New Developments in Massive Gravity

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based on a collaboration with

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Why Higher-Derivative Gravity ?

Einstein Gravity is the unique field theory of interacting massless spin-2 particles around a given spacetime background that mediates the gravitational force

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a \left(R_{\mu\nu}{}^{ab}\right)^2 + b\,(R_{\mu\nu})^2 + c\,R^2 \;:$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no (bulk) massless spin 2 !

$\Rightarrow$ "New Massive Gravity"

Hohm, Townsend + E.B. (2009)

Comparison to Massive Gravity

- Massive Gravity is an IR modification of Einstein gravity that describes a massive spin-2 particle via an explicit mass term

- modified gravitational force

$$V(r) \sim \frac{1}{r} \quad \rightarrow \quad V(r) \sim \frac{e^{-mr}}{r}$$

  - characteristic length scale $r = \frac{1}{m}$

- Cosmological Constant Problem

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3D Einstein-Hilbert Gravity

Deser, Jackiw, 't Hooft (1984)

There are no massless gravitons : "trivial" gravity

Adding higher-derivative terms leads to "massive gravitons"

Free Fierz-Pauli

- $\left(\Box - m^2\right)\tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,,\ \ \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\rm FP} = \frac{1}{2}\tilde{h}^{\mu\nu}G^{\rm lin}_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no obvious non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Higher-Derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \epsilon_\nu{}^{\gamma\delta} \partial_\alpha \partial_\gamma h_{\beta\delta} \equiv G^{\text{lin}}_{\mu\nu}(h)$$

$$\left(\Box - m^2\right) \; G^{\text{lin}}_{\mu\nu}(h) = 0\,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g}\left[-R - \frac{1}{2m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right]$$

"New Massive Gravity" : unitary !

Mode Analysis

- Take NMG with metric $g_{\mu\nu}$, cosmological constant Λ and coefficient $\sigma = \pm 1$ in front of R
- lower number of derivatives from 4 to 2 by introducing an auxiliary symmetric tensor $f_{\mu\nu}$
- after linearization and diagonalization the two fields describe a massless spin 2 with coefficient

$$\bar{\sigma} = \sigma - \frac{\Lambda}{2m^2}$$

and a massive spin 2 with mass

$$M^2 = -m^2\bar{\sigma}$$

Special Cases

- 3D NMG Hohm, Townsend + E.B. (2009)

- $D \geq 3$ "critical gravity" for special value of Λ

  Li, Song, Strominger (2008); Lü and Pope (2011)

What We Now Know

- NMG is (most likely) non-renormalizable

- massive gravitons $\Leftrightarrow$ black holes

What about Critical Gravity?

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The Spectrum

Away from the critical point we have

$$\left(\bar{\Box} - 2\Lambda\right)\left(\bar{\Box} - 2\Lambda - M^2\right) h_{\mu\nu} = 0\,,$$

At the critical point we have

$$\left(\bar{\Box} - 2\Lambda\right)^2 h_{\mu\nu} = 0\,,$$

Massive graviton gets replaced by Log mode

Two-point Functions

For critical gravity we obtain

$$\langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & \mathrm{CFT} \\ \mathrm{CFT} & \mathrm{Log} \end{pmatrix} \qquad \text{i=1,2}$$

See, however, Lu, Pang, Pope (2011)

For tri-critical gravity we obtain

$$\langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & 0 & \mathrm{CFT} \\ 0 & \mathrm{CFT} & \mathrm{Log} \\ \mathrm{CFT} & \mathrm{Log} & (\mathrm{Log})^2 \end{pmatrix} \qquad \text{i=1,2,3}$$

See, however, Apolo, Porrati, (2012)

Conclusion

It is hard to find a unitary CFT

embedded inside a non-unitary LCFT!

An exception is 3D parity-odd gravity

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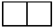
What did we learn?

- two theories can be equivalent at the linearized level (FP and boosted FP) but only one of them allows for a unique non-linear extension i.e. interactions!

- we need massive spin 2 whose massless limit describes 0 d.o.f.

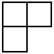
Example :  in 3D

- what about 4D?

Generalized spin-2 FP

standard spin-2 : 

$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 2 \quad \text{d.o.f.} \quad m = 0 \end{cases}$$

generalized spin-2 : 

$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 0 \quad \text{d.o.f.} \quad m = 0 \end{cases}$$

Curtright (1980)

Connection-metric Duality

- Use first-order form with independent fields $e_\mu{}^a$ and $\omega_\mu{}^{ab}$

- linearize around Minkowski: $e_\mu{}^a = \delta_\mu{}^a + h_\mu{}^a$

  and add a FP mass term $-m^2(h^{\mu\nu}h_{\nu\mu} - h^2) \quad \rightarrow$

$$\mathcal{L} \sim \text{“}\, h\,\partial\omega + \omega^2 \,\text{”} - m^2(h^{\mu\nu}h_{\nu\mu} - h^2)$$

- solve for $\omega \;\rightarrow$ standard spin-2 FP

- solve for $h_{\mu\nu} \;\rightarrow$ generalized spin-2 FP

Boosting up the Derivatives

- start with generalized spin-2 FP in terms of

  and subsidiary conditions

$$\tilde{h}_{\mu\nu,\rho}\,\eta^{\nu\rho} = 0\,, \qquad\qquad \partial^{\rho}\,\tilde{h}_{\rho\mu,\nu} = 0$$

- solve for $\partial^{\rho}\,\tilde{h}_{\rho\mu,\nu} = 0 \rightarrow \tilde{h}_{\mu\nu,\rho} = G_{\mu\nu,\rho}(h) \;\rightarrow$ "NMG in 4D" :

$$\mathcal{L}_{\rm NMG} \;\sim\; -\tfrac{1}{2}h^{\mu\nu,\rho}\,G_{\mu\nu,\rho}(h) \;+\; \frac{1}{2m^2}\;\underbrace{h^{\mu\nu,\rho}\;C_{\mu\nu,\rho}(h)}_{\text{"conformal invariance"}}$$

Interactions ?

cp. to Bekaert, Boulanger, Cnockaert (2005)

- compare to Eddington-Schrödinger theory

$$\mathcal{L}'_{\text{ES}} = \sqrt{-\det g}\,[g^{\mu\nu} R_{\mu\nu}(\Gamma) - 2\Lambda] \quad \Leftrightarrow \quad \mathcal{L}_{\text{ES}} = \sqrt{|\det R_{(\mu\nu)}(\Gamma)|}$$

$$g_{\mu\nu} = \frac{(D-2)}{2\Lambda} R_{(\mu\nu)}(\Gamma)$$

"Trivial" Gravity

Consider :  in 3D

- Chern-Simons formulation :

$$I_{CS}\,[h,\omega] = \int d^3x\, \varepsilon^{\mu\nu\rho} \left\{ \omega_\mu{}^a \partial_\nu h_\rho{}^b \eta_{ab} - \frac{1}{2} \omega_\mu{}^a \delta_\nu{}^b \omega_\rho{}^c \varepsilon_{abc} \right\}$$

Achúcarro and Townsend (1986); Witten (1988)

- interactions imposed by CS structure

- generalization to higher spins

4D "Trivial Gravity"

first-order formulation of 4D "trivial" gravity :

- $\left(h_{\mu\nu}{}^{a}\,,\omega_{\mu}{}^{a}\,,\delta_{\mu}{}^{a}\right)$

  Zinoviev (2003); Alkalaev, Shaynkman and Vasiliev (2003)

- "generalised" CS structure for non-Abelian version of Free Differential Algebra?

- "generalised" bi-metric formulation?

  cp. to 4D massive gravity

Outline

Summary

- we introduced the 3D NMG model
- in particular, we discussed the critical gravity case
- moreover, we investigated an extension to 4D
- we did not discuss higher-derivative parity-odd gravity theories

Open Issues

- interactions ?

- extension to higher spins ?

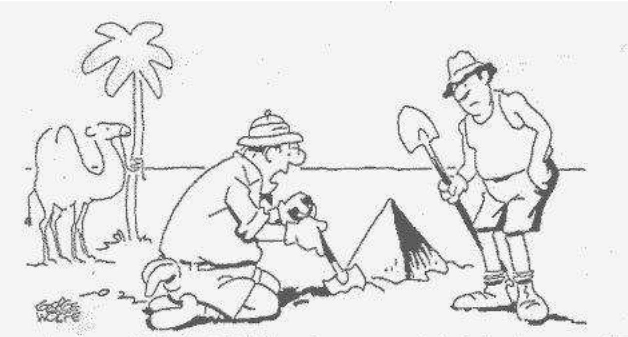
- relation to massive gravity?

Conclusion

New Massive Gravity is an interesting "toy model"

to study issues in gravity


Schatgraven op het strand, Schiermonnikoog



'If it's what I think it is, we've got some work ahead of us.'