Strong Coupling Isotropization Simplified



David Mateos ICREA & University of Barcelona

Work with M.Heller, W. van der Schee, M. Spalinski & D. Trancanelli

- Paul has been a friend and a mentor throughout my career.
- Met at the beginning of my PhD in 1997 -- hard times!
- Paul was visiting Barcelona for a few months.
- Gave lectures on M-theory and things fell into place.
- Wrote a paper before he left Barcelona (with Gomis & Simon).

- In 1998 Paul hosted me as a visiting student at DAMTP... and at his house!
- During that visit we wrote another paper (with Gauntlett, Koehl and Zamaklar).
- In 2000-2002 I joined DAMTP as a postdoc.
- We wrote several papers on "supertubes".

• One day Paul walked into my office with:

Adding flavor to AdS/CFT

Andreas Karch and Emanuel Katz

Department of Physics, University of Washington, Seattle, WA 98195, USA

(karch@phys.washington.edu) (amikatz@phys.washington.edu)

Coupling fundamental quarks to QCD in the dual string representation corresponds to adding the open string sector. Flavors therefore should be represented by space-time filling D-branes in the dual 5d closed string background. This requires several interesting properties of D-branes in AdS. D-branes have to be able to end in thin air in order to account for massive quarks, which only live in the UV region. They must come in distinct sets, representing the chiral global symmetry, with a bifundamental field playing the role of the chiral condensate. We show that these expectations are born out in several supersymmetric examples. To analyze most of these properties it is not necessary to go beyond the probe limit in which one neglects the backreaction of the flavor D-branes.

- Beginning of my interest on applications of gauge/string duality to QCD.
- Today: Applications to out-of-equilibrium physics of QGP.

Last decade: Near equilibrium QGP





Animation by Jeffery Mitchell (Brookhaven National Laboratory). Simulation by the UrQMD Collaboration

Out of equilibrium



Classical Dynamical GR in AdS



• QCD dual is beyond supergravity.

• Do not try to do precision. QCD physics.

• Search for physical insights.

In the context of HIC

- Fast isotropization of the QGP (~ 1 fm/c) remains outstanding challenge.
- Consider simplest possible set-up in AdS/CFT: Isotropization of homogeneous 4D CFT plasma (e.g. N=4 SYM plasma).



In the context of HIC

- Fast isotropization of the QGP (~ 1 fm/c) remains outstanding challenge.
- Consider simplest possible set-up in AdS/CFT: Isotropization of homogeneous 4D CFT plasma (e.g. N=4 SYM plasma).
- Homogeneity: $\partial_{\mu}T^{\mu\nu} = 0 \rightarrow \partial_{t}T^{00} = 0 \rightarrow \mathcal{E}_{i} = \mathcal{E}_{f}$
- No hydrodynamics: $\omega \to 0$ as $q \to 0$
- Only "quasi-normal modes" (QNM).

 $\partial \mathrm{AdS}$ (timelike) t

Equilibrium horizon

$$ds^{2} = \frac{r^{2}}{R^{2}} \left(-f dt^{2} + dx_{1}^{2} + dx_{2}^{2} + dx_{3}^{2} \right) + \frac{R^{2}}{r^{2}f} dr^{2}$$

$$f(r) = 1 - \frac{r_{0}^{4}}{r^{4}}$$

$$f(r) = \frac{r_{0}^{2}}{r^{4}}$$

Equilibrium horizon

Out-of-equilibrium horizon

 $\partial \mathrm{AdS}$ (timelike)

t

Equilibrium horizon

 $\partial \mathrm{AdS}$

t

(timelike)

Out-of-equilibrium horizon

Most general initial state for 5D metric compatible with constraints

Characterized by infinite number of scales.

Out-of-equilibrium horizon

Dynamics

 $\partial \mathrm{AdS}$

t

(timelike)

No sources

Equilibrium horizon

Most general initial state for 5D metric compatible with constraints

Characterized by infinite number of scales.

Source

Vacuum AdS Most general initial state for 5D metric compatible with constraints

Dynamics

Characterized by infinite number of scales.

Compare with Chesler & Yaffe

 $\partial \mathrm{AdS}$ (timelike)

No sources

t

Posing the problem

• Generalized Eddington-Finkelstein coordinates:

 $ds^2 = 2dtd\rho - Adt^2 + \Sigma^2 e^{-2B} dx_{\rm L}^2 + \Sigma^2 e^B d\mathbf{x}_{\rm T}^2$

 A, Σ, B functions of t, ρ only.

• In equilibrium: $A = \rho^2 (1 - \rho_h^4 / \rho^4), \qquad \Sigma = \rho$ $B = 0, \qquad \rho_h = \pi T$

Dynamics

 $\partial \mathrm{AdS}$

Posing the problem

- Generalized Eddington-Finkelstein coordinates:
 - $ds^{2} = 2dtd\rho Adt^{2} + \Sigma^{2}e^{-2B}dx_{\mathrm{L}}^{2} + \Sigma^{2}e^{B}d\mathbf{x}_{\mathrm{T}}^{2}$

 A, Σ, B functions of t, ρ only.

• Near-boundary fall-off determines:

$$\langle T_{\mu\nu} \rangle = \frac{N_{\rm c}^2}{2\pi^2} {\rm diag} \Big[\mathcal{E}, \, \mathcal{P}_{\rm L}(t), \, \mathcal{P}_{\rm T}(t), \, \mathcal{P}_{\rm T}(t) \Big]$$

- In particular, B determines $\Delta \mathcal{P} = \mathcal{P}_{T} \mathcal{P}_{L}$.
- Initial state specified by $B(t = 0, \rho)$.
- CFT: Must specify anisotropy distribution in modes.
- Infinitely-many-scales problem.

∂AdS

Dynamics

Time evolution

• In equilibrium:

$$A = \rho^2 (1 - \rho_{\rm h}^4 / \rho^4), \qquad \Sigma = \rho$$
$$B = 0, \qquad \rho_{\rm h} = \pi T$$

• Evolve initial state according to:

- Full, non-linear EEQs.
- EEQs linearized around final equilibrium state.



Price & Pullin '94





Price & Pullin '94



Price & Pullin '94



Price & Pullin '94



Price & Pullin '94



- Wave-form at infinity accurately reproduced (but perhaps non-asymptotic properties would not be).
- Analog in AdS: Boundary stress-tensor.







- Over 2000 initial profiles.
- May or may not have AH at t=0.
- Ratio of scales gives accuracy: 2/10 ~ 20%
- $\Delta \mathcal{P}(t)/\mathcal{E} \sim 10$ implies far from equilibrium.

• "Entropy" increases during isotropization.



• Isotropization time of order 1/T predicted by LA within 20%. $\Delta \mathcal{P}(t)/\mathcal{E} \leq 0.1$



- Isotropization time of order 1/T predicted by LA within 20%. $\Delta \mathcal{P}(t)/\mathcal{E} \leq 0.1$



Expand in QNMs $\delta B(t,r) = \sum_{i} c_i b_i(r) e^{-i\omega_i t}$

Expand in QNMs (Full / Linear / QNM)

 $\delta B(t,r) = \sum c_i \, b_i(r) e^{-i\omega_i t}$



Discussion

- Gauge: Small perturbations around equilibrium plasma
 → Linear response theory
- Gravity: Small perturbations around equilibrium black hole
 → Linearized Einstein's equations
- In both cases expect linear approx. if $\Delta \mathcal{P}/\mathcal{E} \ll 1$.
- LA in Fourier space:
 - HDMs: $\omega \rightarrow 0$ as $q \rightarrow 0$
 - QNMs: $\omega(0) \neq 0$
- We have studied far-from-equilibrium dynamics of QNMs.

Discussion

- For small perturbations: QNMs relax *linearly* and *independently*, with $t_i^{\rm relax} \sim 1/{\rm Im}\,\omega_i$.
- Extend to not-so-small perturbations by adding interactions.
- Expected to break down for $\Delta \mathcal{P}/\mathcal{E} \sim 1$ but it does not.
- Relaxation still characterized by few frequencies.
- Linear approx. valid for stress tensor 1-point function; other observables probably not well captured.

Discussion

- Next step: Include hydrodynamics (boost-invariant case).
- Preliminary results indicate it works.



Initial state



Initial state

Hydro becomes applicable







Full non-linearity of gravity encoded in the initial horizon.

• Hope it's clear how much respect I have for Paul both as a friend and as a teacher.

• Perhaps only one fault...

• you are too serious.



Thank you Paul, and Happy Birthday!