The Ten-dimensional Multiple Membrane Revisited

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PKT@(60 +1) 2-3 July 2012

1206.6699 with I. Jeon and P. Richmond

Introduction

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"The Eleven-Dimensional Supermembrane Revisited"

claiming that IIA string theory and all its branes arise from an 11D theory on a circle. In particular the 10D membrane is an 11D membrane in a spacetime with a transverse circle.

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... and now has 534 citations and considered a cornerstone paper of M-theory

A lot has changed since then:

- Everyone believes you
- M-theory exists as an 11D theory and is the strong coupling limit of type IIA string theory
- The 11D membrane is called an M2-brane
- The 10D membrane is called a D2-brane

We have also since learnt that the dynamics of multiple D2-branes is given by 3D maximally supersymmetric Yang-Mills Theory [Chan-Paton, Polchinski, Witten]

And that the dynamics of multiple M2-branes is given by a Chern-Simons-Matter Theory [BLG,ABJM]

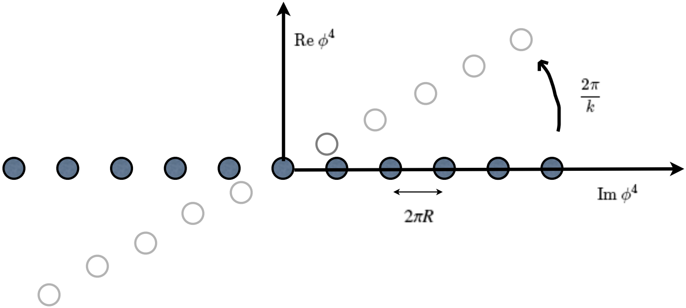
Here we will explore the simple consequence that the ABJM theory of a periodic array of M2-branes should give 3D MSYM.

This kind of array was first considered by [Taylor] for Dp-branes

- Take a vacuum corresponding to an infinite periodic array
- Impose a discrete translational symmetry on the fluctuations
- Infinite tower of massive fields from Higgs mechanism gives a KK-like tower
- T-dual to D$(p+1)$-brane wrapped on S^1

But actually it's not so simple for M2-branes as the shift symmetry is not a symmetry of the lagrangian

- There is a spacetime orbifold present for $k > 1$
- For $k = 1$ monopole operators are required for the shift symmetry



But nevertheless for $k \to \infty$ we should be able to trust the lagrangian system.

Other puzzles and motivations:

- What happens to the KK-like tower of states?
- If we consider a cubic array of M2-branes can this be mapped to M5-branes on $\mathbb{T}^3$?

Note that the connection of the Chern-Simons-Matter theories of M2-branes to the Yang-Mills theories of D2-branes was first established by [Mukhi, Papageorgakis]

- Novel Higgs' mechanism: non-dynamical vector eats a scalar to give Yang-Mills
- Corresponds to going far out on the Coloumb branch where $\mathbb{R}^8 \sim \mathbb{R}^7 \times S^1$
- Only valid at leading order: higher derivative corrections

Our approach should be true 'on the nose' for any coupling.

For other relations of M2's to Dp's see [Naste,Papageorgakis] and [Ho,Matsuo,Shiba],[Kobo, Matsuo, Shiba],[Honma,Zhang] using Lorentzian 3-algebras.

PLAN:

- Introduction
- Evaluating the lagrangian
- Relation to 3D MSYM
- Further T-duality to M5 on $\mathbf{T}^3$
- Conclusions

Evaluating the lagrangian

For M2-branes the vacuum can be described by taking

$$\langle Z^{A'} \rangle = 0\ , \qquad \langle Z^4 \rangle = 2\pi i R \begin{pmatrix} \ddots & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & -1 & \\ & & & & \ddots \end{pmatrix}$$

where each entry is an $M \times M$ matrix.

To evaluate the fluctuations we must impose

$$Z^{A'}_{(m+1)(n+1)} = Z^{A'}_{mn} \qquad Z^4_{(m+1)(n+1)} = 2\pi i R + Z^4_{mn}$$

and similarly for the other fields.

The sum over the infinite array leads to a divergence:

$$\sum_{q\in\mathbb{Z}} 1$$

In the D-brane case this just gives an overall-divergence and can be easily treated.

For M2-branes the fact that translations aren't a symmetry causes additional divergences to arise:

$$\sum_{q\in\mathbb{Z}} q^2$$

And we need to deal with these.

We do this by considering a large but finite array $m, n = -N, ..., +N$ and take $N \to \infty$ in the final expressions.

- We also allow R and k to scale with N.

Thus we take

$$Z^A_{mn} := 2\pi i R \delta^A_4 n 1_{M\times M} \delta_{mn} + \frac{1}{\sqrt{2N}} \phi^4_{n-m} \, ,$$

$$A^{L/R}_{\mu\, mn} := a^{L/R}_{\mu\, n-m} \, , \quad \Psi_{Amn} = \frac{1}{\sqrt{2N}} \psi_{An-m}$$

So that ϕ^A_p and ψ_{Ap} have standard kinetic terms.

However we still find divergences:

From the kinetic term $D_\mu Z^A D^\mu \bar{Z}_A$

$$R^2 \sum_q q^2 \sum_p a^-_{\mu p} a_p^{-\mu} \sim R^2 N^3 \sum_p a^-_{\mu p} a_p^{-\mu}$$

where $a^-_{\mu p} = \frac{1}{2}(a^L_{\mu p} - a^R_{\mu p})$.

Thus we must set these terms to zero:

- $a^L_{\mu p} = a^R_{\mu p}$
    - Gauge group infinitely broken $U(M) \times U(M) \to U(M)$ with all fields in the adjoint.

And from the potential $\frac{(2\pi)^2}{k^2}|[Z^A, Z^B; Z_C]|^2$

$$\frac{4}{3}(2\pi)^6\frac{N^2R^4}{k^2}\sum_p p^2\mathrm{tr}(\phi_p^{A'}\phi^\dagger_{A'p}) + \cdots = M_b^2\sum_p p^2\mathrm{tr}(\phi_p^{A'}\phi^\dagger_{A'p}) + \ldots$$

where $M_b \propto NR^2/k \to \infty$.

What about the fermions? Here we encounter the sum

$$\sum_{q\in\mathbb{Z}} q \sim ??N^2$$

Supersymmetry ($M_b = M_f$) fixes $?? = 2/\sqrt{3}$.

Thus we must set these terms to zero - no infinite KK-like tower.

$$\phi_p^{A'} = 0 \qquad \psi_{Ap} = 0 \qquad p \neq 0$$

However we must also set the sources for these fields to zero.

To cut a long story short:

- the only fields to survive are the zero modes

$$\phi_0^A \qquad \psi_{A0} \qquad a_{\mu 0}^-$$

- other modes have an infinite mass (or are sources)

$$M_b = M_f \sim \frac{2}{\sqrt{3}}(2\pi)^3 \frac{R^2 N}{k} \to \infty$$

- Interactions are controlled by a coupling

$$g \sim \frac{1}{\sqrt{3}}(2\pi)^2 \frac{R\sqrt{N}}{k} \propto \sqrt{\frac{M_b}{k}} < \infty$$

Putting all this into the ABJM lagrangian gives us

$$\mathcal{L}_{array} = -\mathrm{tr}(\nabla_\mu \phi_0^A \nabla^\mu \phi^\dagger_{A0}) - i\mathrm{tr}(\bar{\psi}_0^A \gamma^\mu \nabla_\mu \psi_{A0}) + \mathcal{L}_{Yukawa} - V$$

where $\nabla_\mu = \partial_\mu - i[a^-_{\mu 0}, \ \ ]$

$$\begin{aligned} V &= -\frac{g^2}{2}\mathrm{tr}\left([\phi_0^{A'}, \phi^\dagger_{B'0}][\phi^\dagger_{A'0}, \phi_0^{B'}] + [\phi_0^{A'}, \phi_0^{B'}][\phi^\dagger_{A'0}, \phi^\dagger_{B'0}] \right. \\ &\qquad \left. + 4[\phi_0^{A'}, \mathrm{Im}\ \phi_0^4][\phi^\dagger_{A'0}, \mathrm{Im}\ \phi_0^4]\right) \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{Yukawa} &= \frac{g}{\sqrt{2}}\mathrm{tr}\left(2i\bar{\psi}_0^{A'}[\mathrm{Im}\ \phi_0^4, \psi_{A'0}] - 2i\bar{\psi}_0^4[\mathrm{Im}\ \phi_0^4, \psi_{40}] \right. \\ &\qquad + 2\bar{\psi}_0^{A'}[\phi^\dagger_{A'0}, \psi_{40}] + 2\bar{\psi}_{A'0}[\phi_0^{A'}, \psi_0^4] \\ &\qquad \left. + \varepsilon_{A'B'C'}\bar{\psi}_0^{A'}[\phi_0^{B'}, \psi_0^{C'}] + \varepsilon^{A'B'C'}\bar{\psi}_{A'0}[\phi_{B'0}, \psi_{C'0}]\right) \end{aligned}$$

Relation to 3D MSYM

let us write

$$X^{A'+2} = \frac{1}{\sqrt{2}}(\phi^{A'0} + \phi^\dagger_{A'0}) \qquad X^{A'+5} = \frac{1}{\sqrt{2i}}(\phi^{A'0} - \phi^\dagger_{A'0})$$

$$X^9 = \sqrt{2}\mathrm{Im}\ \phi^4_0 \qquad Y = \sqrt{2}\mathrm{Re}\ \phi^4_0$$

as well as a suitable embedding of ψ_{A0} into Λ then

$$\mathcal{L}_{array} = -\frac{1}{2}\mathrm{tr}\left(\nabla_\mu Y \nabla^\mu Y\right) - \frac{1}{2}\mathrm{tr}\left(\nabla_\mu X^I \nabla^\mu X^I\right) - \frac{i}{2}\mathrm{tr}\left(\bar{\Lambda}\Gamma^\mu \nabla_\mu \Lambda\right) + \frac{g}{2}\mathrm{tr}\left(\bar{\Lambda}\Gamma^{11}\Gamma^I [X^I, \Lambda]\right) + \frac{g^2}{4}\sum_{I,J}\mathrm{tr}([X^I, X^J])^2$$

- 8 scalars and the gauge field has no kinetic term.
- the rest is exactly the lagrangian of 3D MSYM.
- $SU(3)$ has been enhanced to $SO(7)$

The gauge field equation of motion imposes a constraint:

$$i[Y, \nabla_\mu Y] + i[X^I, \nabla_\mu X^I] + \frac{1}{2}[\bar{\Lambda}, \Gamma_\mu \Lambda] = 0$$

And Y has the equation of motion

$$\nabla^2 Y = 0$$

A solution to this equation is

$$\nabla_\mu Y = -\frac{1}{2g}\varepsilon_{\mu\nu\lambda}F^{\nu\lambda}$$

In which case we find that the constraint can be written as

$$\frac{1}{g^2}\nabla^\nu F_{\mu\nu} = i[X^I, \nabla_\mu X^I] + \frac{1}{2}[\bar{\Lambda}, \Gamma_\mu \Lambda]$$

Which is precisely the equation of motion of 3D MSYM.

So every solution of 3D MSYM is a solution the M2-brane equations of motion.

- Y can be thought of as a dual gluon
- there is a hidden ’dynamical’ maximal supersymmetry

But there is slightly more information in the M2-brane description: Consider a flat connection: $A_\mu = ig\partial_\mu g^{-1}$.

$$Y = gY_0g^{-1} \qquad Y_0 = \text{const}$$

Thus there is an additional ’modulus’ Y_0.

- separation of the D2-branes in the 11^{th} dimension

Further T-duality to M5 on $\mathbb{T}^3$

Next consider a cubic array of M2-branes. Following [Taylor]:

- D2-branes with transverse $\mathbb{T}^2 \longleftrightarrow$ D4-branes on $\mathbb{T}^2$.
- Should be the same as M5-branes wrapped on $\mathbb{T}^3$

Starting from $\mathcal{L}_{array}$ we obtain ($\mu', \nu' = 0, 1, 2, ..., 4$, $\alpha, \beta = 3, 4$)

$$
\begin{aligned}
S^{(b)}_{cubic\ array} \quad &= \quad -\mathrm{tr} \int d^5x \, \frac{1}{2}\nabla_{\mu'} X^{I'} \nabla^{\mu'} X^{I'} - \frac{g'^2}{4} \sum_{I',J'} ([X^{I'}, X^{J'}])^2 \\
&\qquad + \frac{1}{2}\nabla_\mu Y \nabla^\mu Y + \frac{1}{2g'^2} F_{\mu\alpha} F^{\mu\alpha} + \frac{1}{4g'^2} F_{\alpha\beta} F^{\alpha\beta}
\end{aligned}
$$

Similar on-shell relation to 5D SYM. But now $\nabla_\mu Y = \frac{1}{2}\varepsilon^{\alpha\beta} H_{\mu\alpha\beta}$.

$$
Y_0 \propto \int_{\mathbb{T}^2} B
$$

Conclusions

We have rederived D-brane dynamics from the eleven-dimensional supermembrane

- novel formulation in terms of a 'dual gluon'
- It is broadly in line with the claim that the $(2,0)$ theory of M5-branes on S^1 is 5D SYM (we could take g'_{YM} large) [Douglas, NL-Papageorgakis-Schmidt-Sommerfeld].

There are some issues I'm not sure about:

- Can we quantize keeping Y?
- Not clear how much of 11D physics is washed away by the $k \to \infty$ limit but clearly some remains.

But I do know that:

- Branes are still mysterious and fun
  - Beautiful interplay between supersymmetry, solitons and geometry.
- It was great to be your student
  - Especially at such an interesting time.

Thanks and Happy Birthday