dynamics of symmetry breaking

with Joe Bhaseen, Ben Simons, Jerome Gauntlett and Toby Wiseman

Julian Sonner
Paul Fest @ Cambridge
3 July 2012

domain walls, janus solutions & cosmologies

- Paul and I worked on a number of projects which were unified by the use of dynamical systems analysis and fake supersymmetry

  [J.S. & Paul. K. Townsend: Dilatonic Domain Walls and Dynamical Systems CQG 23 (2006)
  J.S & Paul. K. Townsend: Recurrent acceleration in dilaton-axion cosmology PRD 74 (2006)
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If you're not having fun doing what you're doing, you're doing something wrong

heeding Paul's advice

4 Feb 2008

Chris Pedder, Julian Sonner and David Tong

Department of Applied Mathematics and Theoretical Physics,
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1. Introduction

Many years ago, Kugo and Townsend [1] pointed out a relationship between supersymmetric field theories with $N = 2, 4, 8$ and 16 supercharges and the four normed division algebras $\mathbb{K} \cong \mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ and $\mathbb{O}$. The key observation is algebraic. Theories

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The Berry Phase of D0-Branes

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Paul, the cook

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aubergine feta penne

serves: one PhD supervisor and student

ingredients:

1 fresh aubergine
250g fresh tomatoes
250g feta cheese (in brine)

happy birthday, paul!

- Many thanks for guiding my PhD and pointing me in the right direction for my subsequent research, Paul.

Happy 60th birthday, Paul !

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weakly-coupled, type IIA description

[Hull & Townsend 1995]

strongly correlated matter at finite density

- strong correlations are hard to deal with. Lots of interest in **strongly correlated** systems: QCD, exotic materials (cuprates), SI transition

- Very hard to make any progress at all. Some control at **quantum-critical** points which have dynamical scaling

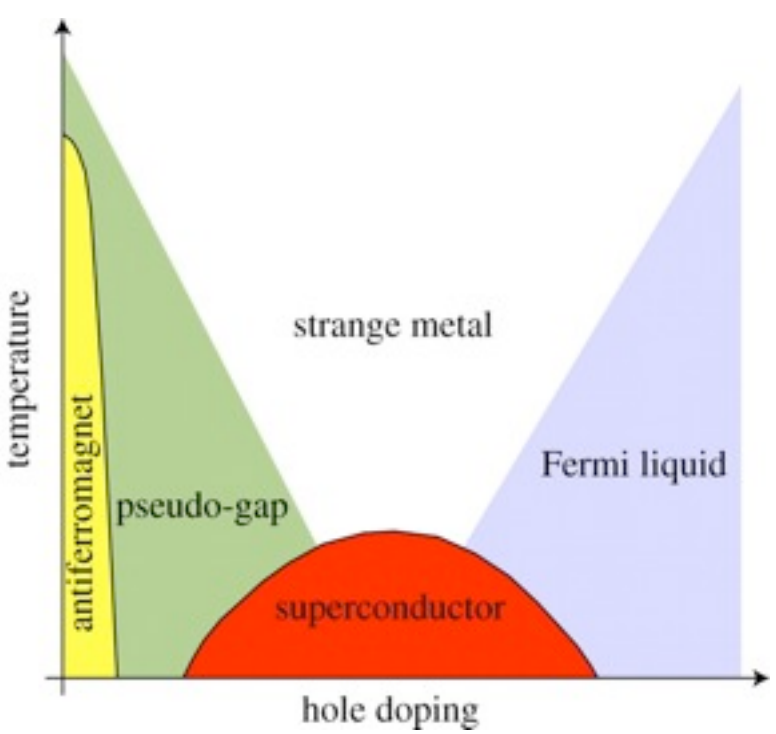
$$\begin{aligned} \xi &\sim (g - g_c)^{-\nu} \\ \Delta &\sim (g - g_c)^{z\nu} \end{aligned}$$

- Two particularly hard areas
  1) Fermionic Quantum Critical Points (at finite density: sign problem)
  [Leiden group & MIT group - top down: J.S. (with Gauntlett and Waldram)]
  2) Out-of Equilibrium phenomena (few if any general principles known)
  [Chesler & Yaffe, Mateos et al.;, ... J.S. & Andrew Green]

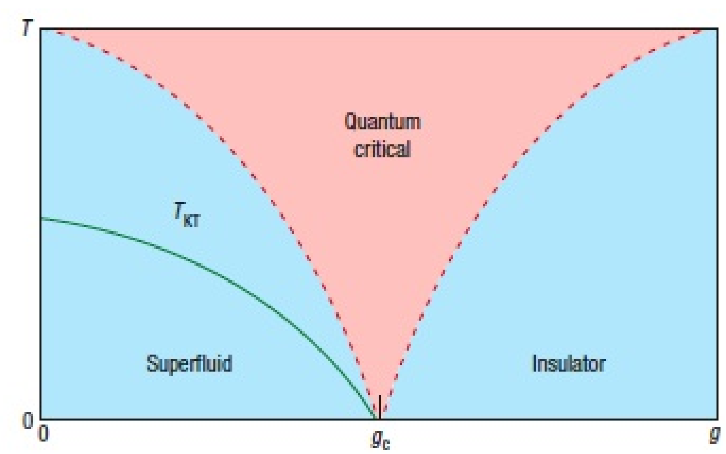
strong correlations in condensed matter

- We will endeavour to model QCPs using holography. **Scaling** symmetries encoded as **isometries** of `dual' spacetime

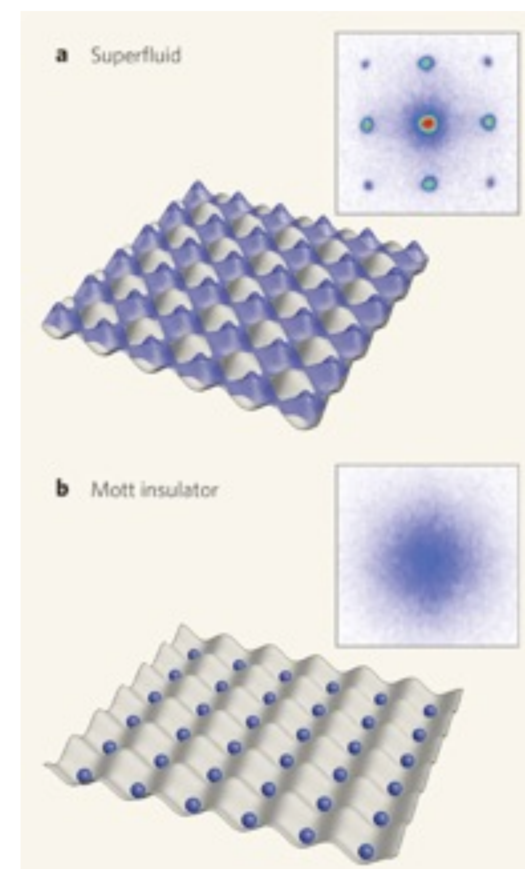
- Continuum theory near QCP is encoded in dynamics of dual string theory



cuprate superconductor



SI transition



realised in cold atoms

outline

1. introduction to ads/cmt
   "quantum criticality encoded in dual spacetime "
2. model and background
   "holographic superconductors, time-dependent BCS"
3. a holographic setup for dynamical symmetry breaking
   "Numerical relativity, structure of quasi-normal modes"
4. conclusions and outlook
   "generic dynamical consequences of symmetry breaking"

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a holographic model of superconductivity

- Superconductivity is a manifestation of symmetry breaking. New results here in a dynamical context are **very general** and extend beyond holography

- Specific example: minimal model of holographic superconductor

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{\ell^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$

- Complex scalar Ψ is dual to symmetry-breaking order parameter
  1) RN: un-condensed normal phase, new hairy BH: s.c. phase

     [Gubser; Hartnoll, Herzog, Horowitz]

  2) leading near-boundary term of Ψ = source; subleading term = vev
  3) M-theory superconducts!

     [Gaunlett, Sonner, Wiseman]

an old dog and a new trick

- BCS theory is the celebrated microscopic explanation of conventional superconductivity. An old story!

BCS hamiltonian: $$\mathcal{H} = \sum_{p,\sigma} \epsilon_p a^\dagger_{p,\sigma} a_{p\sigma} - \frac{\lambda(t)}{2} \sum_{q,p} a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} a_{-q\downarrow} a_{q\uparrow}$$

BCS groundstate: $$|\Psi(t)\rangle = \prod_p \left[u_p(t) + v_p(t) a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} \right] |0\rangle$$

pairing gap function: $$\Delta(t) = \lambda \sum_p u_p(t) v^*_p(t)$$

- Recent (2004 -) new developments: the resulting (non-adiabatic) dynamics can be mapped onto a non-linear integrable system! [Barankov, Levitov & Spivak; Yuzbashyan, Altshuler, Kuznetsov & Enolskii]

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time-dependent BCS pairing problem

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BL phase diagram

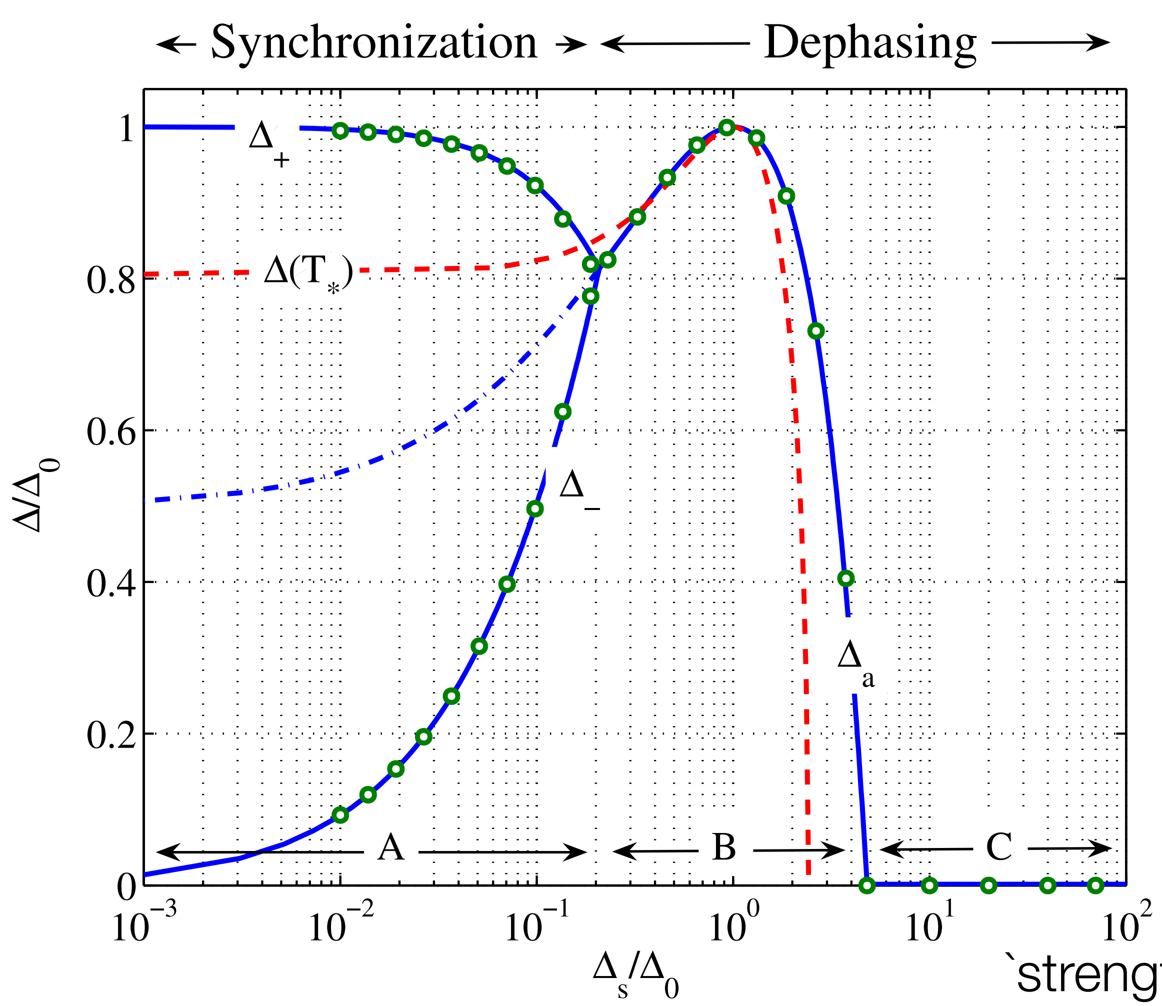
- The dynamics of this quench give rise to **three** distinct regimes

I. Oscillation

II. Decay to finite gap

III. Decay to zero gap

final pairing gap



`strength' of quench

- our achievement is **twofold**: 1) we exhibit analogous phenomena in a strongly-coupled system, with thermal and collisional damping
  2) we identify a new and **completely generic** mechanism within dynamical symmetry breaking leading to this behaviour

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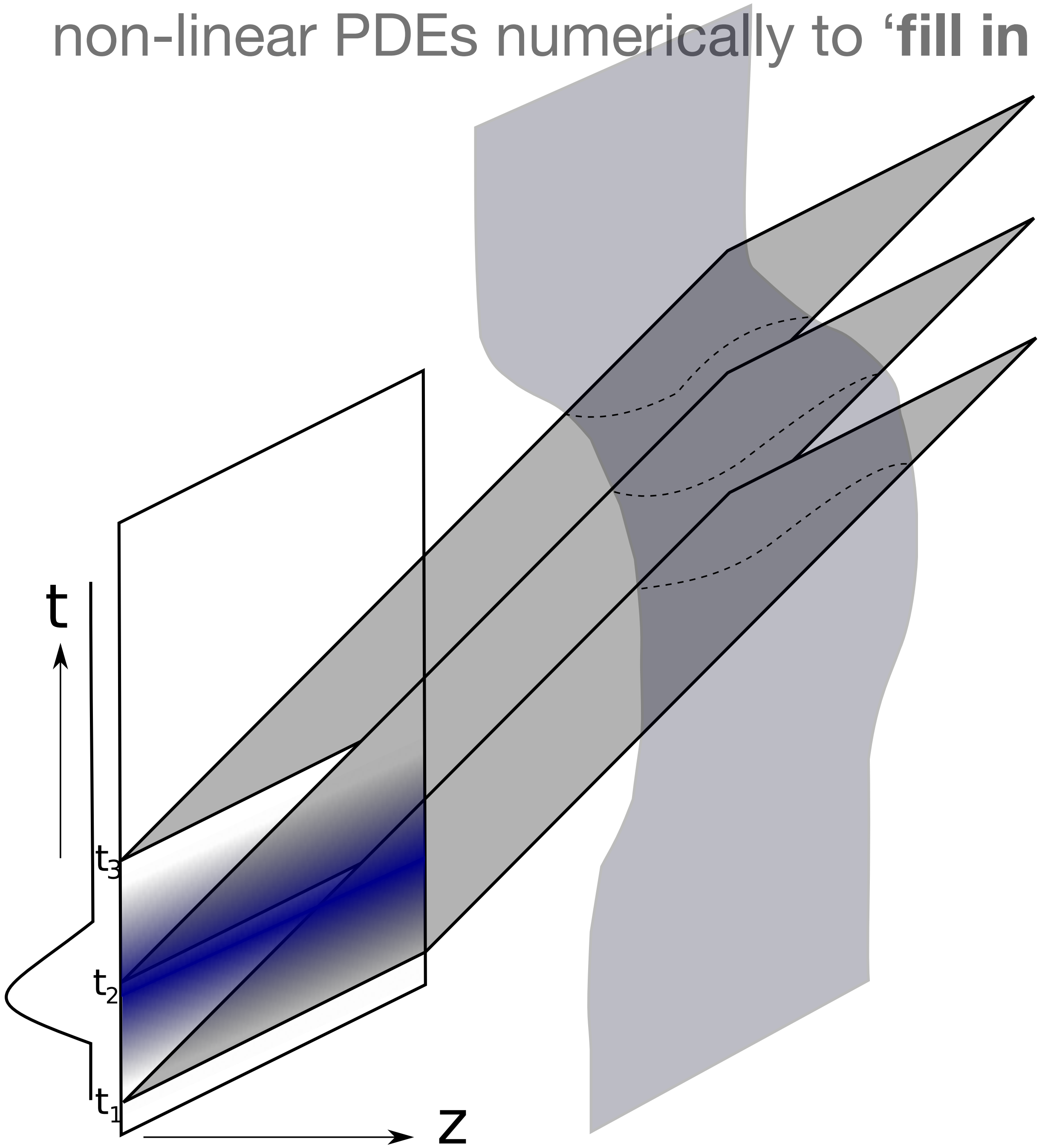
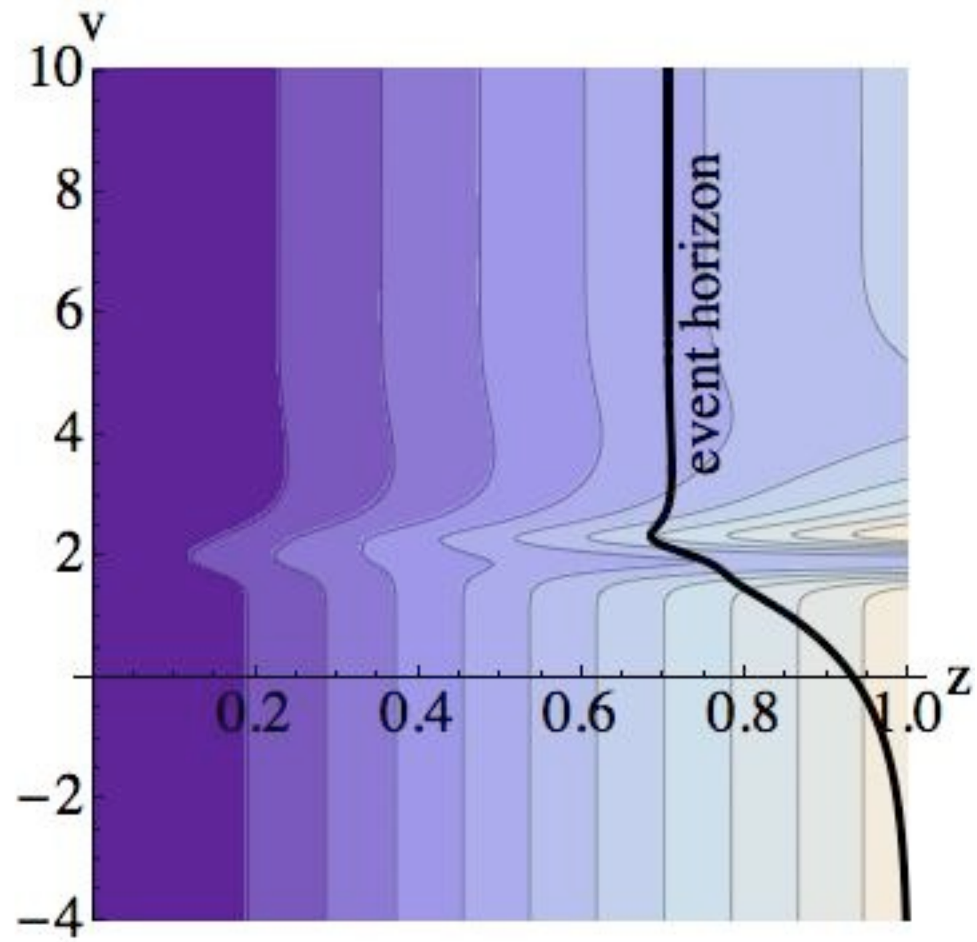
ads/cmt dynamics: numerical relativity

- We wish to model a quench **holographically**: prescribe a sudden change in some physical parameter of the theory on the **boundary** and then evolve the non-linear PDEs numerically to '**fill in the bulk**'





more details of the setup [related work: Murata, Kinoshita & Tanahashi, 2010]

- for simplicity: take **homogeneous** quench

$$ds^2 = \frac{1}{z^2}\left(-T(v,z)\,dv^2 - 2\,dvdz + S(v,z)^2 dx_i^2\right)$$

- then the complex scalar can be expressed as

$$\psi(v,z) \quad = \quad z\left(\psi_1(v) + \hat{\psi}(v,z)\right)$$

- and $\Psi_1(t)$ is the **source** at the boundary. Use a spike in the source to quench the system (can think of different systems and different quenches)

- solve system of **(1+1) non-linear PDE** by a pseudo-spectral method in spatial directions and 'Crank-Nicholson' finite differences in time direction

the resulting dynamics I

- The dynamics of this quench give rise to **three** distinct regimes

I. Oscillation

II.Decay to finite gap

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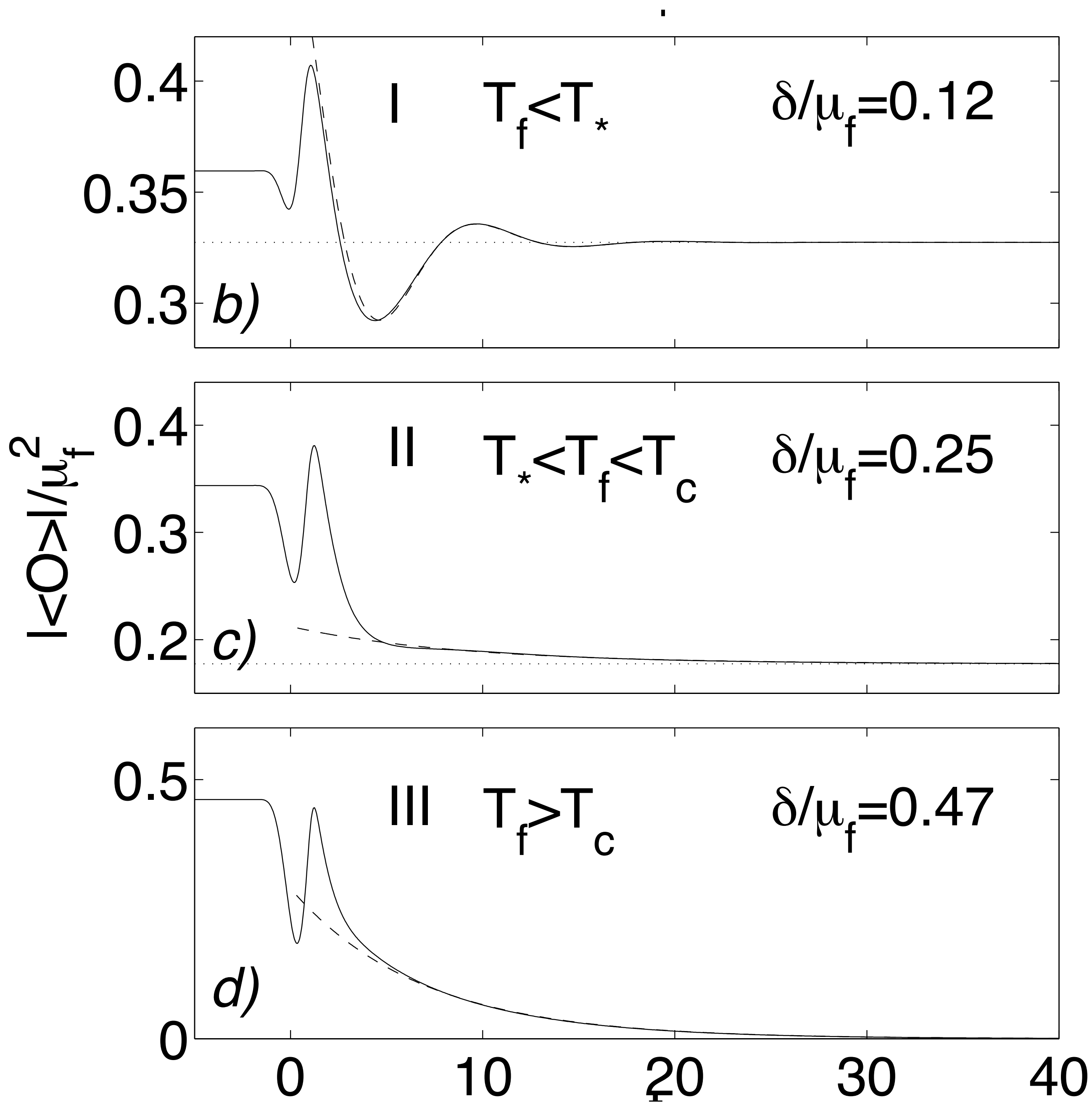
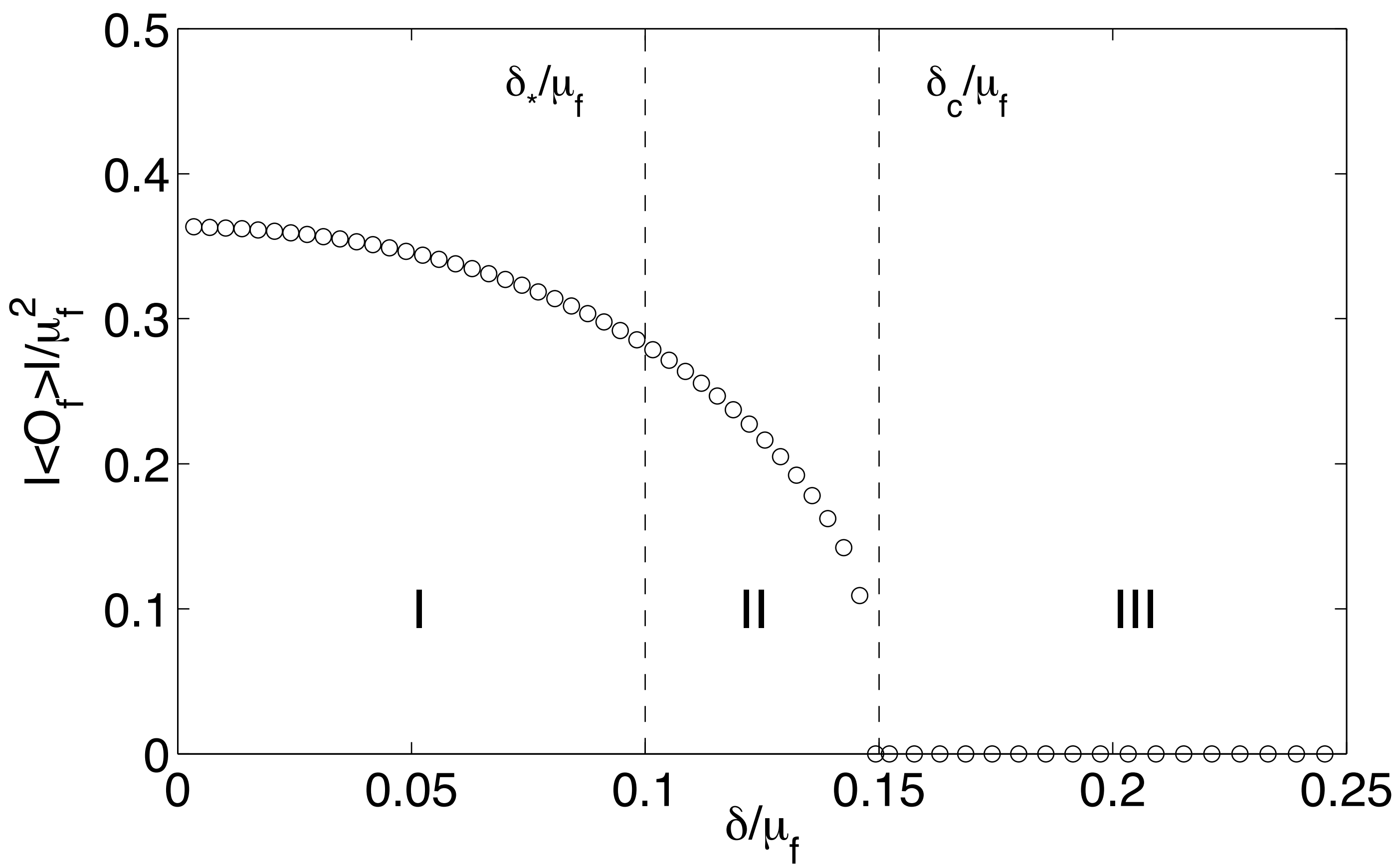


the resulting dynamics II

- we can dress the results up as a **dynamical phase diagram**



$$\psi_1(t) = \delta e^{-10(t-t_0)^2}$$

- BL-type analysis extended to include strong correlations, thermal damping,... we find similar behaviour: great! but why? and how?

clues from quasinormal mode structure

- let us study the structure of **quasi-normal modes** about the final state

$$\begin{aligned} \psi(v,z) &= \psi_0(z) + \delta\psi(v,z) \\ g_{ab}(v,z) &= g_{ab,0}(z) + \delta g_{ab}(v,z) \\ A(v,z) &= A_0(z) + \delta A(v,z) \end{aligned}$$

- deal with diffeo and U(1) gauge symmetry by defining **gauge-invariant variables** (c.f. cosmological perturbation theory)

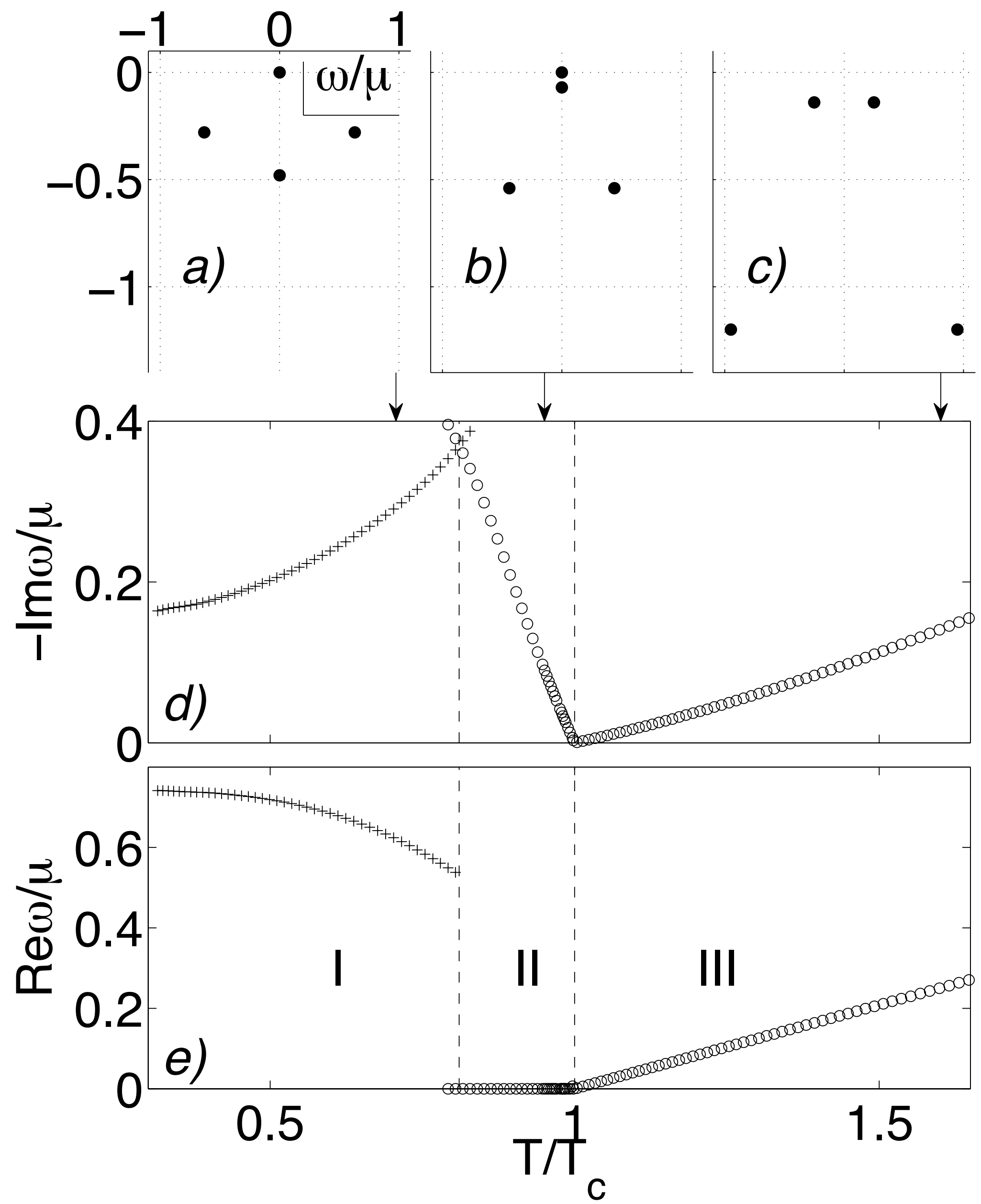
$$\delta\Phi_I(v,z) = e^{-i\omega v}\Phi_I^{\omega}(z)$$

- The **analytic structure** of the Φ tells us about a) late-time behaviour of observables b) poles in two-point functions of dual operators

quasi-normal mode structure

$$|\langle\mathcal{O}(t)\rangle| = |\langle\mathcal{O}_f\rangle + ce^{-i\omega_L t}|$$

- Off-axis poles lead to **oscillations** in **broken** phase
- Dynamics very well approximated by **leading QNM**
- Very good **quantitative agreement** with non-linear PDE code



quasi-normal pole dance

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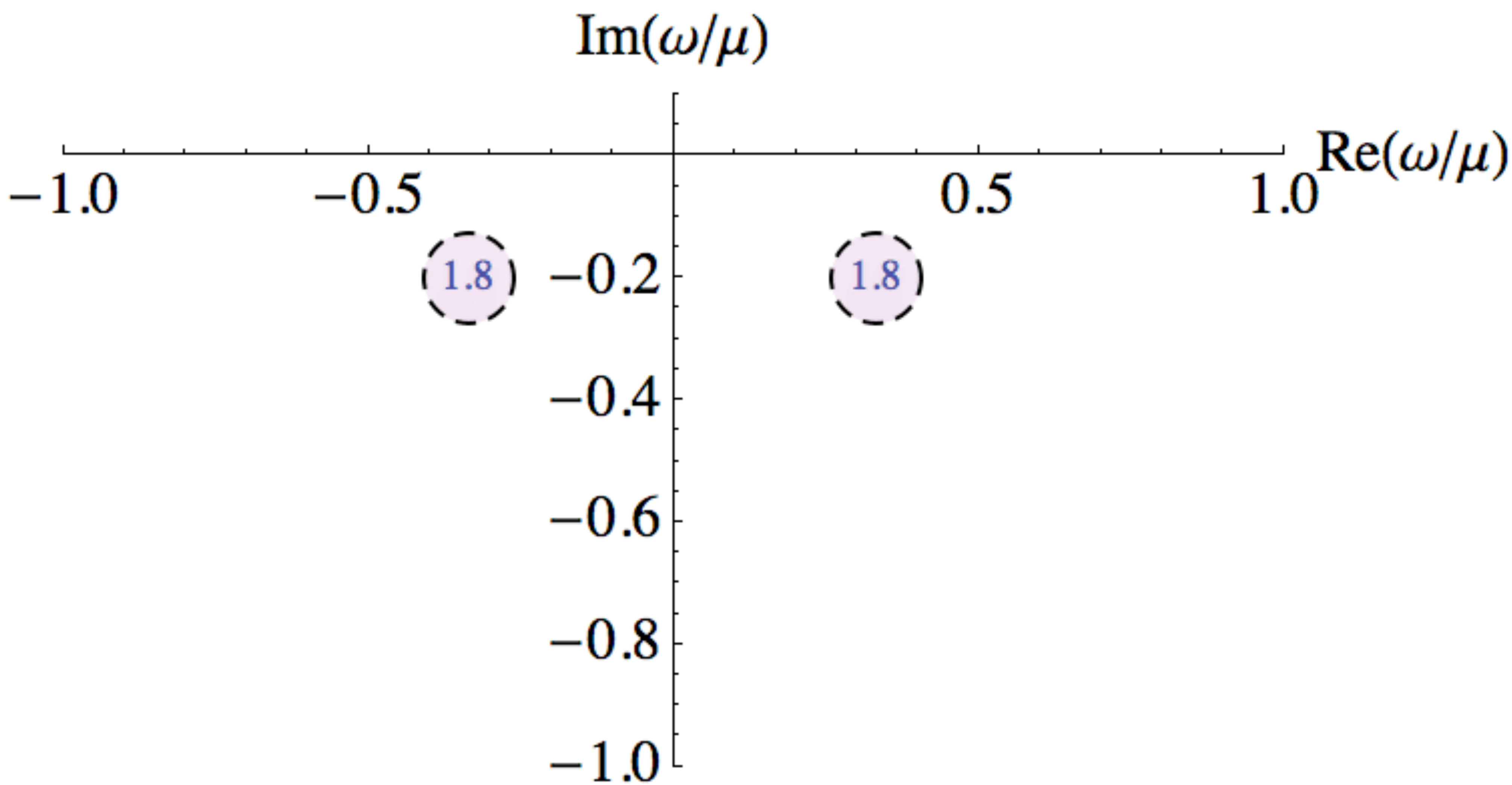

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dynamics of symmetry breaking

- T-reversal invariance means collective mode spectrum (manifested in our example as QNMs) must be **symmetric** under

$$\omega \to -\omega^*$$

- Poles in spectral function (and other observables) come in two varieties:
  a) pairs of poles **off** imaginary axis
  b) single poles **on** imaginary axis

1. S.c. phase transition: coalescence of two poles at TC at $\omega = 0$

2. Broken U(1) $\Rightarrow$ Single pole (i.e. mode) at $\omega = 0$ (Goldstone mode)

3. At T=0 no source of dissipation $\Rightarrow$ leading poles are oscillatory in nature

1 + 2 + 3 = BL dynamical phase diagram!

conclusions

- very interesting **far-from-equilibrium** problems are accessible at the intersection of numerical relativity and AdS/CFT.
  speculative comment: exact non-linear PDE methods may well be brought to bear on non-equilibrium field theory!

- simulated a **quantum quench** in ads/cmt: persistence of **BL phenomena** to strong coupling and in systems that thermalise makes it more likely to be observed in actual experiments

- in fact: our analysis shows that BL-type behaviour is completely **generic** for dynamical breaking of a local symmetry. This makes the **experimental point** even more emphatically.

- Are there different contexts? Higgs mechanism, early universe, you name it...