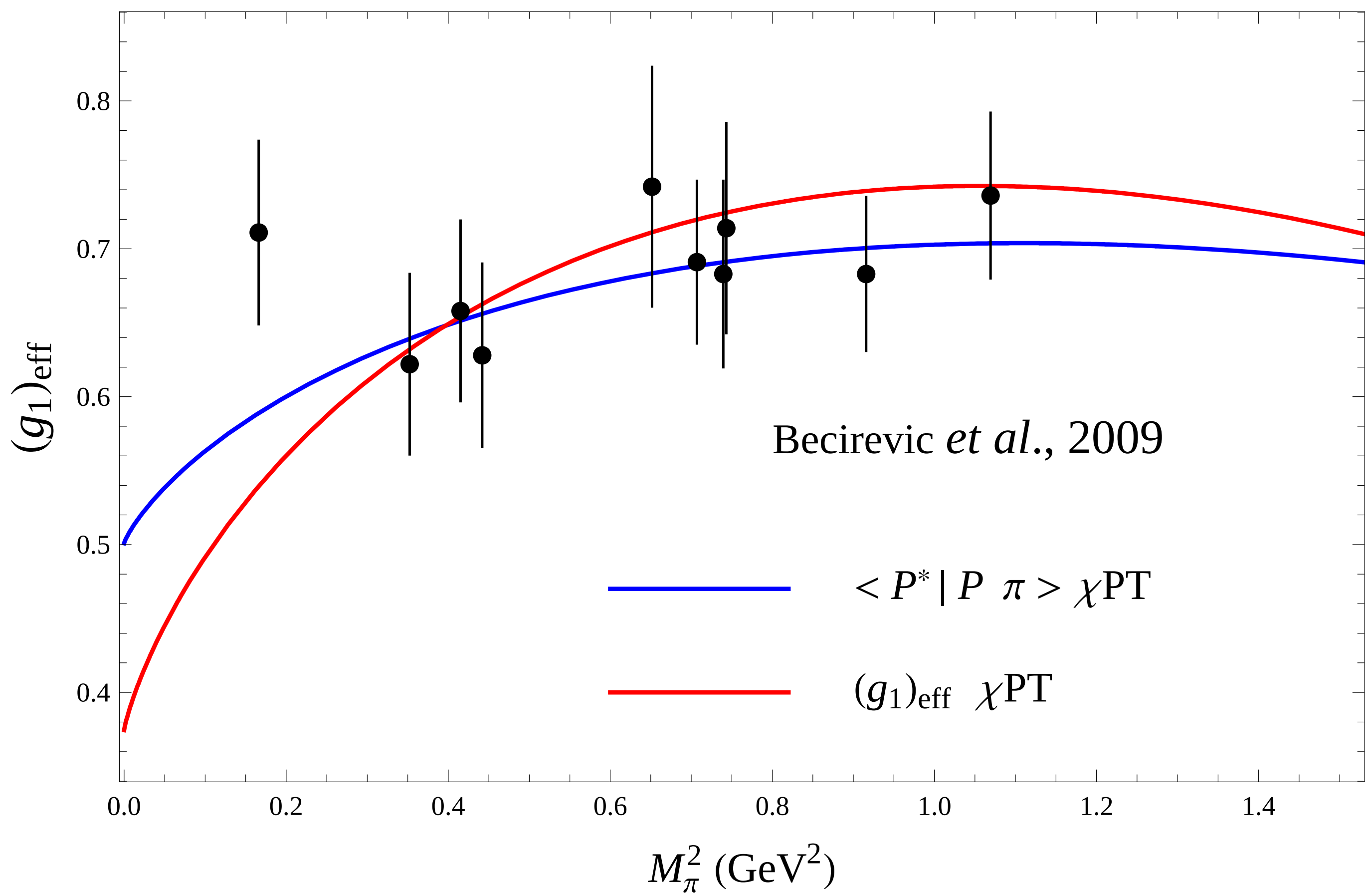
- For $(g_1)_{\text{eff}}$ and $\langle P^*|P\pi\rangle$,

$$
\begin{aligned}
(g_1)_{\text{eff}} &= g_1\left[1 - 2\left(\frac{M_\pi^2}{4\pi f}\right)\log\left(\frac{M_\pi^2}{\mu^2}\right) - 4g_1^2\left(\frac{M_\pi^2}{4\pi f}\right)\log\left(\frac{M_\pi^2}{\mu^2}\right) + c(\mu)M_\pi^2\right], \\
\langle P^*|P\pi\rangle &= g_1\left[1 \qquad\qquad - 4g_1^2\left(\frac{M_\pi^2}{4\pi f}\right)\log\left(\frac{M_\pi^2}{\mu^2}\right) + c'(\mu)M_\pi^2\right].
\end{aligned}
$$

Impact on recent numerical computations



Impact on recent numerical computations



Conclusion and outlook

- Correct χPT formulae for $(g_{1,2,3})_{\text{eff}}$ obtained at NLO.
- Non-negligible effects on the recent, previous determination of g_1.
- Our numerical results are reported in Will Detmold's talk.
- Use these couplings to control chiral extrapolations for heavy-hadron physics.