B-meson physics with dynamical domain-wall light quarks and nonperturbatively tuned relativisitc b-quarks

Oliver Witzel
for the RBC and UKQCD collaborations

Boston University, Center for Computational Science

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Outline

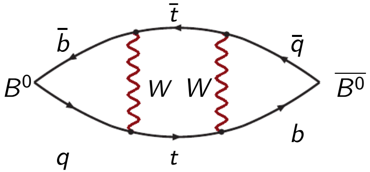
- Introduction
- Actions
- Nonperturbative tuning
- Lattice perturbation theory
- Results for bottomonium
- B-physics
- Conclusions

Phenomenological Importance

- $B-\bar{B}$-mixing allows us to determine CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

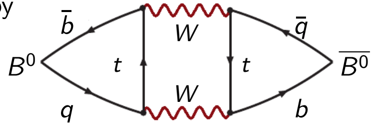
$$\left.\begin{array}{r}|V_{td}^* V_{tb}|\,\text{for}\, B_d\text{−mixing}\\ |V_{ts}^* V_{tb}|\,\text{for}\, B_s\text{−mixing}\end{array}\right\} \Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

- Non-perturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$



- CKM matrix elements are extracted by

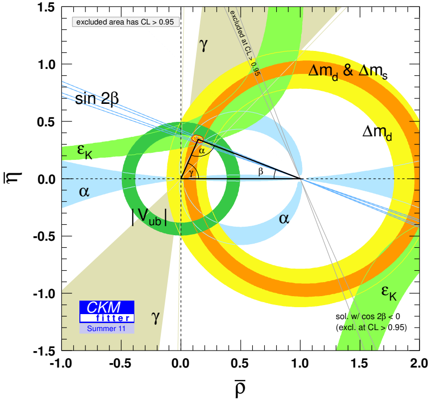
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



- Experimental error of Δm_q is better than a percent; lattice uncertainty for ξ is about 3%

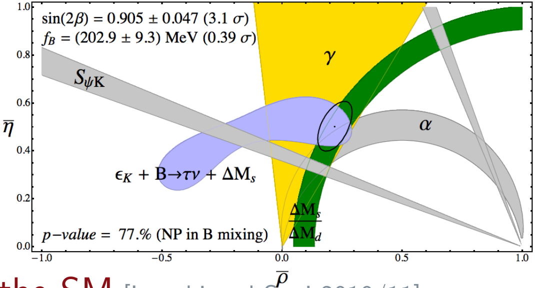
Constraining the CKM Unitarity Triangle

- The apex of the unitarity triangle is constrained by the ratio of B_s to B_d oscillation frequencies (Δm_q)
- Δm_q is experimentally measured to better than a percent [BABAR, Belle, CDF]
- Dominant error comes from the uncertainty on the lattice QCD calculation of the ratio ξ ($\sim 3\%$)
- A precise determination is needed to help constrain physics beyond the Standard Model



Unitarity Fit without Semileptonic Decays [Lunghi and Soni 2009]

- Avoids 2-3 σ tension between inclusive and exclusive determinations of both V_{ub} and V_{cb}
- Requires precise determination of f_B (and also of BR($B \to \tau\nu$) and ΔM_s)



Possible Deviations from the SM [Lunghi and Soni 2010/11]

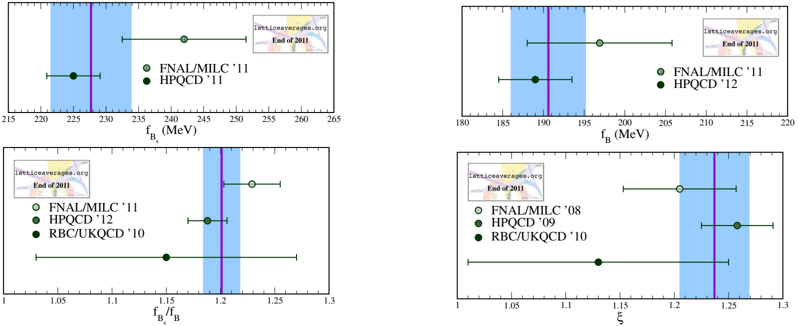
- Experimental value for sin(2β) is 3.3σ lower than SM expectation
- Measured value for BR($B \to \pi l \nu$) is 2.8σ lower than predicted
- Most likely source of deviation in $B_{d(s)}$ mixing and sin(2β);less likely in $B \to \tau\nu$

Latest Results (End of 2011) [http://www.latticeaverages.org]

- New physics in $B \to \tau\nu$ decay prefered less so in in B-mixing

See also: http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it

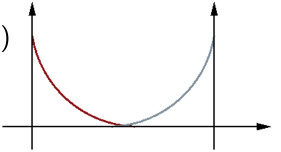
2+1 Flavor Lattice Calculations of f_{B_s}, f_B, f_{B_s}/f_B, ξ



- HPQCD and FNAL-MILC result both based on the asqtad-improved staggered ensembles generated by MILC
- RBC/UKQCD result only exploratory study computed on 16^3 lattices and using static approximation for the b-quarks
- This project aims for an independent cross-check at high precision using domain-wall light-quarks and relativistic heavy quarks

2+1 Flavor Domain-Wall Gauge Field Configurations

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992, Shamir 1993]
- Iwasaki gauge action [Iwasaki 1983]



L	a(fm)	m_l	m_s	m_π(MeV)	approx. # configs.	# time sources
24	≈ 0.11	0.005	0.040	331	1636	1
24	≈ 0.11	0.010	0.040	419	1419	1
32	≈ 0.08	0.004	0.030	307	628	2
32	≈ 0.08	0.006	0.030	366	889	2
32	≈ 0.08	0.008	0.030	418	544	2

[C. Allton et al. 2008, Y. Aoki et al. 2010]

Relativistic Heavy Quark Action for the b-Quarks

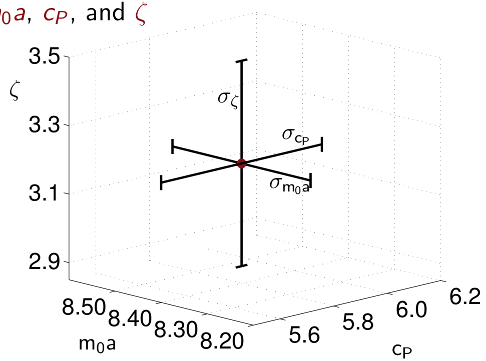
- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the b-quarks in 2-point and 3-point correlation functions [Christ, Li, Lin 2007; Lin and Christ 2007]
- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie 1997] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation [Aoki, Kuramashi, Tominaga 2003]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

Tuning the Parameters for the RHQ Action

$$S = \sum_{n,n'} \bar{\Psi}_n \left\{ m_0 + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a \left(\vec{D} \right)^2}{2} \right] - a \sum_{\mu\nu} \frac{i c_P}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \Psi_{n'}$$

- Start from an educated guess for $m_0 a$, c_P, and ζ

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix} \pm \begin{bmatrix} \sigma_{m_0 a} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sigma_{c_P} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \sigma_\zeta \end{bmatrix}$$



- Compute for all seven parameter sets

spin-averaged mass	$\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$	$\rightarrow$	5403.1(1.1) MeV
hyperfine-splitting	$\Delta_M = (M_{B_s^*} - M_{B_s})$	$\rightarrow$	49.0(1.5) MeV
ratio	$\frac{M_1}{M_2} = M_{\text{rest}}/M_{\text{kinetic}}$	$\rightarrow$	1

- Assuming linearity

$$Y_r = \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}_r = J^{(3\times3)} \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_r + A^{(3\times1)} \qquad (r = 1, \ldots, 7)$$

and defining

$$J = \left[\frac{Y_3 - Y_2}{2\sigma_{m_0 a}}, \frac{Y_5 - Y_4}{2\sigma_{c_P}}, \frac{Y_7 - Y_6}{2\sigma_\zeta}\right] \qquad A = \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}_1 - J \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_1$$

- We extract the RHQ parameters and iterate until result is inside uncertainties

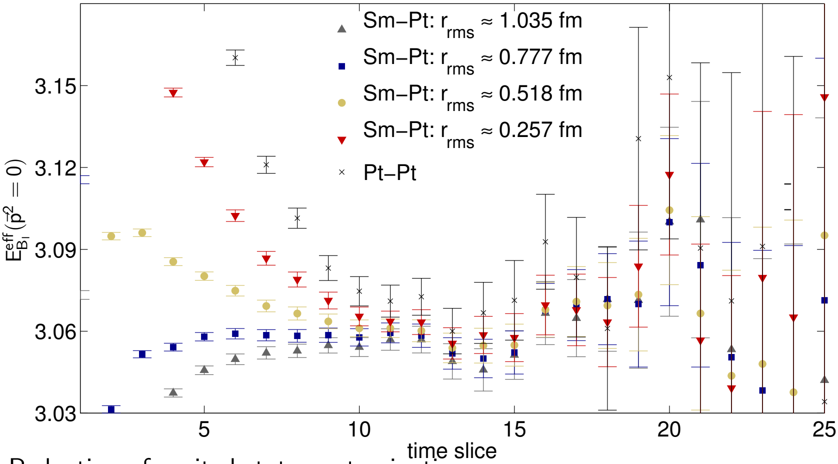
$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\text{RHQ}} = J^{-1} \times \left(\begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}^{\text{PDG}} - A \right)$$

Improvement of Tuning

- Tuning method pioneered on 24^3 ($a \approx 0.11$fm) by Min Li [M. Li 2009]
  Further studies by Hao Peng on 32^3 ($a \approx 0.08$fm) [H. Peng 2010]
  Exploratory studies; results not suitable for production

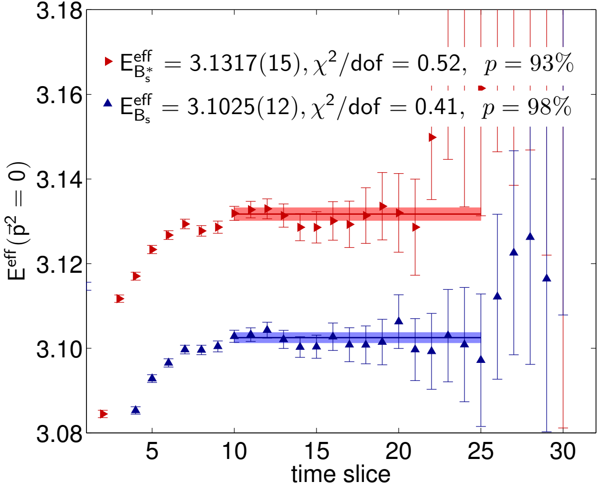
- Improvements and new setup
  - Use of point-source strange quark operators
    and Gaussian-smeared heavy quarks
  - Performed optimization study of smearing parameters
  - Significantly increased statistics
  - Only use of heavy-light quantities
  - Check on linearity assumption

Improving the Signal by Smearing of Source



- Reduction of excited state contamination
- 818 measurements, $m^l_{sea} = m^l_{val} = 0.005$, $m_0 a = 7.38$, $c_P = 3.89$, $\zeta = 4.19$

Effective Masses for Pseudoscalar and Vector State

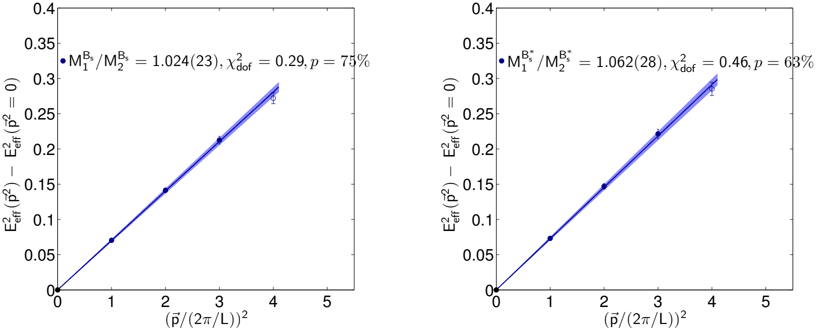


- $L = 24$
  $m^l_{sea} a = 0.005$
  $m_0 a = 8.40$
  $c_P = 5.80$
  $\zeta = 3.20$
- 1636 measurements

Determination $\frac{M_1}{M_2}$

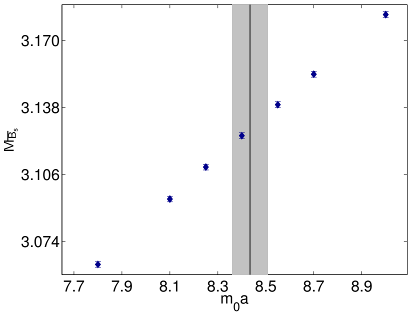
- Compute effective masses for momenta $p^2 = 0, 1, 2, 3$
- Obtain $\frac{M_1}{M_2}$ from fit to dispersion relation

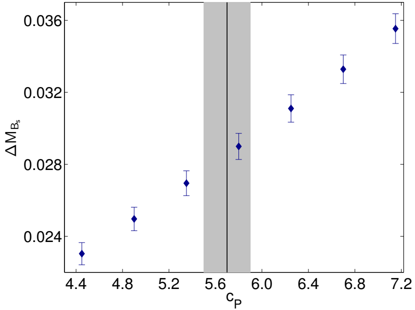
$$E_p^2 = \frac{M_1}{M_2}\, p^2/(2\pi L)^2 + E_0^2$$

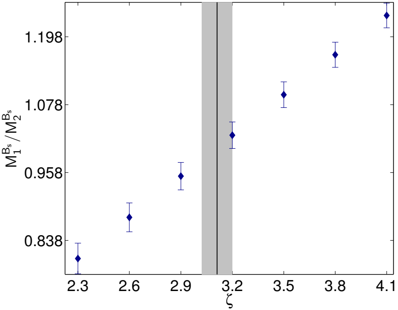


Test of Linearity

- Run simulation with 19 different RHQ parameter sets
- 1 center point and use variations roughly 1.5σ, 3σ and 4.5σ (based on initial iteration)
- Consistent result







Preliminary Parameters of the RHQ Action

Non-perturbatively tuned

m^l_{sea}	m_0a	c_P	ζ
0.005	8.43(7)	5.7(2)	3.11(9)
0.010	8.47(9)	5.8(2)	3.1(2)
average	8.45(6)	5.8(1)	3.10(7)

m^l_{sea}	m_0a	c_P	ζ
0.004	4.07(6)	3.7(1)	1.86(8)
0.006	3.97(5)	3.5(1)	1.94(6)
0.008	3.95(6)	3.6(1)	1.99(8)
average	3.99(3)	3.57(7)	1.93(4)

RHQ Lattice Perturbation Theory I [C. Lehner]

Motivation

- Knowing the RHQ parameters nonperturbatively we can compare the outcome with lattice perturbation theory
- Helps to build confidence that lattice perturbation theory is working also in cases where we do not have fully non-perturbative matching (e.g. decay constants, form factors)

Method

- Computation at 1-loop order
- Mean field improved
- Use nonperturbative inputs for $\langle P \rangle$, $\langle R \rangle$, $\langle L \rangle$ and $m_0 a$
- Predict: c_P and ζ
- Naive $\alpha_S^2 \sim 5\%$ power-counting estimate

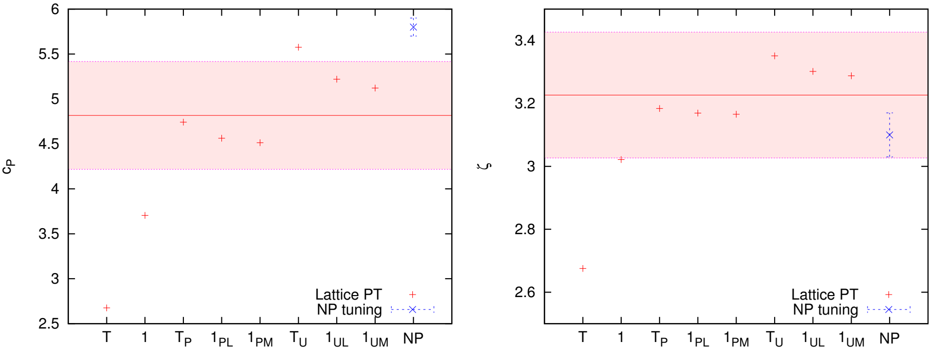
RHQ Lattice Perturbation Theory II [C. Lehner]

cP
- Match lattice quark-gluon vertex to the continuum counterpart in the on-shell limit
- At intermediate steps infrared divergences are regulated with a nonzero gluon mass λ
- Final results are obtained in the limit $\lambda \to 0$

ζ
- Extract the lattice heavy-quark dispersion relation from momentum dependence of the pole in the heavy-quark propagator at 1-loop
- Require that this dispersion relation agrees with the continuum

Mean Field Improvement – two methods:

- Use $u_0 = \langle P \rangle^{1/4}$ to resum tadpole contributions
- Estimate u_0 from spatial link field in Landau gauge $\langle L \rangle$
- The maximum of the spread between both values and a naive α_s^2 estimate is used to estimate the systematic error



- Central values: average of one-loop mean-field improved values computed with u_0 obtained from the plaquette and from the spatial Landau link
- Error on perturbative c_P: difference between mean field methods dominates
- Error on perturbative ζ: naive power-counting dominates
- Nonperturbative values statistical errors only
- Agreement within in 2σ – MF improved LPT is working!

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Results RHQ Lattice Perturbation Theory [C. Lehner]

a in fm	$\langle P\rangle$	$\langle R\rangle$	$\langle L\rangle$	m_0a	c_P	ζ
0.11	0.58803	0.34350	0.8439	8.45	4.8(6)	3.2(2)
0.086	0.61558	0.37984	0.8609	3.99	3.0(3)	2.1(1)

Predictions from seven RHQ parameter sets

- Compute quantity Q on all seven RHQ parameter sets
- Build-up prediction matrix J_p and vector A_p

$$J_p = \left[\frac{Q_3 - Q_2}{2\sigma_{m_0 a}}, \frac{Q_5 - Q_4}{2\sigma_{c_P}}, \frac{Q_7 - Q_6}{2\sigma_{\zeta}} \right] \qquad A_p = Q_1 - J_p \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_1$$

- By linearity we can predict Q for the tuned parameter set

$$Q^{\mathrm{RHQ}} = J_p^{(1\times 3)} \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\mathrm{RHQ}} + A_p$$

- Statistical errors in predicted value also reflect statistical uncertainty in the tuned RHQ parameters and account for statistical correlations between the three RHQ parameters

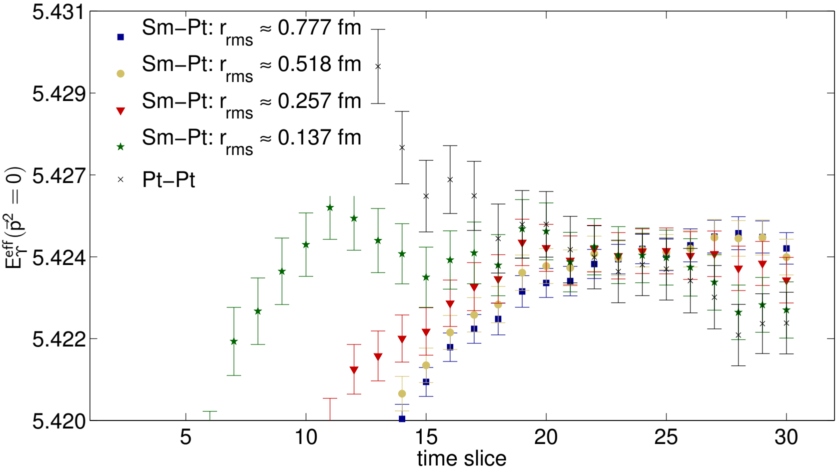
Computing Heavy-Heavy States

η_b pseudoscalar
Υ vector
χ_{b0} scalar
χ_{b1} axial
h_b tensor

$\Delta(\eta_b, \Upsilon)$
$\Delta(\chi_{b0}, \chi_{b1})$

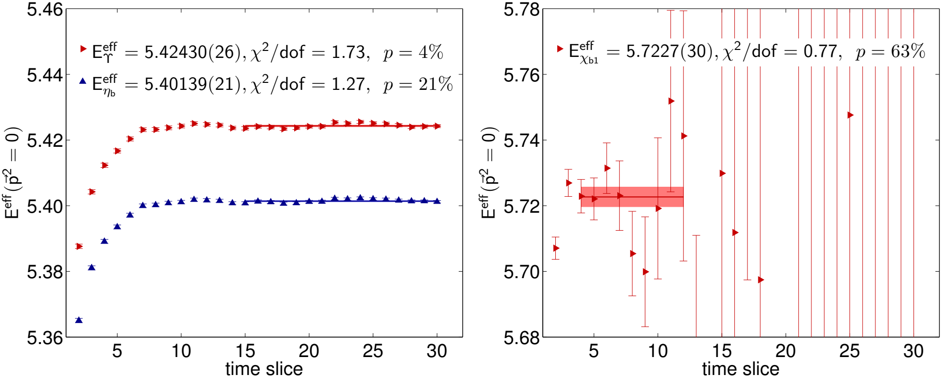
- For a good signal need different source smearing
- Higher sensitivity to non-linearity effects

Source Smearing for Heavy-Heavy States e.g. Υ



- 818 measurements, $m^l_{\text{sea}} = 0.005$, $m_0 a = 8.40$, $c_P = 5.80$, $\zeta = 3.20$

Effective Mass Plots for η_b, Υ and χ_{b1}



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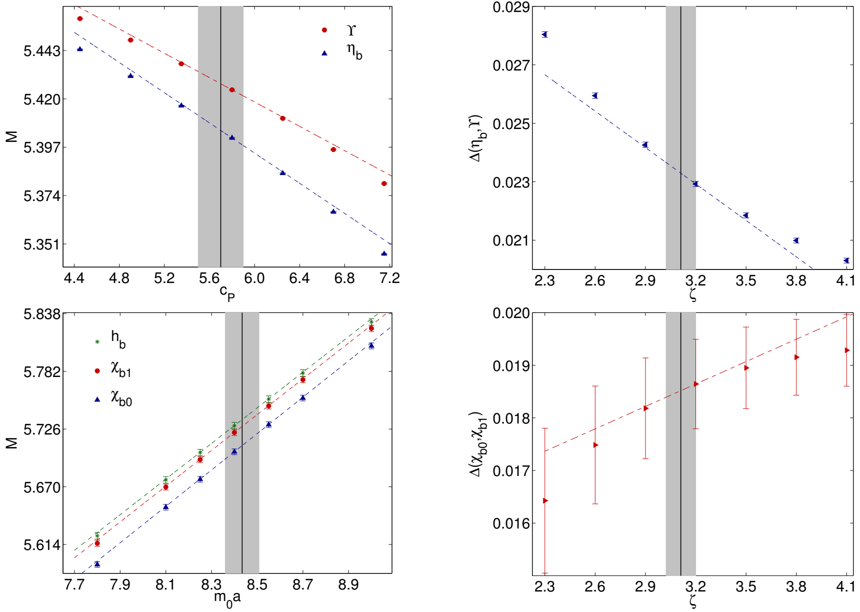
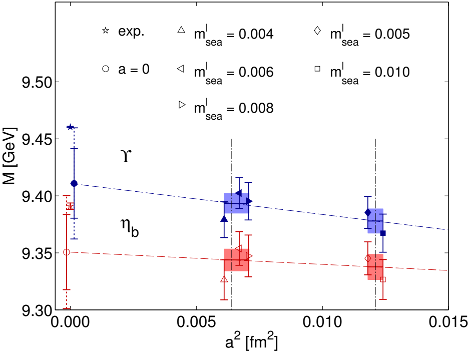
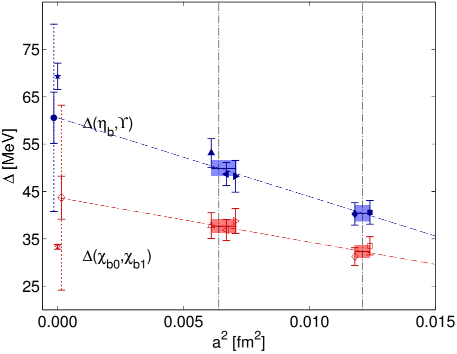
Higher Sensitivity to Non-Linearity Effects



Preliminary Predictions for the Heavy-Heavy States

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the B_s system we can predict bottomonium states and mass splittings



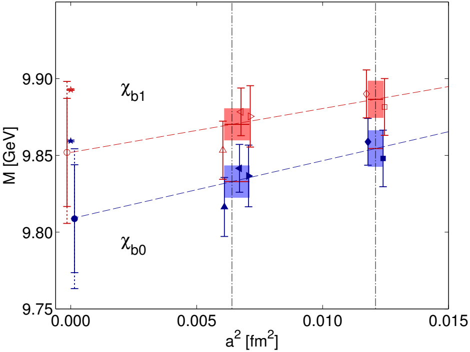


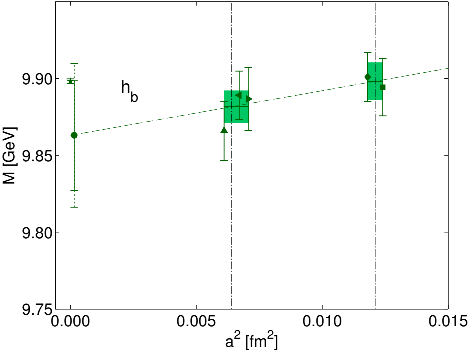
$\eta_b = 9350(33)(37)$ MeV

$\Upsilon = 9410(30)(38)$ MeV

$\Delta(\eta_b, \Upsilon) = 60(05)(19)$ MeV

$\Delta(\chi_{b0}, \chi_{b1}) = 44(05)(19)$ MeV





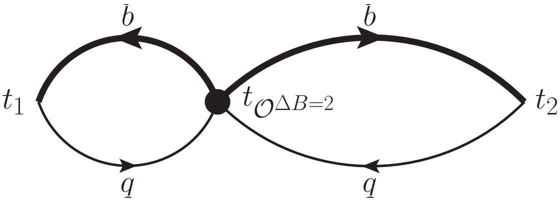
$\chi_{b0} = 9808(35)(29)$ MeV

$\chi_{b1} = 9851(35)(30)$ MeV

$h_b = 9862(36)(30)$ MeV

- Publication on tuning and bottomonium spectroscopy is in preparation

$B^0 - \overline{B^0}$ Mixing Matrix Element Calculation



- Location of four-quark operator is fixed
- Location of B-mesons is varied over all possible time slices
- Need: one point-source light quark and one point-source heavy quark originating from operator location
- Propagators can be used for B- and $\overline{B}$-meson
- Project out zero-momentum component using a Gaussian sink

Mostly Nonperturbative Renormalization

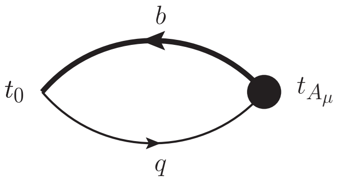
For f_{B_d}, f_{B_s} and $B \to \pi$ we plan to compute mostly non-perturbative renormalization factors á la [El Khadra et al. 2001]

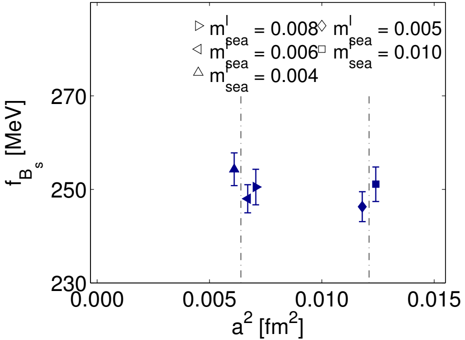
$$\varrho^{bl} = \frac{Z_V^{bl}}{\sqrt{Z_V^{bb} Z_V^{ll}}}$$

- Compute Z_V^{ll} and Z_V^{bb} non-perturbatively and only ϱ^{bl} perturbatively
- Enhanced convergence of perturbative serious of ϱ^{bl} w.r.t. Z_V^{bl} because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- ϱ^{bl} is expected to be of $\mathcal{O}(1)$; receiving only small corrections
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know Z_V^{ll} [Y. Aoki et al. 2011]
- Mostly nonperturbative renormalization not yet computed for B–$\bar{B}$ mixing

B-meson Decay Constant Calculation

- Re-use: point-source light quark and generate Gaussian smeared-source heavy quark
- Final result will use mostly nonperturbative renormalization





- Very preliminary result for f_{B_s}
- Renormalization to be improved:
  nonperturbative Z_V^{ll}
  perturbative Z_V^{bb} (1-loop)
  $\varrho_{bl} = 1$
- Scaling violations observed to be small

$B \to \pi l \nu$ form factor

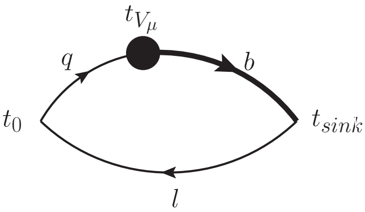
- Allows to determine the CKM matrix element V_{ub} from the experimental branching ratio

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} |f_+(q^2)|^2$$

- Tension between exclusive determination and inclusive determinations of V_{ub} is greater than 3σ

$B \to \pi l \nu$ form factor

- Compute matrix element of the $b \to u$ vector current between B-meson and pion
- Fix location of pion at t_0 and B meson at $T - t_{\text{sink}} - t_0$
- Vary operator location t_{V_μ} in that range
- B-meson is at rest, inject momentum on pion side
- Using partially quenched daughter quark-masses should help to better resolve quark-mass dependence and pion-energy dependence



$B \to \pi l \nu$ form factor

- f_+ is a linear combination of $f_\parallel$ and $f_\perp$

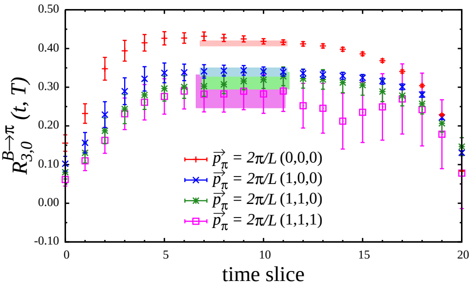
$$f_+(q^2) = \frac{1}{\sqrt{2m_0^B}} \left[f_\parallel(E_\pi) + (m_0^B - E_\pi) f_\perp(E_\pi) \right]$$

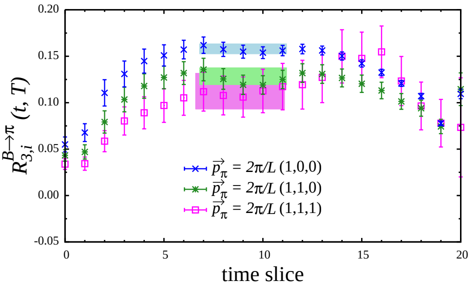
- Compute $f_\parallel$ and $f_\perp$ from the ratio

$$R_{3,\mu}^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{\sqrt{C_2^\pi(t) C_2^B(T-t)}} \sqrt{\frac{2E_0^\pi}{\exp(-E_0^\pi t)\exp(-m_0^B t)}}$$

with $\quad f_\parallel^{\text{lat}} = \lim_{\substack{t-t0\to\infty \\ T-t\to\infty}} R_{3,0}^{B\to\pi} \quad$ and $\quad f_\perp^{\text{lat}} = \lim_{\substack{t-t_0\to\infty \\ T-t\to\infty}} \frac{1}{p_\pi^i} R_{3,i}^{B\to\pi}$

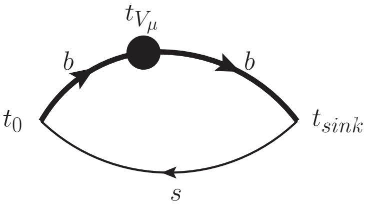
First Results for $B \to \pi l \nu$ [T. Kawanai]

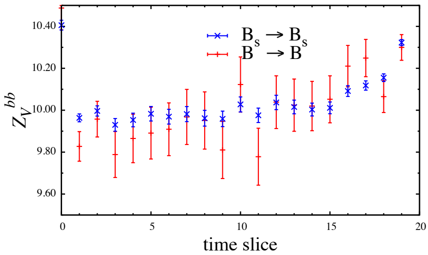




- 1636 measurements $m^l_{\text{sea}} = m^l_{\text{val}} = 0.005,\ m_0 a = 8.40,\ c_P = 5.80,\ \zeta = 3.20$
- $t_0 = 0,\ T = t_{\text{sink}} = 20$

Computing Z_v^{bb} [T. Kawanai]





- Computation of $B \to B$ (to get Z_v^{bb}) similar to $B \to \pi$
- Independent of the light spectator quark mass
- Significantly reduce statistical uncertainty by using strange quark and considering $B_s \to B_s$
- 1636 measurements $m^l_{\text{sea}} = 0.005$, $m^l_{\text{val}} = 0.005, 0.0343$ and $T = 20$ $m_0 a = 8.40$, $c_P = 5.80$, $\zeta = 3.20$

Conclusion

- We have completed tuning the parameters of the RHQ action for b-quarks, and find good agreement between our predictions for bottomonium masses and fine splittings with experiment.
- Given this success, we are now using this method for B-meson quantities such as decay constants and form factors, and expect to obtain errors competitive with other groups.
- The RHQ action can also be used for charm quarks, and Hao Peng is currently performing the necessary parameter tuning.
- We should have results for decay constants, mixing parameters, and form factors within the next year, and maybe sooner!