PARTITION FUNCTIONS OF $\mathcal{N}=(2,2)$ GAUGE THEORIES ON S^2 AND VORTICES

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Based on [arXiv: 1206.2356] with F. Benini

(Overlap with 1206.2606 Doroud, Gomis, Le Floch, Lee)

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THE PATH INTEGRAL OF QFT IS <u>HARD</u>.

→ PERTURBATIVE COMPUTATIONS AT WEAK COUPLING.

SUPERSYMMETRIC FIELD THEORIES ARE EASIER:
<u>EXACT COMPUTATIONS</u>.

IDEAL THEORETICAL LABS TO EXPLORE THE BEHAVIOUR OF QFT AT STRONG COUPLING.

INSIGHTS INTO:

- GAUGE THEORIES
- STRING THEORY
- GEOMETRY

EXACT RESULTS IN SUSY QFT

- Past : CHIRAL / HOLOMORPHIC $\qquad \bar{Q}_{\dot\alpha} \mathcal{O} = 0$

- Present : REAL / "HOLOM $\times$ $\overline{\text{HOLOM}}$." $\qquad$ [Pestun '07; ...]

① RIGID SUSY QFT ON CURVED MANIFOLDS

② SUPERSYMMETRIC LOCALISATION

$$\boxed{\langle \mathcal{O}_{BPS} \rangle \overset{\text{exactly}}{=} \text{Semiclassical approximation}}$$

$\mathcal{Q}\mathcal{O}_{BPS} = 0$

$Q\mathcal{O}' = \bar{Q}\mathcal{O}' = 0$

$Q = \epsilon^\alpha Q_\alpha \ , \ \bar{Q} = (\epsilon^c)^{\dot\alpha} \bar{Q}_{\dot\alpha}$

$\mathcal{Q} = Q + \bar{Q}$

ϵ: Killing spinor

NP : $\mathcal{Q} \longrightarrow \mathcal{Q}^N$ $\qquad \longleftarrow$ A - twist

SP : $\mathcal{Q} \longrightarrow \mathcal{Q}^S = (\mathcal{Q}^N)^\dagger$ $\qquad \longleftarrow$ $\bar{A}$ - twist

2d $\mathcal{N}=(2,2)$ SUPERSYMMETRY ON S^2

- $U(1)_{R_V}$
- VECTOR + CHIRAL MULTIPLETS

MOTIVATIONS

- Factorisation of partition functions
- String Theory : $\mathcal{N}=(2,2)$ GLSM $\xrightarrow{RG}$ NLSM
- 2d/4d : vortex strings and surface operators in 4d $\mathcal{N}=2$
- 2+4=6 : M5-branes on 4-manifolds

PLAN OF THE TALK

$\boxed{1}$ RIGID SUSY ON CURVED MANIFOLDS

1.1 - 2d $\mathcal{N}=(2,2)$ Supersymmetry with $U(1)_{R_V}$ on S^2

$\boxed{2}$ LOCALISATION

2.1 - Localisation on the Coulomb branch

2.2 - Localisation on the Higgs branch: vortex $\times$ $\overline{\text{vortex}}$

$\boxed{3}$ APPLICATIONS

RIGID SUSY ON CURVED MANIFOLDS

ⓐ BRUTE FORCE : correct flat space δ^{SUSY} and $\mathcal{L}$ in $1/r$ expansion.

$$\boxed{\begin{aligned} \delta^{SUSY}_{curved} &= \delta^{(0)}_{\nabla} + \sum_{n\geq 1} \frac{1}{r^n}\,\delta^{(n)} \\ \mathcal{L}^{SUSY}_{curved} &= \mathcal{L}^{(0)}_{\nabla} + \sum_{n\geq 1} \frac{1}{r^n}\,\mathcal{L}^{(n)} \end{aligned}}$$

(stops at $n=1$)

(stops at $n=2$)

ⓑ SYSTEMATIC [Festuccia-Seiberg '11]

- Couple FT to SUGRA
- Take RIGID LIMIT $M_{Pl}\to\infty$ with $g_{\mu\nu}$ fixed (scaling aux. fields).

Gravity multiplet OFF-SHELL, but SUSY:

$\boxed{\delta\psi^{\mu}_{\alpha}=0}$ $\longrightarrow$ KILLING SPINOR EQNS

$$\boxed{\begin{aligned} \delta^{SUGRA}(\text{matter}) &\longrightarrow \delta^{SUSY}_{curved}(\text{matter}) \\ \mathcal{L}^{SUGRA} &\longrightarrow \mathcal{L}^{SUSY}_{curved} \end{aligned}}$$

$1/r$ expansion : expansion in auxiliary fields.

§1.1 - 2d $\mathcal{N}=(2,2)$ SUSY WITH $U(1)_{R_V}$ ON S^2

- FT R-multiplet : $T^{\mu\nu}$, S^μ_α , R^μ , J , $\widetilde{J}$ [Dumitrescu Seiberg '11]
- SUGRA "New minimal" 2d : $g_{\mu\nu}$, ψ^α_μ , A^R_μ , H , $\widetilde{H}$

aux.

KILLING SPINOR EQNS

$$\boxed{\begin{aligned}(\nabla_\mu - iA^R_\mu)\,\varepsilon &= -\tfrac{1}{2}H\gamma_\mu\varepsilon - \tfrac{i}{2}\widetilde{H}\gamma_\mu\gamma_3\,\varepsilon \\ (\nabla_\mu + iA^R_\mu)\,\bar\varepsilon &= -\tfrac{1}{2}H\gamma_\mu\bar\varepsilon + \tfrac{i}{2}\widetilde{H}\gamma_\mu\gamma_3\,\bar\varepsilon\end{aligned}}$$

[Closset, Dumitrescu, Festuccia, Komargodski to appear]

ROUND S^2_r :

$$ds^2 = r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$A^R_\mu = 0\,, \qquad H = -\frac{i}{r}\,, \qquad \widetilde{H} = 0$$

$$\longrightarrow \quad \boxed{\begin{aligned}\nabla_\mu\varepsilon &= \frac{i}{2r}\gamma_\mu\varepsilon \\ \nabla_\mu\bar\varepsilon &= \frac{i}{2r}\gamma_\mu\bar\varepsilon\end{aligned}}$$

$\underline{SU(2|1)} \supset SU(2)_J \times U(1)_{R_V}$

[Benini, SC '12]

FIELD THEORY MULTIPLETS

- VECTOR $V=V^\dagger$:

$$\left|\begin{array}{l} A_\mu\,,\ \sigma+i\eta \\ D \end{array}\right. \qquad \begin{array}{c}\lambda \\ \bar\lambda\end{array} \qquad \longrightarrow \quad \mathcal{L}_{SYM}\,,\ \mathcal{L}_{FI,\theta}$$

- CHIRAL Φ : $\bar{D}_\alpha \Phi = 0$

$$\left|\begin{array}{l} \varphi \\ F \end{array}\right. \qquad \psi \qquad \longrightarrow \quad \begin{array}{l} \mathcal{L}_{matter}\,,\ \mathcal{L}_W \\ W(\Phi)\quad R_V\text{-charge } 2 \\ \mathcal{L}_W = \int d^2\theta\, W(\Phi) + c.c. \\ \qquad d^2\theta = d\theta^+ d\theta^- \end{array}$$

- TWISTED CHIRAL $\tilde{\Phi}$: $\bar{D}_+\tilde{\Phi} = D_-\tilde{\Phi} = 0$

$$\left|\begin{array}{l} \tilde\varphi \\ \tilde F \end{array}\right. \qquad \tilde\psi \qquad \longrightarrow \quad \begin{array}{l} \mathcal{L}_{matter}\,,\ \mathcal{L}_{\tilde W} \\ \mathcal{L}_{\tilde W} = \int d^2\tilde\theta\, \tilde{W}(\tilde\Phi) + c.c. \\ \qquad d^2\tilde\theta = d\theta^+ d\bar\theta^- \end{array}$$

VECTOR V $\longrightarrow$ TWISTED CHIRAL $\Sigma \sim \bar{D}_+ D_- V$:

$$\left|\begin{array}{l} \sigma+i\eta \\ D-iF_{12} \end{array}\right. \qquad \lambda$$

$$\left(F_{12} = \frac{1}{2}\varepsilon^{\mu\nu}F_{\mu\nu}\right)$$

SUPERSYMMETRIC ACTIONS ON S^2

- $\mathcal{L}_{SYM} = \frac{1}{2}\mathrm{Tr}\left\{ \left(F_{12}-\frac{\eta}{r}\right)^2 + \left(D+\frac{\sigma}{r}\right)^2 + (D_\mu\sigma)^2 + (D_\mu\eta)^2 - [\sigma,\eta]^2 + \text{fermions} \right\}$

$$\bar{\epsilon}\epsilon \int d^2x\sqrt{g}\,\mathcal{L}_{SYM} = \begin{matrix}\delta_\epsilon\delta_{\bar\epsilon}\\ \delta_{\bar\epsilon}\delta_\epsilon\end{matrix} \int d^2x\sqrt{g}\,\mathrm{Tr}\left(\frac{1}{2}\bar\lambda\lambda - 2D\sigma - \frac{1}{r}\sigma^2\right)$$

- $\mathcal{L}_{FI,\theta} = -i\xi D + i\frac{\theta}{2\pi}F_{12}$

$$\mathcal{L}_{\widetilde{W}} = i\widetilde{W}'(\sigma+i\eta)\cdot\left(D - iF_{12} + \frac{\sigma+i\eta}{r}\right) - \frac{i}{r}\widetilde{W}(\sigma+i\eta) - \frac{i}{2}\widetilde{W}''(\sigma+i\eta)\cdot\bar\lambda(1+\gamma_3)\lambda + \text{c.c.}$$

- $\mathcal{L}_{matter} = D_\mu\bar\Phi D^\mu\Phi + \bar\Phi\left(\sigma^2+\eta^2+iD+i\frac{R[\Phi]}{r}\sigma + \frac{R[\Phi](2-R[\Phi])}{4r^2}\right)\Phi + \bar{F}F + \text{fermions}$

$$\bar{\epsilon}\epsilon \int d^2x\sqrt{g}\,\mathcal{L}_{matter} = \begin{matrix}\delta_\epsilon\delta_{\bar\epsilon}\\ \delta_{\bar\epsilon}\delta_\epsilon\end{matrix} \int d^2x\sqrt{g}\left(\bar\Psi\Psi - 2i\bar\Phi\sigma\Phi + \frac{R[\Phi]-1}{r}\bar\Phi\Phi\right)$$

- $\mathcal{L}_W = \frac{\partial W}{\partial\phi^j}F^j - \frac{1}{2}\frac{\partial^2 W}{\partial\phi^j\partial\phi^k}\psi^j\psi^k + \text{c.c.}$

- <u>TWISTED MASSES</u>: couple to background V^{ext} ($\to\Sigma^{ext}$) for G_{global}.

$$\widetilde{m}^{ext} = \sigma^{ext} + i\eta^{ext} \quad \in \quad \mathrm{Cartan}(G_{global})$$

$$\left(D^{ext} - iF_{12}^{ext}\right)$$

LOCALISATION

$\underline{Q \text{ FERMIONIC SYMMETRY}: \quad QS[X] = 0}$. $\qquad Q^2 = L$

$$\rightarrow \langle QY \rangle = \int \mathcal{D}X\, (QY)\, e^{-S[X]} = \int \mathcal{D}X\, Q\left(Y e^{-S[X]}\right) = 0$$

$$\rightarrow \boxed{\langle O_{BPS} \rangle = \int \mathcal{D}X\, e^{-S[X]} O_{BPS} = \int \mathcal{D}X\, e^{-S[X] - tQ\mathcal{P}_F[X]}\, O_{BPS}}$$

$\forall t$ and $\mathcal{P}_F[X]$ s.t. $Q^2 \mathcal{P}_F[X] = L\mathcal{P}_F[X] = 0$

$Q O_{BPS} = 0$

- $\underline{t \to +\infty}$: saddle pts determined by $S_{loc}[X] = Q\mathcal{P}_F[X]$!

<u>If</u> $\mathcal{L}_{loc}\big|_{bosonic} = |Q\psi_x|^2 + |Q\psi_x^\dagger|^2$: saddle pts of S_{loc} = BPS config's $Q\psi_x = Q\psi_x^\dagger = 0$

$$X = \underset{BPS}{X_0} + \frac{1}{\sqrt{t}} \delta X$$

$$\boxed{\langle O_{BPS} \rangle = \int_{BPS} dX_0\, e^{-S[X_0]}\, O_{BPS}\big|_{X_0} \Big/ \mathrm{Sdet}\begin{bmatrix}\text{Quadratic} \\ \text{Kinetic op.}\end{bmatrix}}$$

- LOCALISATION SCHEME : choice of Q and S_{loc}.

LOCALISATION OF 2d $\mathcal{N}=(2,2)$ THEORIES WITH $U(1)_{R_V}$ ON S^2

- **AIM**: Compute $\quad Z_{S^2}[V^{ext}] = \int \mathcal{D}\Phi\, \mathcal{D}V\, e^{-S[\Phi, V, V^{ext}]}$.

- **LOCALISING SUPERCHARGE**

Select a Killing spinor ϵ_+: $\quad \nabla_\mu \epsilon_+ = \frac{i}{2r}\gamma_\mu \epsilon_+ \quad , \quad \epsilon_+^\dagger \epsilon_+ = 1$.

$\rightarrow$ Killing vector $v^\mu = \epsilon_+^\dagger \gamma^\mu \epsilon_+$: $\quad v = \frac{1}{r}\partial_\varphi$

$U(1)_n \subset SU(2)_n$

$\rightarrow$ Supercharge $\quad Q = \epsilon_+^\alpha Q_\alpha + (\epsilon_+^c)^\alpha Q_\alpha^\dagger$

SUPERALGEBRA $\quad \boxed{Q^2 = \left(M + \frac{R_V}{2}\right) + i\Lambda}$ $\qquad \underline{SU(1|1)} \supset U(1)_{M+\frac{R_V}{2}}$

gauge transf. $\Lambda = -\sigma + i\cos\theta\, \eta$

§ 2.1 - LOCALISATION ON THE COULOMB BRANCH

$$\boxed{S_{loc} = \int d^2x \sqrt{g} \left(\mathcal{L}_{SYM} + \mathcal{L}_{\psi} \right)}$$

$$\mathcal{L}_{SYM} = \frac{1}{4} Q \,\mathrm{Tr} \left(\overline{(Q\lambda)}\lambda + \lambda^\dagger \overline{(Q\lambda^\dagger)} \right) \longrightarrow \mathcal{L}_{SYM}\Big|_{bosonic} = \frac{1}{4} \mathrm{Tr} \left(|Q\lambda|^2 + |Q\lambda^\dagger|^2 \right)$$

$$\mathcal{L}_{\psi} = \frac{1}{2} Q \left(\overline{(Q\psi)}\psi + \psi^\dagger \overline{(Q\psi^\dagger)} \right) \longrightarrow \mathcal{L}_{\psi}\Big|_{bosonic} = \frac{1}{2} \left(|Q\psi|^2 + |Q\psi^\dagger|^2 \right)$$

<u>BPS CONFIGURATIONS</u>

- $\boxed{Q\lambda = Q\lambda^\dagger = 0}$: $\sigma + i\eta$ CONSTANT $\in$ Cartan(G), $D - iF_{12} = -\frac{\sigma + i\eta}{r}$.

  $\sigma \propto D$ CONTINUOUS $\longrightarrow \int \left(\prod_{i=1}^{r} d\sigma_i \right)$

  $\eta \propto F_{12}$ QUANTISED $\longrightarrow \sum_{m \text{ GNO}}$

  $\left(m = \frac{1}{2\pi} \int_{S^2} F = 2r^2 F_{12} \right)$

- $\boxed{Q\psi = Q\psi^\dagger = 0}$: $\phi = \bar{\phi} = F = \bar{F} = 0$. (smooth)

- **CLASSICAL** :

$$Z_{class}(\sigma, m; \xi, \theta) = e^{-S[X_0]} = e^{-4\pi i \xi \operatorname{Tr}\sigma - i\theta \operatorname{Tr} m}$$

- **1-LOOP DETERMINANTS** :

$$Z_{1\text{-}loop}(\sigma, m; \tau, n) = Z_{vector}(\sigma, m) \prod_{\Phi_i \, chiral} Z_{\Phi_i}(\sigma, m; \tau, n)$$

$$Z_{vector}(\sigma, m) = \prod_{\alpha \in \Delta_+} \left(\frac{\alpha(m)^2}{4} + \alpha(\sigma)^2 \right)$$

$$Z_{\Phi_i}(\sigma, m; \tau, n) = \prod_{\rho \in R_{\Phi_i}} \frac{\Gamma\left(\left(\frac{R[\Phi_i]}{2} - i\rho(\sigma) - i f^a[\Phi_i]\tau_a \right) - \frac{\rho(m) + f^a[\Phi_i] n_a}{2} \right)}{\Gamma\left(1 - \left(\quad '' \quad \right) - \quad '' \quad \right)}$$

$$\boxed{Z_{S^2}(\xi, \theta; \tau, n) = \frac{1}{|W_G|} \sum_{\substack{m \\ GNO}} \int \left(\prod_{i=1}^{r} d\sigma_i \right) Z_{class}(\sigma, m; \xi, \theta) \, Z_{1\text{-}loop}(\sigma, m; \tau, n)}$$

§ 2.2 – LOCALISATION ON THE HIGGS BRANCH

Evaluate Coulomb branch representation of Z_{S^2} for $U(N)$:

$$\boxed{Z_{S^2} = \sum_{\substack{\text{Isolated}\\ \text{vacua}\\ \text{(roots of Higgs branch)}}} e^{-S}\, Z'_{1\text{-loop}}\left(\begin{smallmatrix}\text{massive}\\ \text{fields}\end{smallmatrix}\right) Z_{\text{vortex}}(\ldots)\, Z_{\overline{\text{vortex}}}(\ldots)}$$

similar to [Pasquetti '11] for $U(1)$ on S^3_b

This FACTORISATION can be proven more generally : new LOCALISATION SCHEME .

- If the gauge group has a central $U(1)$:

$$\boxed{S'_{loc} = \int d^2x \sqrt{g}\, (\mathcal{L}_{SYM} + \mathcal{L}_\psi + \mathcal{L}_H)}\,, \qquad \boxed{\mathcal{L}_H = Q\, \mathrm{Tr}\left[\frac{\epsilon_+^\dagger \lambda - \lambda^\dagger \epsilon_+}{2i}\,(\phi\phi^\dagger - \chi \mathbb{1})\right]}$$

χ : "FAKE" FI PARAMETER

- Change path integration contour of D and integrate it out :

$$\boxed{\left(D + \frac{\sigma}{r}\right) + i(\phi\phi^\dagger - \chi\mathbb{1}) = 0}$$

$\rightarrow \mathcal{L}'_{loc}\Big|_{\text{bosonic}}$ reduces to a sum of squares.

SADDLE POINTS OF S'_{loc}

① $\boxed{\text{$\chi$-deformed COULOMB BRANCH}}$:

σ CONSTANT $\in$ Cartan(G)
m QUANTISED $\in$ Cartan(G)

$\rightarrow \displaystyle\sum_{\substack{m \\ \text{GNO}}} \int \left(\prod_{i=1}^{r} d\sigma_i \right)$

$\eta = r\chi\cos\theta + \dfrac{m}{2r}$
$F_{\eta} = 2\chi\cos\theta + \dfrac{m}{2r^2}$

$\phi = \bar{\Phi} = F = \bar{F} = 0$.

$Z^{\text{Coulomb}}_{\text{1-loop}}(\chi;\ldots)$ depends on χ due to $Z^{\text{Coulomb}}_{\Phi_i}(\chi;\ldots)$.

<u>LIMITS</u>

- $\chi \rightarrow 0$: $Z^{\text{Coulomb}}_{\text{1-loop}}(\chi;\ldots) \longrightarrow Z_{\text{1-loop}}(\ldots)$

- $\chi \rightarrow \begin{cases} \operatorname{sign}\left(\sum_i Q_i\right)\cdot\infty & , \text{ if } \sum_i Q_i \neq 0 \\ \pm\infty & , \text{ if } \sum_i Q_i \;\; 0 \end{cases}$: $Z^{\text{Coulomb}}_{\text{1-loop}}(\chi;\ldots) \longrightarrow 0$!

(Q_i : charge of Φ_i under $U(1)_{\text{gauge}}$)

② $\boxed{\text{HIGGS BRANCH}}$: $\sigma + i\eta$ CONSTANT $\in$ Cartan(G) $\quad F_{12} = \frac{\eta}{r}$

$(\chi \neq 0)$ $\quad F = 0 \;,\quad D_\mu \phi = 0 \;,\quad \eta\phi = 0 \;,\quad \underline{(\sigma + \tau)\phi = 0 \;,\quad \phi\phi^\dagger = \chi \mathbb{1}}$.

(→ Continuous σ and nontrivial magnetic flux only for unbroken subgroup of G.
We are interested in complete breaking; no mixed branches.)

* Example: U(N) with N_f fundamentals Φ, N_a antifundamentals $\tilde{\Phi}$.

• $\chi > 0$: $\quad \Phi = \sqrt{\chi}\,\left(\mathbb{1}_{N\times N},\; 0_{N\times(N_f - N)}\right)$

$\underline{\sigma_a = -\tau_{\ell_a}}$, $\vec{\ell} \in C(N, N_f)$

Roots of Higgs branch

if $\underline{N_f \geq N}$.

$\tilde{\Phi} = 0_{N_a \times N}$.

• $\chi < 0$: $\quad \tilde{\Phi} = \sqrt{-\chi}\begin{pmatrix} \mathbb{1}_{N\times N} \\ 0_{(N_a - N)\times N} \end{pmatrix}$

$\underline{\sigma_a = \tilde{\tau}_{\ell_a}}$ $\quad \vec{\ell} \in C(N, N_a)$

if $\underline{N_a \geq N}$.

$\Phi = 0_{N \times N_f}$.

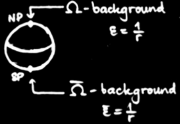
We are interested in the $\chi \to \infty$ limit which suppresses $Z^{\text{Coulomb}}(\chi; \dots)$: path integral localizes onto isolated Higgs vacua, along with

③ $\boxed{\text{POINTLIKE } \frac{\text{VORTICES}}{\overline{\text{VORTICES}}} \text{ AT } \begin{matrix}\text{NP} \\ \text{SP}\end{matrix}}$ $(|\chi| \to \infty)$

NP $\theta = 0$: $\boxed{\left(D + \frac{\sigma}{r}\right) + i\left(\phi\phi^\dagger - \chi \mathbb{1}\right) = +iF_{12} \,, \quad D_-\phi = 0}$ BPS vortices

SP $\theta = \pi$: $\boxed{\left(D + \frac{\sigma}{r}\right) + i\left(\phi\phi^\dagger - \chi \mathbb{1}\right) = -iF_{12} \,, \quad D_+\phi = 0}$ $\overline{\text{vortices}}$

(σ as in Higgs branch)



Size $L \sim \frac{r}{\sqrt{|\chi|}} \xrightarrow[\chi \to \infty]{} 0$ pointlike

$\xrightarrow[\chi \to 0]{} \infty$ $(\phi \to 0)$ Coulomb

$$\boxed{Z_{S^2} = \sum_{\substack{\vec{\ell}:\text{ isolated} \\ \text{roots of} \\ \text{Higgs branch}}} e^{-S} \, Z'_{1\text{-loop}}\left(-i m^{\vec{\ell}}_{\text{eff}}\right) Z_{\text{vort}}\left((-1)^N z; \epsilon = \tfrac{1}{r}; -i m^{\vec{\ell}}_{\text{eff}}\right) Z_{\text{vort}}\left((-1)^N \bar{z}; \epsilon = -\tfrac{1}{r}; +i m^{\vec{\ell}}_{\text{eff}}\right)}$$

$$z = e^{-2\pi\xi - i\theta} = S^{\text{BPS}}_{1\text{-vortex}}$$

APPLICATIONS

- **2d $\mathcal{N}=(2,2)$ GAUGE THEORIES**

  - Compute new observables
  - Test / Infer dualities
  - Study phases

[Hanany-Hori '97, Hori-Tong '06]

- **SUPERSTRINGS ON CY_3**

2d $\mathcal{N}=(2,2)$ GLSM $\xrightarrow{\text{RG flow}}$ 2d $\mathcal{N}=(2,2)$ NLSM

[Witten '93]

Complexified FI — Complexified Kähler

Gauge instantons (vortices) — Worldsheet instantons

$$\boxed{Z_{S^2}\Big|_{v^{ext}=0} = e^{-K}}$$

[Jockers, Kumar, Lapan, Morrison, Romo '12]

[Gomis, Lee '12]

K: KÄHLER POTENTIAL ON KÄHLER MODULI SPACE, EXACT IN α' (CLASSICAL IN g_s).

$\rightarrow$ Genus 0 GW invariants.

NO NEED OF MIRROR SYMMETRY!

• 2d/4d CORRESPONDENCES

[Dorey '98; Hanany-Tong '04; ...]

Worldsheet dynamics of $\frac{1}{2}$-BPS vortex strings / surface operators in 4d $\mathcal{N}=2$.

- Probes of 4d physics

- Relation with Toda CFT via AGT for surface operators by decoupling 4d dynamics: [Alday, Gaiotto, Gukov, Tachikawa, Verlinde '09]

$$Z_{S^2} \quad = \quad \text{4-pt correlator } A_{N-1} \text{ Toda on sphere}$$

[Doroud et al]

2d $\mathcal{N}=(2,2)$ SQED

N flavours (charges ± 1)

(punctures: 2 maximal
1 semi-deg.
1 deg)

from 4d $\mathcal{N}=2$ U(N) with 2N hypers, 1-vortex string.

SPHERICAL PARTITION FUNCTIONS OF 2d $\mathcal{N}=(2,2)$ THEORIES :

- EASY TO EVALUATE

- ENCODE A GREAT DEAL OF INFORMATION ON
  - 2d Physics
  - 4d Physics
  - Type II String
  - Enumerative Geometry of Kähler manifolds

MORE TO BE UNCOVERED !