Hadronic Effects in Exclusive $b \to s\,\ell^+\ell^-$ Transitions

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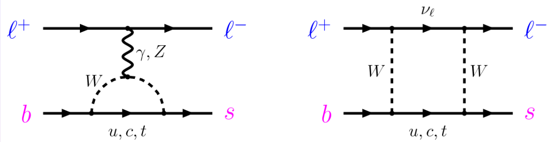
Basics

[see also Alex Khodjamirian's Talk]

- In the SM, $b \to s\,\ell^+\ell^-$ transitions are induced by penguin and box diagrams:



...etc.

- At energies below M_W, described by **Effective Hamiltonian:***

$$H_{\rm eff} \propto -V_{tb}V_{ts}^* \left(C_7(\mu)\, \boxed{\mathcal{O}_7^{(\gamma)}} + C_9(\mu)\, \boxed{\mathcal{O}_9^{(\ell\ell)V}} + C_{10}(\mu)\, \boxed{\mathcal{O}_{10}^{(\ell\ell)A}} \right)$$

$$+ \sum_{i=1}^{2} C_i(\mu) \left(V_{ub}V_{us}^*\, \boxed{\mathcal{O}_i^{(u\bar u)}} + V_{cb}V_{cs}^*\, \boxed{\mathcal{O}_i^{(c\bar c)}} \right) - V_{tb}V_{ts}^* \left(\sum_{i=3}^{6} C_i(\mu)\, \boxed{\mathcal{O}_i^{(q\bar q)}} + C_8(\mu)\, \boxed{\mathcal{O}_8^{(g)}} \right)$$

Complication: Hadronic Operators for $b \to sg$, $b \to sq\bar q$ contribute as well !

* Possible NP effects can be accounted for by different values for Wilson coefficients $C_i(M_W)$ and/or additional operators.

Factorization

$$\langle X_s\, \ell^+\ell^- | H_{\rm eff} | X_b \rangle \;=\; ???$$

- Ignoring QED corrections, matrix elements of $\mathcal{O}_7^{(\gamma)}$ and $\mathcal{O}_{9,10}^{(\ell\ell)}$ *factorize:*

$$\langle X_s\, \ell^+\ell^- | \mathcal{O}_7^{(\gamma)} | X_b \rangle \;\propto\; 2m_b \, \langle \ell^+\ell^- | F_{\mu\nu}^{(\gamma)} | 0 \rangle_{\rm QED} \times \langle X_s | \bar s_L \, \sigma^{\mu\nu}\, b_R | X_b \rangle_{\rm QCD}$$

$$\langle X_s\, \ell^+\ell^- | \mathcal{O}_{9,10}^{(\ell\ell)} | X_b \rangle \;\propto\; \langle \ell^+\ell^- | \bar\ell_R \gamma_\mu \ell_R \pm \bar\ell_L \gamma_\mu \ell_L | 0 \rangle_{\rm QED} \times \langle X_s | \bar s_L \, \gamma^\mu \, b_L | X_b \rangle_{\rm QCD}$$

$\Rightarrow$ Hadronic effects accounted for by $X_b \to X_s$ Transition Form Factors

(for Tensor and $(V-A)$ currents)

- Matrix elements of Hadronic Operators ($b \to sg$, $b \to sq\bar q$) *Do Not Factorize !!!*
  - essential to compensate scale-dependence from $C_i(\mu)$ from factorizable operators (!)
  - part of the effects are universal and can be absorbed via $C_9^{(\ell\ell)}(\mu) \to C_9^{\rm eff}(\mu, q^2)$ $\surd$
  - part of the effects can be reduced to simpler objects in the limit $m_b \gg \Lambda_{\rm QCD}$ $(\surd)$
  - irreducible systematic hadronic uncertainties remain (?)

Calculation of Heavy-to-light Form Factors

- Lattice QCD:
  - in principle: straight-forward
  - in practice: difficult/costly to simulate fast light hadrons on a lattice
  - typically: reliable predictions for *intermediate* momentum transfer

[see A. Kronfeld's talk]

- (conventional) Light-cone Sum Rules:
  - Study Correlation Functions with interpolating current for *heavy* hadron
  - Factorization of hard and soft scales
    $\rightarrow$ Non-perturbative input from Universal LCDAs for light hadrons
  - Dispersion relations $\oplus$ Continuum Model [Ball/Zwicky; Khodjamirian et al., ...]

- **Sum Rules within Soft-Collinear Effective Theory (SCET):**
  - Exchange role of light and heavy hadrons
  - Factorization of hard scales ($\mu^2 \sim M_b^2$) from decay-current matching.
  - Factorization of "hard-collinear" scales ($\mu^2 \sim \Lambda M_b$)
    and soft scales ($\mu^2 \sim \Lambda^2$) in SCET correlation functions.

[De Fazio/TF/Hurth 05; Khodjamirian/Mannel/Offen 05]

- QCD Factorization (BBNS):
  - Soft form factors
    - ★ Non-Factorizable (i.e. irreducible, non-perturbative) ingredients in Factorization Theorems
    - ★ obey Symmetry Relations (see below)
  - Spectator-scattering corrections (at large recoil) [Beneke/TF 2000]
  - non-perturbative power corrections ($\rightarrow$ systematic uncertainties)

Some SCET Jargon (for *b*-decays)

- "hard modes":
  virtual quarks and gluons with virtualities of order m_b^2,
  will be integrated out by matching QCD $\to$ SCET.
- "hard-collinear modes": jet-like configurations,
  with large energies ($\sim m_b/2$), but intermediate virtualities ($\sim \Lambda m_b$).
  can be treated perturbatively in SCET $\to$ "Jet Functions, etc."
- "collinear modes":
  constituents of exclusive configurations with large energies and small virtualities,
  if factorization works $\to$ light-cone distribution amplitudes etc.
- "soft modes":
  constituents of low-energy exclusive configurations,
  $\to$ soft functions (*b*-quark pdf in *B*-meson, *B*-meson distribution amplitudes, etc.)

Problem:

Perturbative separation of soft and collinear modes does not always work !

... bad for pQCD missionaries ... good for non-perturbative QCD freaks ...

Outline

1. Form Factors (main part)
   - Form Factors for $B \to K^{(*)}\ell^+\ell^-$ Transitions
   - Form Factors for $\Lambda_b \to \Lambda\,\ell^+\ell^-$ Transitions
2. Other Hadronic Effects (not form-factor like)
   - $B \to K^{(*)}\mu^+\mu^-$ at small q^2
   - $B \to K^{(*)}\mu^+\mu^-$ at large q^2
3. Summary/Outlook

Based on:

- *Form Factors for $\Lambda_b \to \Lambda$ Transitions in SCET*, TF and M.W.Y. Yip, **PRD 85** (2012) 014035.
- *Theory of $B \to K^{(*)}\ell^+\ell^-$ decays at high q^2 ...*, G. Buchalla, M. Beylich, TF, **EPJC 71** (2011) 1635.
- *Theoretical and Phenomenological Constraints on FFs ...*, A. Bharucha, TF, M. Wick, **JHEP 1009** (2010) 090.

[some overlap with A. Khodjamirian's and Chr. Bobeth's talks, (partly) by intention]

Form Factors (main part)

Form-Factor Definitions

Traditional Conventions for $B \to K$:

$$\langle K(k)|\bar{q}\gamma_\mu b|B(p)\rangle = \left(p_\mu + k_\mu - q_\mu \,\frac{m_B^2 - m_K^2}{q^2}\right) f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2}\, q_\mu\, f_0(q^2)\,,$$

$$\langle K(k)|\bar{q}\sigma_{\mu\nu}q^\nu b|B(p)\rangle = \frac{i}{m_B + m_K}\left(q^2(p+k)_\mu - (m_B^2 - m_K^2)\, q_\mu\right) f_T(q^2,\mu)\,, \qquad (f_0(0) = f_+(0))$$

More Convenient Definition ("Helicity-Based FFs"): [Boyd/Savage 97, Bharucha/TF/Wick 10]

$$\mathcal{A}_{V,\sigma}(q^2) = \sqrt{\frac{q^2}{\lambda}}\, \varepsilon_\sigma^{*\mu}(q)\, \langle K(k)|\bar{q}\,\gamma_\mu\, b|\bar{B}(p)\rangle$$

$$\mathcal{A}_{T,\sigma}(q^2) = (-i)\sqrt{\frac{1}{\lambda}}\, \varepsilon_\sigma^{*\mu}(q)\, \langle K(k)|\bar{q}\,\sigma_{\mu\nu}q^\nu\, b|\bar{B}(p)\rangle$$

- $\varepsilon_\sigma(q)$: transverse, longitudinal, or time-like polarization vectors, $\sigma = \{\pm; 0; t\}$ or $\{1{,}2; 0; t\}$
- normalization: $\lambda = \left((m_B - m_K)^2 - q^2\right)\left((m_B + m_K)^2 - q^2\right) \equiv (t_- - q^2)(t_+ - q^2)$

- Similarly, for $B \to K^*$ form factors ($\mathcal{B}_{V,\sigma}$ for V-A; $\mathcal{B}_{T,\sigma}$ for T)

Advantages of Helicity-Based FFs

- diagonalization of unitarity relations ($\to$ simpler expressions for unitarity bounds)
- definite spin-parity ($\to$ simple implementation of b-resonance contributions)
- simple form of symmetry relations for small/large recoil

HQET Limit:

For $B \to K$:

$$2m_B\sqrt{q^2}\,\mathcal{A}_{T,0} = (m_B^2+q^2)\,\mathcal{A}_{V,0} - (m_B^2-q^2)\,\mathcal{A}_{V,t}$$

For $B \to K^*$:

$$2m_B\sqrt{q^2}\,\mathcal{B}_{T,0} = (m_B^2+q^2)\,\mathcal{B}_{,0} + (m_B^2-q^2)\,\mathcal{B}_{V,t}$$

$$2m_B\sqrt{q^2}\,\mathcal{B}_{T,1} = (m_B^2+q^2)\,\mathcal{B}_{V,1} + (m_B^2-q^2)\,\mathcal{B}_{V,2}$$

$$2m_B\sqrt{q^2}\,\mathcal{B}_{T,2} = (m_B^2+q^2)\,\mathcal{B}_{V,2} + (m_B^2-q^2)\,\mathcal{B}_{V,1}$$

SCET Limit:

For $B \to K$:

$$\mathcal{A}_{V,0} \simeq \mathcal{A}_{V,t} \simeq \frac{m_B}{\sqrt{q^2}}\,\mathcal{A}_{T,0}$$

For $B \to K^*$:

$$\mathcal{B}_{V,0} \simeq \mathcal{B}_{V,t} \simeq \frac{m_B}{\sqrt{q^2}}\,\mathcal{B}_{T,0}$$

$$\mathcal{B}_{V,1} \simeq \mathcal{B}_{V,2} \simeq \frac{m_B}{\sqrt{q^2}}\,\mathcal{B}_{T,1} \simeq \frac{m_B}{\sqrt{q^2}}\,\mathcal{B}_{T,2}$$

- relatively simple expressions for $B \to K^{(*)}\ell^+\ell^-$ observables in factorization approximation

[Bharucha/TF/Wick 10]

Series Expansion for generic form factor $F^{H\to L}(t = q^2)$:

[Boyd, Grinstein, Lebed, Savage, Caprini, Lellouch, Neubert, Becher, Hill, ...] [also: A. Kronfeld's talk]

- Conformal Mapping:

$$z = z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_- - t_0}}{\sqrt{t_+ - t} + \sqrt{t_- - t_0}}\,, \qquad |z| \ll 1$$

with $t_\pm = (m_H \pm m_L)^2$ and $0 \leq t_0 < t_-$.

- (truncated) Series Expansion:

$$F(t) = \text{(pre-factor)(t)} \times \sum_{i=0}^{N} \alpha_i \cdot z^i$$

(pre-factor contains analytic structure from resonances outside the decay region)

- Coefficients α_i constrained by "Dispersive Bounds":

$$\sum_{i=0}^{N} |\alpha_i|^2 \leq 1$$

(from calculation of correlation functions with the corresponding decay currents)

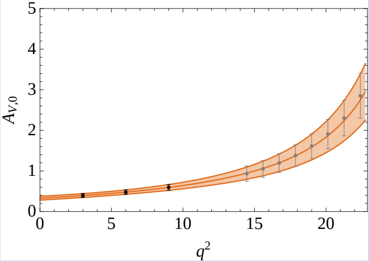
(Heavy-to-light) Form Factor Fits with Series Expansion

[here e.g. from: Bharucha/TF/Wick — Status 2010]

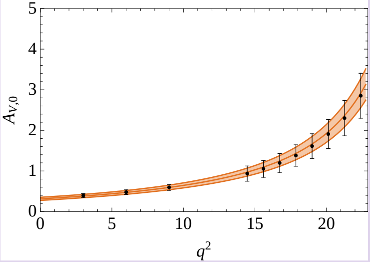
- FF at small momentum transfer $t = q^2$: from LCSR approach [Ball/Zwicky 04]
- FF at large momentum transfer $t = q^2$: Lattice QCD estimates [QCDSF 0903.1664]
- Interpolation: Truncated Series Expansion $(N = 1)$

Example: $B \to K$ transition form factors (vector current: $A_{V,0}(q^2) \equiv f_+(q^2)$)

LCSR only

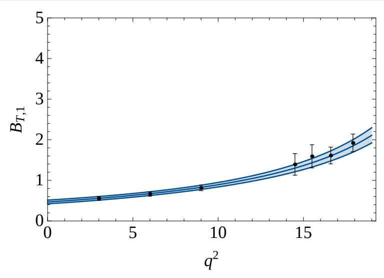


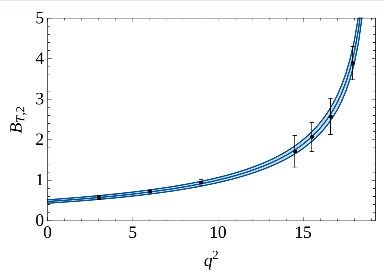
LCSR + Lattice



New Lattice data for $B \to K^*$ Tensor FFs — PRELIMINARY —

[Liu et al, 1101.2726, 0911.2370, see also M. Wingate's talk]





- Combined Fit with LCSR estimates at low q^2
  (issue with LCSR normalization re-adjusted from $B \to K^* \gamma$)
- using simplified series expansion (again, with $N = 1$)

[Aoife Bharucha, private communication]

Form Factors for $\Lambda_b \to \Lambda\, \ell^+ \ell^-$ Transitions

Example: Vector Form Factors

$$\langle \Lambda(p', s')|\bar{q}\,\gamma_\mu\, b|\Lambda_b(p,s)\rangle$$
$$= \bar{u}_\Lambda(p', s') \Big\{ f_0(q^2)\,(M_{\Lambda_b} - m_\Lambda)\,\frac{q_\mu}{q^2}$$
$$+ f_+(q^2)\,\frac{M_{\Lambda_b} + m_\Lambda}{s_+} \left(p_\mu + p'_\mu - \frac{q_\mu}{q^2}\,(M^2_{\Lambda_b} - m^2_\Lambda)\right)$$
$$+ f_\perp(q^2) \left(\gamma_\mu - \frac{2m_\Lambda}{s_+}\,p_\mu - \frac{2M_{\Lambda_b}}{s_+}\,p'_\mu\right) \Big\}\, u_{\Lambda_b}(p,s)$$

(similar for 3 axial FFs: $g_{0,+,\perp}$; 4 tensor FFs: $h_{+,\perp}, \tilde{h}_{+,\perp}$)

- $f_0, f_+, f_\perp$ correspond to time-like, longitudinal, transverse polarization
- For $f_0, f_+, f_\perp \to 1$, one obtains

$$\langle \Lambda(p', s')|\bar{q}\,\gamma_\mu\, b|\Lambda_b(p,s)\rangle$$
$$= \bar{u}_\Lambda(p', s')\,\gamma_\mu\, u_{\Lambda_b}(p,s)$$

$$s_+ = (M_{\Lambda_b} + m_\Lambda)^2 - q^2$$

[TF/Yip 11]

Form Factors for $\Lambda_b \to \Lambda\,\ell^+\ell^-$ Transitions

Example: Vector Form Factors

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\begin{aligned}
&\langle \Lambda(p',s')|\bar q\,\gamma_\mu\, b|\Lambda_b(p,s)\rangle \\
&= \bar u_\Lambda(p',s') \left\{ f_0(q^2)\,(M_{\Lambda_b}-m_\Lambda)\,\frac{q_\mu}{q^2} \right. \\
&\quad + f_+(q^2)\,\frac{M_{\Lambda_b}+m_\Lambda}{s_+}\left(p_\mu + p'_\mu - \frac{q_\mu}{q^2}\,(M^2_{\Lambda_b}-m^2_\Lambda)\right) \\
&\quad \left. + f_\perp(q^2)\left(\gamma_\mu - \frac{2m_\Lambda}{s_+}\,p_\mu - \frac{2M_{\Lambda_b}}{s_+}\,p'_\mu\right)\right\} u_{\Lambda_b}(p,s)
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\end{aligned}
$$

$s_+ = (M_{\Lambda_b}+m_\Lambda)^2 - q^2$ [TF/Yip 11]

HQET limit: Reduction from 10 $\to$ 2 independent form factors

$$\langle \Lambda(p',s')|\bar q\,\Gamma\, b|\Lambda_b(p,s)\rangle \simeq \bar u_\Lambda(p',s')\Big\{A(v\cdot p') + \not{v}\,B(v\cdot p')\Big\}\Gamma\, u_{\Lambda_b}(p,s)$$

which implies

$$f_0(q^2) \simeq A(v\cdot p') + B(v\cdot p')\,, \qquad f_+(q^2) \simeq f_\perp(q^2) \simeq A(v\cdot p') - B(v\cdot p') \qquad \text{etc.}$$

[e.g. Mannel/Recksiegel 97]

Form Factors for $\Lambda_b \to \Lambda\, \ell^+ \ell^-$ Transitions

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$$
\langle \Lambda(p', s') | \bar q\, \gamma_\mu\, b | \Lambda_b(p, s)\rangle = \bar u_\Lambda(p', s')\,\gamma_\mu\, u_{\Lambda_b}(p, s)
$$

$$s_+ = (M_{\Lambda_b} + m_\Lambda)^2 - q^2$$

[TF/Yip 11]

SCET limit: Reduction from 10 $\to$ 1 independent form factor

$$
\langle \Lambda(p', s') | \bar q\, \Gamma\, b | \Lambda_b(p, s)\rangle \simeq \xi_\Lambda(n_+ p')\, \bar u_\Lambda(p', s')\,\Gamma\, u_{\Lambda_b}(p, s)
$$

which implies

$$
f_0(q^2) \simeq f_+(q^2) \simeq f_\perp(q^2) \simeq \xi_\Lambda(n_+ p') \qquad \text{etc.}
$$

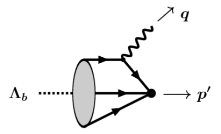
[TF/Yip 11; see also Mannel/Wang 11]

SCET Sum Rules for Baryonic Transitions

[TF/Yip 11]

Correlator:

($n_\pm^\mu$: light-like reference vectors)



- For appropriate currents, correlator takes the form

$$\Pi(n_- p') = f_{\Lambda_b}^{(2)} \frac{\not{n}_-}{2} u_{\Lambda_b} \int_0^\infty d\omega \, \frac{\phi_{\Lambda_b}^{(4)}(\omega;\mu)}{\omega - n_- p' - i\epsilon} + \mathcal{O}(\alpha_s)$$

$\phi_{\Lambda_b}^{(4)}(\omega;\mu)$: LCDA of light scalar diquark component in Λ_b

After Borel transformation ($\to \omega_M$) and continuum subtraction ($\to \omega_s$):

$$e^{-m_\Lambda^2/(n_+p')\omega_M} \, (n_+p') \, f_\Lambda \, \xi_\Lambda(n_+p',\mu) = f_{\Lambda_b}^{(2)} \int_0^{\omega_s} d\omega \, \phi_{\Lambda_b}^{(4)}(\omega,\mu) \, e^{-\omega/\omega_M} + \ldots$$

Theoretical Uncertainties / Hadronic Input:

- Values for light and heavy hadron decay constants.
- Shape of LCDAs for Λ_b.
- Reasonable choice of continuum threshold parameter $\omega_s = s_0/(n_+p')$.
- Reasonable range for Borel parameter $\omega_M = M^2_{\rm Borel}/(n_+p')$.
- Logarithmically enhanced radiative corrections $\sim \alpha_s \ln \omega_{s,M}/\omega$.
  ($\leftrightarrow$ endpoint divergences in QCDF $\leftrightarrow$ non-factorizable dependence on continuum model)

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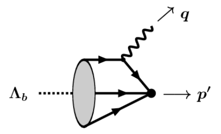

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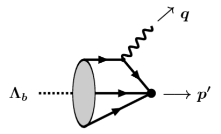

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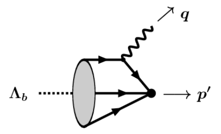

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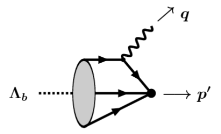

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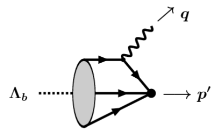
- Shape of LCDAs for Λ_b.
- Reasonable choice of continuum threshold parameter $\omega_s = s_0/(n_+p')$.
- Reasonable range for Borel parameter $\omega_M = M^2_{\rm Borel}/(n_+p')$.
- Logarithmically enhanced radiative corrections $\sim \alpha_s \ln \omega_{s,M}/\omega$.
  ($\leftrightarrow$ endpoint divergences in QCDF $\leftrightarrow$ non-factorizable dependence on continuum model)

SCET Sum Rules for Baryonic Transitions

[TF/Yip 11]

Correlator:

($n_\pm^\mu$: light-like reference vectors)



- For appropriate currents, correlator takes the form

$$\Pi(n_- p') = f^{(2)}_{\Lambda_b} \frac{\not n_-}{2} u_{\Lambda_b} \int_0^\infty d\omega \, \frac{\phi^{(4)}_{\Lambda_b}(\omega;\mu)}{\omega - n_- p' - i\epsilon} + \mathcal{O}(\alpha_s)$$

$\phi^{(4)}_{\Lambda_b}(\omega;\mu)$: LCDA of light scalar diquark component in Λ_b

After Borel transformation $(\to \omega_M)$ and continuum subtraction $(\to \omega_s)$:

$$e^{-m_\Lambda^2/(n_+p')\omega_M} (n_+ p') \, f_\Lambda \, \xi_\Lambda(n_+ p', \mu) = f^{(2)}_{\Lambda_b} \int_0^{\omega_s} d\omega \, \phi^{(4)}_{\Lambda_b}(\omega, \mu) \, e^{-\omega/\omega_M} + \ldots$$

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  ($\leftrightarrow$ endpoint divergences in QCDF $\leftrightarrow$ non-factorizable dependence on continuum model)

Numerical estimates for Baryonic Form Factor ξ_Λ (LO)

Input Parameters (default values):

Threshold $s_0 = \omega_s \, (n_+ p')$	2.55 GeV^2
Borel $M^2_{\rm Borel} = \omega_M \, (n_+ p')$	2.5 GeV^2
Decay constant f_Λ	$6 \cdot 10^{-3}$ GeV^2
Decay constant $f^{(2)}_{\Lambda_b}$	0.030 GeV^3
LCDA par. ω_0	300 MeV

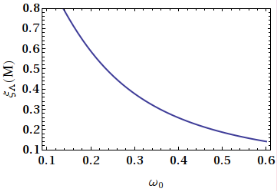
Model for the Λ_b LCDA (for illustration):

$$\phi^{(4)}_{\Lambda_b}(\omega) := \frac{\omega}{\omega_0^2} \, e^{-\omega/\omega_0}$$

Central Value (LO, max. recoil)

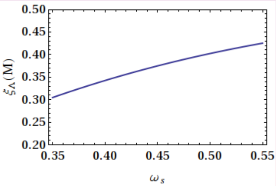
$$\xi_\Lambda(n_+ p' = M_{\Lambda_b}) \approx 0.38$$

ω_0 dependence



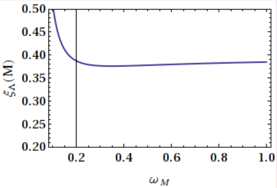
(very sensitive)

ω_s dependence



(moderate)

ω_M dependence



(weak)

[TF/Yip 11]

Application: FBA-Zero in $\Lambda_b \to \Lambda\,\mu^+\mu^-$ [TF/Yip 11]

Zero of Forward-Backward Asymmetry, for values of q^2 satisfying

$$\mathrm{Re}\left[C_9^{\rm eff}(q^2)\right] + \frac{M_b\,(M_{\Lambda_b}+m_\Lambda)\,C_7^{\rm eff}}{q^2}\,\frac{h_\perp}{f_\perp} + \frac{M_b\,(M_{\Lambda_b}-m_\Lambda)\,C_7^{\rm eff}}{q^2}\,\frac{\tilde h_\perp}{g_\perp}\Big|_{q^2=q_0^2} \stackrel{!}{=} 0$$

- Symmetry Limit: Recover LO result from inclusive/mesonic case $\surd$
- Perturbative spectators corrections additionally suppressed by m_Λ/M_{Λ_b} (!)
- Corrections from hard-vertex corrections (model-independent)

$$q_0^2 \simeq \begin{cases} 3.6\ \mathrm{GeV}^2 & \text{(symmetry limit)}, \\ 3.4\ \mathrm{GeV}^2 & \text{(incl. corrections)}. \end{cases}$$

- Power Corrections to form-factor relations ($\to$ SR/Lattice)
- Non-Factorizable Corrections from long-distance photons ($\to$ future work)

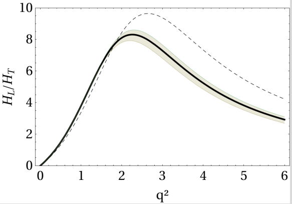
Differential Decay Rates at Large Recoil Energy

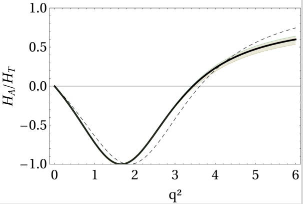
- Double differential rate decomposed into transverse, longitudinal and FB-asymmetric term

$$\frac{d^2\Gamma(\Lambda_b \to \Lambda \ell^+ \ell^-)}{dq^2\, d\cos\theta} \equiv \frac{3}{8}\left\{(1+\cos^2\theta)\, H_T(q^2) + 2(1-\cos^2\theta)\, H_L(q^2) + 2\cos\theta\, H_A(q^2)\right\}$$

- Ratios take relatively simple form in factorization + symmetry limit.

Ratios of observables H_L/H_T and H_A/H_T





- - - dashed line: symmetry limit

— solid line/grey error band: default estimates for radiative corrections to FF ratios [TF/Yip 11]

Summary Part I: Form Factors for $X_b \to X_s \ell^+\ell^-$ Transitions

Small Recoil Energy $E_{K^{(*)}}$ – Large Momentum Transfer $q^2 = m_{\ell\ell}^2$

- Form Factors from ChPT + HQET (via resonance contributions, e.g. $B \to B^* K^{(*)}$)
- HQET Symmetry Relations in the limit $m_b \to \infty$

Intermediate Energies and Momentum Transfer

- Form Factor Calculations from the Lattice

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Other Hadronic Effects (not form-factor like)

General Discussion of $B \to K^{(*)}\ell\ell$ [$\to$ **Alex Khodjamirian's Talk**]

- Hadronic amplitude A_{10}^μ, multiplying the lepton <u>axial-vector</u> current, entirely from local operator $\mathcal{O}_{10}$ in $H_{\rm eff}$ $\longrightarrow C_{10} \times$ (form factor)
- Hadronic amplitude A_9^μ, multiplying the lepton <u>vector</u> current,

$$A_9^\mu = C_9 \, \langle \bar K^{(*)} | \bar s \gamma^\mu (1-\gamma_5) b | \bar B \rangle \qquad \longrightarrow C_9 \times \text{(form factor)}$$

$$+ \, C_7 \, \frac{2 i m_b \, q_\lambda}{q^2} \, \langle \bar K^{(*)} | \bar s \sigma^{\lambda\mu} (1+\gamma_5) b | \bar B \rangle \qquad \longrightarrow C_7 \times \text{(form factor)}$$

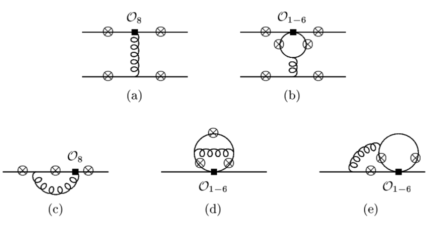
$$+ \, \langle \bar K^{(*)} | \, \mathcal{K}_H^\mu(q) \, | \bar B \rangle \qquad \longrightarrow \begin{cases} q^2 \ll 4 m_c^2 & : \quad \text{QCD factorization} \\ q^2 \gg 4 m_c^2 & : \quad \text{OPE} \end{cases}$$

  - $\mathcal{K}_H^\mu(q)$ from time-ordered product (non-local) between electromagnetic current and hadronic part of $H_{\rm eff}(b \to s)$.
  - "Non-factorizable contributions": require further non-perturbative hadronic input !

Potential Issue:

Violation of Parton-Hadron Duality,
in particular from $B \to V(\to \mu^+\mu^-) K^*$, with $V = J/\psi, \psi', \ldots$

$B \to K^*\mu^+\mu^-$ at small q^2



QCD Factorization at large recoil, $E_{K^*} \gg \Lambda_{\rm QCD}$:

[Beneke/TF/Seidel 01,04]

$$\langle K^*\mu^+\mu^-|\mathcal{O}_i|\bar{B}\rangle\Big|_{\rm hard} = \xi_{\perp,\parallel}^{(B\to K^*)}(q^2)\left(1+\frac{\alpha_s}{4\pi}\,t_i^{\perp,\parallel}(q^2,\mu)+\ldots\right),$$

$$\langle K^*\mu^+\mu^-|\mathcal{O}_i|\bar{B}\rangle\Big|_{\rm spect.} = f_{K^*}^{\perp,\parallel} f_B \int du\, d\omega \left(\frac{\alpha_s}{4\pi}\,h_i^{\perp,\parallel}(u,\omega,q^2,\mu)+\ldots\right)\phi_{K^*}^{\perp,\parallel}(u,\mu)\,\phi_B(\omega,\mu)$$

in terms of universal form factors, decay constants, LCDAs.

Potential Duality Violation

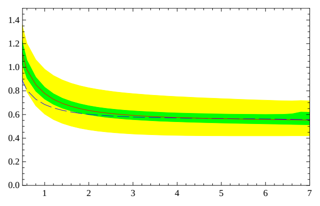
[$\to$ more in talk by Alex Khodjamirian]

$\leftrightarrow$ Enhanced perturbative μ and m_c dependence near partonic threshold, $q^2 \lesssim 4m_c^4$

Example: $B \to K^{*(0)}\mu^+\mu^-$ at large recoil

[following Beneke/TF/Seidel 04]

$d\mathrm{Br}/dq^2$



- theory predictions
  (using input values as in [TF @ CKM'08])
- dashed line: LO approximation
  (incl. some tree-level $1/m_b$ corrections)
- yellow band: "all" (i.e. parametric) uncertainties,
  but $A_0(q^2 = 4\,\mathrm{GeV}^2)$ factored out.
- green band:
  w/o error on form factors and CKM input.

$\dfrac{\frac{d\Gamma^F - d\Gamma^B}{dq^2}}{\frac{d\Gamma^F + d\Gamma^B}{dq^2}}$

- tensor form factor adjusted to $B \to K^*\gamma$
- dashed line: LO approximation
  (incl. some tree-level $1/m_b$ corrections)
- green band: all parametric uncertainties.
  (form factor dependence decreases around q_0^2)

FBA zero q_0^2: $4.36^{+0.33}_{-0.31}$ GeV^2
(slightly lower for K^{*+} mode)

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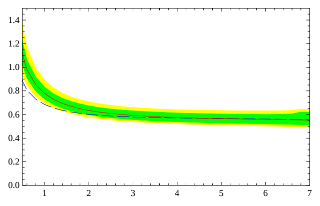

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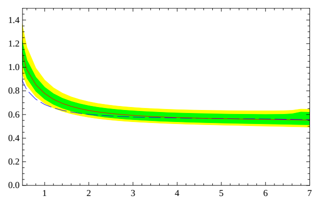

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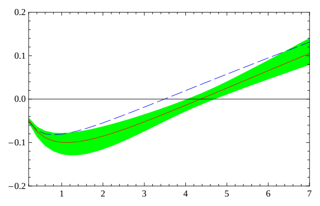

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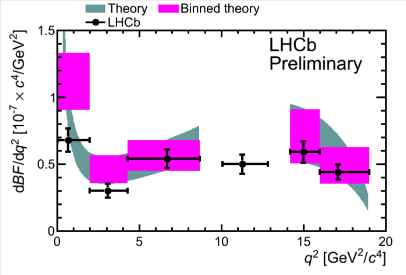
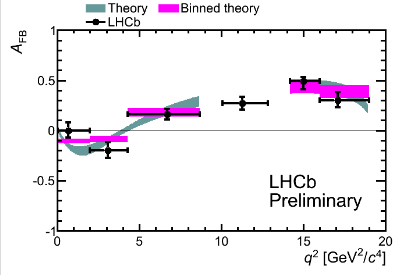
State of the Art: Recent LHCb measurements [LHCb-CONF-2012-008, tomorrow's talk]

Differential Rate and FB Asymmetry in $B \to K^* \mu^+ \mu^-$





FBA zero: $q_0^2 = (4.9^{+1.1}_{-1.3})\ \text{GeV}^2$ (in line with SM expectation)

[updated theory estimates from Bobeth/Hiller/van Dyk 11, see references therein]

Excursion: Duality violation in $e^+e^- \to$ hadrons

Duality Violation:

- Exponentially suppressed terms in the Euclidean region.
- Do not appear in any order of the OPE.
- Correspond to oscillating terms in the physical spetral function.

Example: Shifman Model for Vector Correlator

- Asymptotic result for vector-current correlator from OPE in QCD:

$$\Delta(q^2) = 2\pi^2 \left(\Pi(q^2) - \Pi(0)\right) = -\frac{N_c}{6} \ln \frac{-q^2 - i\epsilon}{\lambda^2} \qquad \text{(large } q^2\text{)}$$

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- Can be reproduced by a function that re-sums an infinite tower of vector resonances:

$$\Delta(q^2) = -\frac{N_c}{6(1-b/\pi)}\left[\psi(z+1)+\gamma_E\right], \qquad z = (-q^2/\lambda^2 - i\epsilon)^{1-b/\pi}$$

  - λ : mass^2-distance; $b = \Gamma_n/M_n$ width-to-mass ratio of resonances
  - correct analytic behaviour (cut for positive q^2)

- For $2\pi b q^2 \gg \lambda^2$ and $\frac{b}{\pi}\ln\frac{q^2}{\lambda^2} \ll 1$:

$$\mathrm{Re}\Delta(q^2)\Big|_{\rm dual.\ viol.} \approx -\frac{N_c\pi}{3}\exp(-2\pi b\frac{q^2}{\lambda^2})\sin(2\pi\frac{q^2}{\lambda^2})$$

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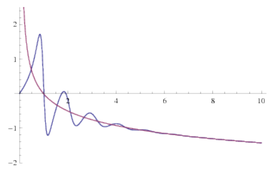


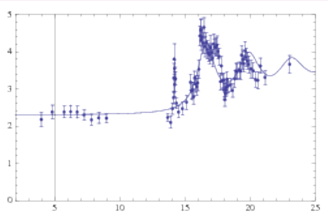
Modelling duality violation from charm-loop in $e^+e^- \to$ hadrons

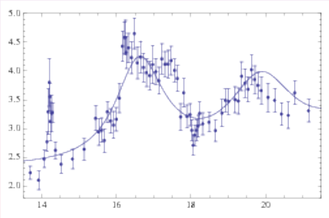
- Assume trajectory of charmonium resonances: $M_n^2 = n\lambda^2 + M_0^2$.
  (narrow resonances to be considered separately)

$\to$ Ansatz for R-ratio in the $c\bar{c}$-region:

$$R = R_{\rm light} - \frac{4}{3} \frac{1}{(1-b/\pi)\,\pi} \, \mathrm{Im}\,\psi(3+z)\,, \qquad z = \left(-\frac{q^2 - 4m_c^2 + i\epsilon}{\lambda^2}\right)^{1-b/\pi}$$

- (crude) Fit to BES data :
  - $R_{\rm light} = 2.31$, from below charm threshold.
  - $m_c = 1.33$ GeV.
  - $\lambda^2 = 3.08$ GeV^2, from average distance of (broad) resonances.
  - $b = 0.082$, from average width of (broad) resonances.





- Use same parameters to describe charm-contribution to $\langle \mathcal{K}_H^\mu \rangle$
  (assuming pessimistic scenario – all resonances (including non-factorizable ones!) contribute coherently)

[Beylich/Buchalla/TF 11]

Duality Violation in $B \to K^{(*)}\mu^+\mu^-$ at high-q^2

[Beylich/Buchalla/TF 11] [see also Buchalla/Isidori 98, Grinstein/Pirjol 04, Khodjamirian et al. 10]

OPE for high-q^2 region: (above $c\bar{c}$ resonances)

- leading term in OPE from dim-3 operators, ($\to$ standard form factors)
  α_s corrections to $\langle \mathcal{K}_H^\mu \rangle_{\rm dim-3}$ known (and important) [...Seidel 04, Greub/Pilipp/Schüpbach 08]
- contributions from dim-4 operators suppressed $\alpha_s \frac{m_s}{m_b} \sim 0.5\%$
- contributions from dim-5 operators $\langle \bar{s} G^{\mu\nu} b \rangle$ estimated $< 1\%$
- dim-6 operators include weak annihilation effects, negligible at high-q^2 $\mathcal{O}(0.1\%)$

Duality-violating effects at high-q^2:

- Estimated on the basis of a model for an inifinite series of charm resonances, fitted to experimental R-ratio [Shifman 2000]
- Uncertainty on partially integrated decay rate ($q^2 \geq 15$ GeV^2) $\pm 2\%$

Duality violation for differential rate (point-by-point) remains model-dependent.

High-q^2 region of $B \to K^{(*)}\mu^+\mu^-$ under (very) good theoretical control

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  $\rightarrow$ factorizable form-factor contributions dominate
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  (non-factorizable charm-loops deserve further investigations)

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- $c\bar{c}$ threshold and sharp J/ψ and ψ' resonances
  $\rightarrow$ no theoretical control on the region $4m_c^2 \lesssim q^2 \lesssim M_{\psi'}^2$.

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Summary Part II: Non-Factorizable Effects

Small Recoil Energy $E_{K^{(*)}}$ – Large Momentum Transfer $q^2 = m_{\ell\ell}^2$

- OPE converges reasonably well
  $\rightarrow$ factorizable form-factor contributions dominate
- Duality-violating effects estimated to be small
  (non-factorizable charm-loops deserve further investigations)

Intermediate Energies and Momentum Transfer

- $c\bar{c}$ threshold and sharp J/ψ and ψ' resonances
  $\rightarrow$ no theoretical control on the region $4m_c^2 \lesssim q^2 \lesssim M_{\psi'}^2$.

Large Recoil Energy – Small Momentum Transfer

- Part of non-factorizable effects reducible in the large-energy limit
  (symmetry form factors, LCDAs, perturbative coefficient functions)
- Parametric uncertainties (scale-dependencies, hadronic input) still sizeable.
- Reliable estimate of power corrections remains an open issue ($\rightarrow$ sum rules, lattice?)

Summary/Outlook

Outlook

- Improved estimates of Hadronic Input
  - Transition Form Factors
  - Light-cone Distribution Amplitudes (2-particle, 3-particle, ...)
  - Decay Constants and Hadronic Coupling Constants
  - Matrix Elements of higher-dimensional Operators in OPE
- Higher-order computations in QCDF / SCET
- Higher-order computations from Sum Rules ($\rightarrow$ Alex Khodjamirian's talk)
- Improved Lattice-QCD estimates (e.g. directly for helicity-FF ratios).
- Inclusion of hadronic parameters in Global SM Fits. [see Chr. Bobeth's talk]

- Better theoretical control on Non-Factorizable Effects
- Better theoretical control on Duality-violating Effects

Needs High Precision to test SM / look for NP !

Backup Slides

Relation between Helicity-Based FFs and Traditional Convention

$B \to P$:

$$\mathcal{A}_{V,0}(q^2) = f_+(q^2)\,, \qquad \mathcal{A}_{V,t}(q^2) = \frac{m_B^2 - m_P^2}{\sqrt{\lambda}}\, f_0(q^2)\,, \qquad \mathcal{A}_{T,0}(q^2) = \frac{\sqrt{q^2}}{m_B + m_P}\, f_T(q^2)$$

$B \to V$:

$$\mathcal{B}_{V,0}(q^2) = \frac{(m_B + m_V)^2\,(m_B^2 - m_V^2 - q^2)\,A_1(q^2) - \lambda\, A_2(q^2)}{2m_V\sqrt{\lambda}\,(m_B + m_V)}\,, \qquad \mathcal{B}_{V,t}(q^2) = A_0(q^2)\,,$$

$$\mathcal{B}_{V,1}(q^2) \equiv -\frac{\mathcal{B}_{V,-} - \mathcal{B}_{V,+}}{\sqrt{2}} = \frac{\sqrt{2\,q^2}}{m_B + m_V}\, V(q^2)\,,$$

$$\mathcal{B}_{V,2}(q^2) \equiv -\frac{\mathcal{B}_{V,-} + \mathcal{B}_{V,+}}{\sqrt{2}} = \frac{\sqrt{2\,q^2}\,(m_B + m_V)}{\sqrt{\lambda}}\, A_1(q^2)$$

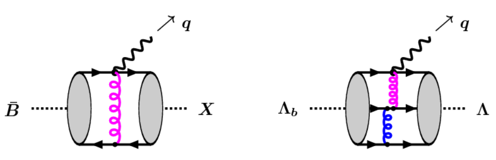
and

$$\mathcal{B}_{T,0}(q^2) = \frac{\sqrt{q^2}\,(m_B^2 + 3m_V^2 - q^2)}{2m_V\sqrt{\lambda}}\, T_2(q^2) - \frac{\sqrt{q^2\,\lambda}}{2m_V\,(m_B^2 - m_V^2)}\, T_3(q^2)\,,$$

$$\mathcal{B}_{T,1}(q^2) = \sqrt{2}\, T_1(q^2)\,, \qquad \mathcal{B}_{T,2}(q^2) = \frac{\sqrt{2}\,(m_B^2 - m_V^2)}{\sqrt{\lambda}}\, T_2(q^2)$$

Calculation of perturbative Corrections $\Delta\xi_\Lambda$

QCD Factorization: Convolution of LCDAs and spectator-scattering kernels



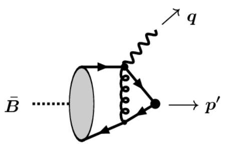
(improved version of Efremov-Radyushkin-Brodsky-Lepage picture)

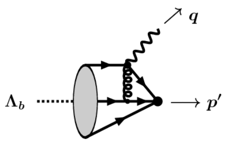
- Factorization for $B \to X = \pi, \rho \ldots$ in the large recoil limit [Beneke/TF 2000]
- For $\Lambda_b \to \Lambda$ form factors, factorization spoiled by soft-gluon exchange between spectator quarks

...

SCET Sum Rules: Correlators involving "factorizable" part of SCET currents

$$\bar{q}\,\Gamma_i\, Q_v \longrightarrow C_{ij}\,\bar{\xi}\,\tilde{\Gamma}_j\, h_v \overbrace{-\frac{1}{n_+p'}\,\bar{\xi}\, g\not{A}_\perp \frac{\not{n}_+}{2}\,\Gamma_i\, h_v - \frac{1}{M_b}\,\bar{\xi}\,\Gamma_i\,\frac{\not{n}_-}{2}\, g\not{A}_\perp\, h_v} + \ldots$$





- Reproduces QCDF for $B \to X$ in the large recoil limit [De Fazio/TF/Hurth 05]
- Predictions for violation of symmetry relations through $\Delta\xi_\Lambda \sim \mathcal{O}(\alpha_s)$

[TF/Yip 11]

Features of $\Delta\xi_\Lambda$

In the limit $\omega_{s,M} \ll \omega_0$, the sum rule takes the form

$$e^{-m_\Lambda^2/(n_+p')\omega_M}\, f_\Lambda\, m_\Lambda\, \Delta\xi_\Lambda$$
$$\simeq \frac{\alpha_s C_F}{2\pi}\, \underbrace{\frac{f_{\Lambda_b}^{(2)}}{M_{\Lambda_b}} \int_0^\infty \frac{d\omega}{\omega}\, F(\omega,1)}\, \times\, \underbrace{\omega_M \left(\omega_M - e^{-\omega_s/\omega_M}(\omega_M+\omega_s)\right)}$$

- Formally, $\Delta\xi_\Lambda$ and ξ_Λ scale with the same power of M_b.
- Dependence on Λ_b-Properties and Sum-Rule Parameters factorizes !
- Proportional to inverse moment of the effective LCDA

$$F(\omega,1) \equiv \int_0^\omega d\eta\, \frac{\eta}{\omega} \left(\tilde\psi_4(\eta,u=1) - \tilde\psi_2(\eta,u=1)\right)$$

- Dependence on decay constants cancels in ratio $\Delta\xi_\Lambda/\xi_\Lambda$.
- Contributions to individual form factors take the form

$$f_{0,\perp}(q^2) \simeq C_{f_{0,\perp}}\, \xi_\Lambda(n_+p') \mp \frac{2M}{n_+p'}\, \Delta\xi_\Lambda(n_+p')$$
$$f_+(q^2) \simeq C_{f_+}\, \xi_\Lambda(n_+p') - 2\left(2 - \frac{M}{n_+p'}\right) \Delta\xi_\Lambda(n_+p') \qquad \text{etc.}$$

[TF/Yip 11]