Probing strongly coupled gauge theories with AdS/CFT: the violation of the η/s bound

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0812.3572 , 0903.3244, 0910.5159

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Window into Strong Coupling

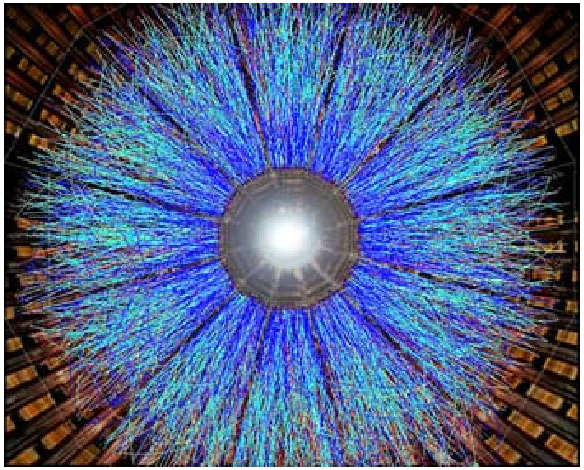
More than a decade of AdS/CFT:

- Deeper insight into gauge/gravity duality
  (e.g. microscopic constituents of black holes)

- A new way of thinking about strongly coupled gauge theories

Powerful tool to investigate
thermal and hydrodynamic properties
of field theories at strong coupling

Probing non-equilibrium strongly coupled gauge theories



- RHIC → probing behavior of strongly coupled QCD plasma (dynamics, transport coefficients)
- Theoretical tools for studying such systems limited :
  - Lattice simulations work well for static (equilibrium) processes
  - Dynamics? Lattice methods much less effective.
- Why AdS/CFT?

  window into *non-equilibrium* processes

Weak/strong coupling duality :

$\boxed{\text{Type IIB on } AdS_5 \times S^5}$ $\longleftrightarrow$ $\boxed{D=4\ \mathcal{N}=4 \text{ SYM}}$

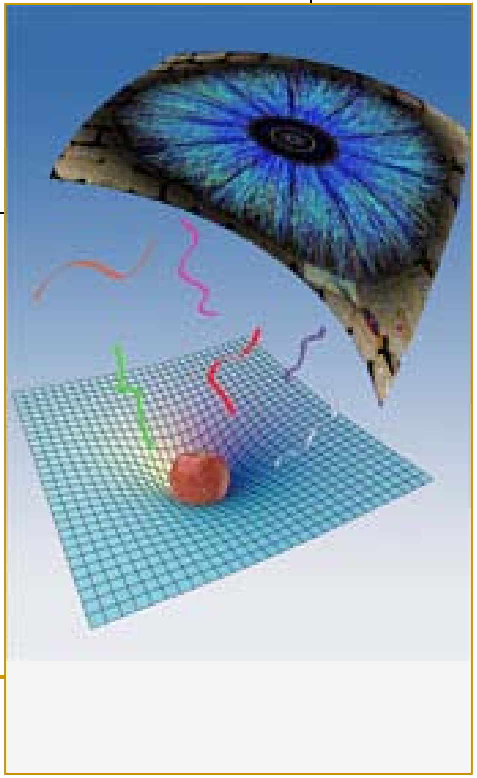
Insight into the Quark Gluon Plasma?

Can we use CFTs to study properties of QCD?

$\mathcal{N} = 4$ SYM at finite temperature is NOT QCD but:

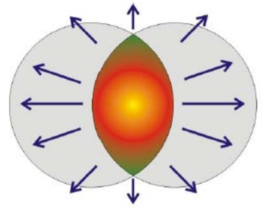
- **Some features *qualitatively* similar to QCD (for T ~ T_c - $3T_c$)**
  - **nearly conformal (small bulk viscosity away from T_c)**
- **Some properties of the plasma may be *universal***

shear viscosity to entropy ratio
bulk viscosity bound

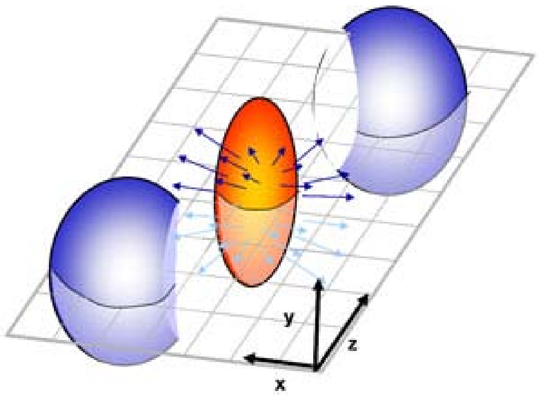


such generic relations might provide
INPUT into realistic simulations of sQGP

Elliptic Flow at RHIC



Off-central heavy-ion collisions at RHIC:



Anisotropic Flow
(large pressure gradient
in horizontal direction)

"Elliptic flow" ability of matter to flow freely locally

shear viscosity

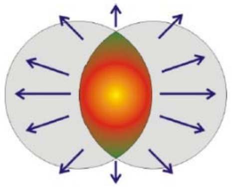
Well described by hydrodynamical calculations with
very small shear viscosity/entropy density ratio -- "perfect fluid"

RHIC data favors $0 < \eta/s < 0.3$

- D. Teaney nucl-th/0301099
- Luzum, Romatschke 0804.4015
- H. Song, U.W. Heinz 0712.3715

(different fireball initial conditions)

Nearly ideal, strongly coupled QGP



Contrast to <u>weak coupling</u> calculations in thermal gauge theories (Boltzmann eqn)

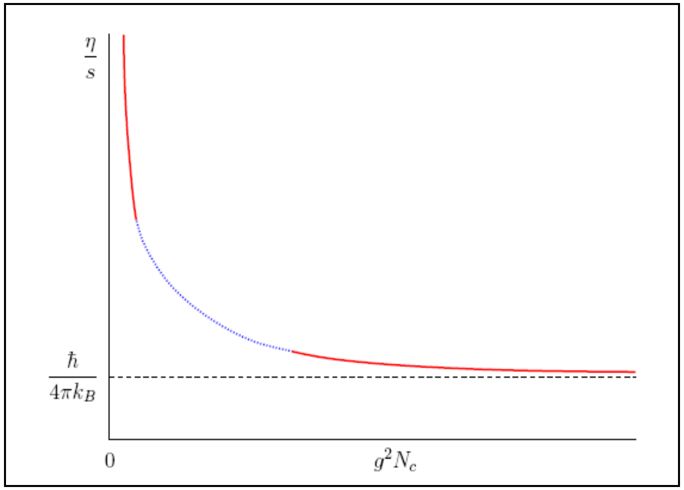
$$\eta \sim \frac{N_c T^3}{\lambda^4 \log 1/\lambda^2}$$

$$S \sim N_c T^3 V_3$$

$$\boxed{\frac{\eta}{s} \sim \frac{1}{\lambda^4 \log 1/\lambda^2} \gg 1}$$

Weak Coupling Prediction

η/s << 1 **Strong Coupling Regime**



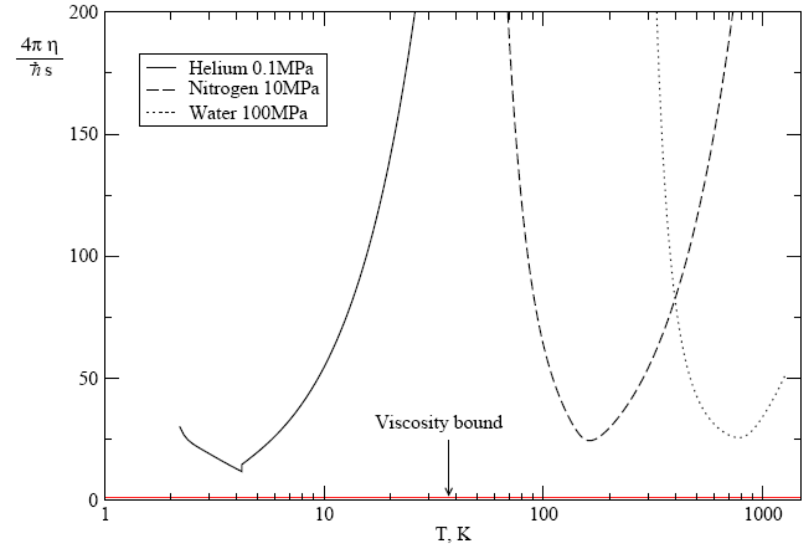
Strong coupling → natural setting for AdS/CFT applications

Shear Viscosity/Entropy Bound

Evidence from AdS/CFT:

- **Conjectured lower bound for field theory at finite T (Kovtun, Son, Starinets 0309213)**

$$\boxed{\frac{\eta}{s} \geq \frac{1}{4\pi}}$$



Fundamental in nature?

lower than any observed fluid

- **Gauge theories with Einstein GR dual saturate the bound (Buchel, Liu th/0311175)**

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi} \sim .08}$$

The RHIC value is at most a few times

Corrections to the Bound

Bound saturated in leading SUGRA approximation

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim .08$$

String theory corrections ?

- **Leading α' correction on $AdS_5 \times S^5$ ($\mathcal{N} = 4$ SYM) increased the ratio**

  (Buchel, Liu, Starinets th/0406264)

$$\frac{\eta}{s} = \frac{1}{4\pi}[1 + 15\zeta(3)\lambda^{-3/2} + \ldots]$$

- **Possible bound violations ? YES**

  Brigante et al, arXiv:0712.0805; Kats & Petrov, arXiv:0712.0743

$$I = \frac{1}{16\pi G_N}\int d^5x \sqrt{-g}\left[R - 2\Lambda + \frac{\lambda_{GB}}{2}L^2(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\right]$$

$$\Longrightarrow \quad \frac{\eta}{s} = \frac{1}{4\pi}[1 - 4\lambda_{GB}]$$

String Construction Violating the Bound

Kats & Petrov (arXiv:0712.0743)

- Type IIB on $\mathbf{AdS_5 \times S^5/\mathbb{Z}_2}$
- Decoupling limit of N D3's sitting inside 8 D7's coincident on O7 plane

$$S = \int d^D x \sqrt{-g} \left(\frac{R}{2\kappa} - \Lambda + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right)$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[1 - 4(D-4)(D-1) \frac{c_3}{L^2/\kappa} \right]$$

Violation

for $c_3 > 0$

- Couplings determined by (fundamental) matter content of the theory

(Buchel et al. arXiv:0812.2521 for more examples of CFTs violating bound)

Outline for rest of talk

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- Explore string theory corrections with finite (R-charged) chemical potential (D=5 N = 2 gauged SUGRA, SUSY completion of R^2 terms)

- Effects on thermodynamics and hydrodynamics (shear viscosity)

- At two-derivative level, chemical potential does not affect η/s
  - With higher derivatives?
  - Is bound restored for sufficiently large chemical potential?

- Bound is violated AND R-charge makes violation worse

- Possible connection with fundamental GR constraints (weak GR conjecture)

Why explore higher derivative corrections?

$$\mathcal{L} = R - \frac{1}{2n!} F_n^2 + \ldots + \alpha' R^2 + \alpha'^2 R^3 + \alpha'^3 R^4 + \ldots$$

Supergravity is only an effective low-energy description of string theory

- Higher derivative corrections are natural from the point of view of EFT
- Interesting applications to black hole physics (smoothing out singularities of small black holes)

From more "phenomenological" point of view:

- Corrections might bring observable quantities closer to observed values

Pathologies of higher derivative gravity?

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2 + \alpha_3 R_{\mu\nu\rho\sigma}^2 + \ldots$$

Higher derivative corrections can lead to undesirable features:

- **Modify graviton propagator**
- **ill-poised Cauchy problem (no generalization of Gibbons-Hawking term)**

Both issues related to presence of <u>four-derivative terms</u>.

However:

- **pathologies show up only at the Planck scale**
- **α_i perturbative parameters →** **generalization of Gibbons-Hawking term, boundary counterterms**

$$\boxed{\mathcal{L} = R - \frac{1}{4}F^2 - \Lambda + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2 + \alpha_3 R_{\mu\nu\rho\sigma}^2}$$

arXiv:0910.5159

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Corrections to η/s at finite chemical potential

arXiv:0903.3244
S.C., K. Hanaki,
J.Liu, P. Szepietowski

Role of R-charge chemical potential on η/s ?

D=5 $\mathcal{N}=2$ gauged SUGRA

- To leading order:

$$\mathcal{L}_0 = -R - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}F_{\mu\nu}F_{\rho\lambda}A_\sigma + 12g^2$$

gauged SUGRA coupling constant

- Corrections start at R^2 (sensitive to amount of SUSY)
  → include mixed gauge-gravitational CS term $\mathbf{A} \wedge \mathbf{Tr}(\mathbf{R} \wedge \mathbf{R})$

- R^2 terms in principle can be derived directly from string theory
- → would require specific choice of string compactification (Sasaki-Einstein)

SUSY R^2 terms in 5D

- Instead make use of SUSY (Hanaki, Ohashi, Tachikawa, hep-th/0611329)

SUSY completion of mixed CS term

$$\mathbf{A} \wedge \mathbf{Tr}(\mathbf{R} \wedge \mathbf{R})$$

coupled to arbitrary # of vector multiplets

Off-shell formulation of N=2, D=5 SUGRA (superconformal formalism)

gauge invariance under superconformal group → enlarging the symmetry facilitates construction of invariant action

End result

off shell action, lots of auxiliary fields, supersymmetric curvature-squared term in 5D

Role of SUSY-complete R^2 terms on bound violation ?

Off-shell Lagrangian, N=2, D=5 gauged SUGRA

Physical fields $g_{\mu\nu}$ A^I_μ M^I $\mathcal{N} = \frac{1}{6} c_{IJK} M^I M^J M^K$

Auxiliary fields D $v_{\mu\nu}$ V^{ij}_μ Y^I_{ij} $\mathcal{A}^\alpha_i$

$$\mathcal{L}_0 = \boxed{\tfrac{1}{4} D(2\mathcal{N} + \mathcal{A}^2)} + \boxed{R\left(\tfrac{3}{8}\mathcal{A}^2 - \tfrac{1}{4}\mathcal{N}\right)} + v^2(3\mathcal{N} - \tfrac{1}{2}\mathcal{A}^2)$$
$$+2\mathcal{N}_I v^{\mu\nu} F^I_{\mu\nu} + \mathcal{N}_{IJ}\left(\tfrac{1}{4} F^I_{\mu\nu} F^{J\,\mu\nu} - \tfrac{1}{2}\mathcal{D}^\mu M^I \mathcal{D}_\mu M^J\right) + \tfrac{1}{24} c_{IJK}\epsilon^{\mu\nu\rho\lambda\sigma} A^I_\mu F^J_{\nu\rho} F^K_{\lambda\sigma}$$
$$-\mathcal{N}_{IJ} Y^I_{ij} Y^{J\,ij} + 2\left[\mathcal{D}^\mu \mathcal{A}^{\bar{\alpha}}_i \mathcal{D}_\mu \mathcal{A}^i_\alpha + \mathcal{A}^{\bar{\alpha}}_i (g\,M)^2 \mathcal{A}^i_\alpha + 2g Y^{ij}_{\alpha\beta} \mathcal{A}^{\bar{\alpha}}_i \mathcal{A}^\beta_j\right].$$

D equation of motion

Canonical EH term

Scalars parametrize a *very special* manifold

Integrating out auxiliary fields →

$$\mathcal{L} = -R - \tfrac{3}{4} F^2_{\mu\nu} + \tfrac{1}{4}\epsilon^{\mu\nu\rho\lambda\sigma} A_\mu F_{\nu\rho} F_{\lambda\sigma} + 12 g^2$$

Off-shell Lagrangian, N=2, D=5 gauged SUGRA

Physical fields $g_{\mu\nu} \quad A^I_\mu \quad M^I \qquad \mathcal{N} = \frac{1}{6} c_{IJK} M^I M^J M^K$

Auxiliary fields $D \quad v_{\mu\nu} \quad V^{ij}_\mu \quad Y^I_{ij} \quad \mathcal{A}^\alpha_i$

$$
\begin{aligned}
e^{-1}\mathcal{L}_1^{\rm ungauged} = \frac{1}{24} c_{2I} \Big[& \frac{1}{16}\epsilon_{\mu\nu\rho\lambda\sigma} A^{I\,\mu} R^{\nu\rho\alpha\beta} R^{\lambda\sigma}{}_{\alpha\beta} + \frac{1}{8} M^I C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{1}{12} M^I D^2 + \frac{1}{6} F^I_{\mu\nu} v^{\mu\nu} D \\
& - \frac{1}{3} M^I C_{\mu\nu\rho\sigma} v^{\mu\nu} v^{\rho\sigma} - \frac{1}{2} F^{I\,\mu\nu} C_{\mu\nu\rho\sigma} v^{\rho\sigma} + \frac{8}{3} M^I v_{\mu\nu} \nabla^\nu \nabla_\rho v^{\mu\rho} \\
& - \frac{16}{9} M^I v^{\mu\rho} v_{\rho\nu} R^\nu_\mu - \frac{2}{9} M^I v^2 R + \frac{4}{3} M^I \nabla^\mu v^{\nu\rho} \nabla_\mu v_{\nu\rho} + \frac{4}{3} M^I \nabla^\mu v^{\nu\rho} \nabla_\nu v_{\rho\mu} \\
& - \frac{2}{3} M^I \epsilon_{\mu\nu\rho\lambda\sigma} v^{\mu\nu} v^{\rho\lambda} \nabla_\delta v^{\sigma\delta} + \frac{2}{3} F^{I\,\mu\nu} \epsilon_{\mu\nu\rho\lambda\sigma} v^{\rho\delta} \nabla_\delta v^{\lambda\sigma} + F^{I\,\mu\nu} \epsilon_{\mu\nu\rho\lambda\sigma} v^\rho{}_\delta \nabla^\lambda v^{\sigma\delta} \\
& - \frac{4}{3} F^{I\,\mu\nu} v_{\mu\rho} v^{\rho\lambda} v_{\lambda\nu} - \frac{1}{3} F^{I\,\mu\nu} v_{\mu\nu} v^2 + 4 M^I v_{\mu\nu} v^{\nu\rho} v_{\rho\lambda} v^{\lambda\mu} - M^I (v^2)^2 \Big] , \\
e^{-1}\mathcal{L}_1^{\rm gauged} = \frac{1}{24} c_{2I} \Big[& -\frac{1}{12}\epsilon_{\mu\nu\rho\lambda\sigma} A^{I\,\mu} R^{\nu\rho\,ij}(U) R^{\lambda\sigma}_{ij}(U) \\
& - \frac{1}{3} M^I R^{\mu\nu\,ij}(U) R_{\mu\nu\,ij}(U) - \frac{4}{3} Y^I_{ij} v_{\mu\nu} R^{\mu\nu\,ij}(U) \Big] ,
\end{aligned}
$$

On-shell Lagrangian (minimal SUGRA)

arXiv:0812.3572

S.C., K. Hanaki, J.Liu, P. Szepietowski

Truncation to minimal SUGRA

$$M^I = \bar{M}^I + c_2 \hat{M}^I, \qquad A^I_\mu = \bar{M}^I A_\mu, \qquad c_2 \equiv c_{2I}\bar{M}^I$$

$$\begin{aligned}
\mathcal{L} \; = \; & -R - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\left(1 - \frac{1}{6}c_2 g^2\right)\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu F_{\nu\rho}F_{\lambda\sigma} + 12g^2 \\
& + \frac{c_2}{24}\left[\frac{1}{48}RF^2 + \frac{1}{576}(F^2)^2\right] + \mathcal{L}_1^{\text{ungauged}},
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_1^{\text{ungauged}} = \frac{c_2}{24}\Big[& \boxed{\frac{1}{16\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} + \frac{1}{8}C^2_{\mu\nu\rho\sigma}} + \frac{1}{16}C_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda} - \frac{1}{3}F^{\mu\rho}F_{\rho\nu}R^\nu_\mu \\
& - \frac{1}{24}RF^2 + \frac{1}{2}F_{\mu\nu}\nabla^\nu\nabla_\rho F^{\mu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\mu F_{\nu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\nu F_{\rho\mu} \\
& + \frac{1}{32\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}F^{\mu\nu}(3F^{\rho\lambda}\nabla_\delta F^{\sigma\delta} + 4F^{\rho\delta}\nabla_\delta F^{\lambda\sigma} + 6F^\rho{}_\delta\nabla^\lambda F^{\sigma\delta}) \\
& + \frac{5}{64}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} - \frac{5}{256}(F^2)^2\Big]. \qquad (55)
\end{aligned}$$

Physical Meaning of c_2?

- **Parameters of 5D action contain info about 10D description (<u>string theory inputs</u>)**

$$\mathcal{L} = \frac{1}{16\pi G_5}\Big[R - \frac{1}{4}F^2 + 12g^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}A_\mu F_{\nu\rho}F_{\sigma\lambda} + \frac{c_2}{96\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} + \ldots\Big]$$

- **Ungauged case (e.g. D=11 SUGRA on CY_3) c_2 related to topological data (2^{nd} Chern class)**

- **Gauged case:**

  $c_2 = 0$ for IIB on S^5 (no R^2 terms with maximal sugra)

  For us: IIB on Sasaki-Einstein → meaning of c_2 less clear

<u>We can use AdS/CFT to relate c_2 to central charges of dual CFT via:</u>

- **Holographic trace anomaly**
- **R-current anomaly**

Using the dual CFT ($\mathcal{N}$=1)

- **4D CFT central charges $\boxed{a\ ,\ c}$ defined in terms of trace anomaly:**
  (CFT coupled to external metric)

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} C - \frac{a}{16\pi^2} E$$

$$\langle T^\mu_\mu \rangle = \frac{1}{16\pi^2}\Big[\Big(\frac{c}{3} - a\Big) R^2 + (4a - 2c) R^2_{\mu\nu} + (c - a) R^2_{\mu\nu\rho\sigma}\Big]$$

$$\Updownarrow$$

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R^2_{\mu\nu} + \alpha_3 R^2_{\mu\nu\rho\sigma} + \dots$$

sensitive to higher derivative corrections

Extracting c_2 : the holographic trace anomaly

- **Prescription for obtaining trace anomaly for higher derivative gravity**

Blau, Narain, Gava (th/9904179), Nojiri, Odintsov (th/9903033)

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(-R + 12g^2 + \alpha R^2 + \beta R_{\mu\nu}^2 + \gamma R_{\mu\nu\rho\sigma}^2 + \cdots\right)$$

$$\langle T^\mu_\mu \rangle = \frac{2L}{16\pi G_5}\left[\left(-\frac{L}{24} + \frac{5\alpha}{3} + \frac{\beta}{3} + \frac{\gamma}{3}\right)R^2 + \left(\frac{L}{8} - 5\alpha - \beta - \frac{3\gamma}{2}\right)R_{\mu\nu}^2 + \frac{\gamma}{2}R_{\mu\nu\rho\sigma}^2\right]$$

$$g = \frac{1}{L}\left[1 - \frac{1}{6L^2}(20\alpha + 4\beta + 2\gamma)\right]$$

$$a = \frac{\pi L^3}{8G_5}, \qquad c = \frac{\pi L^3}{8G_5}\left(1 + \frac{c_2}{24L^2}\right), \qquad g = \frac{1}{L}.$$

$$\Rightarrow \quad c_2 = \frac{24}{g^2}\,\frac{c-a}{a}$$

Thermodynamics of R-charged black holes

Given higher derivative action, we can find near-extremal D3-brane solution

- Lowest order theory admits a two-parameter family of solutions [Behrndt, Cvetic, Sabra]

$$ds^2 = H^{-2} f dt^2 - H\left(f^{-1}dr^2 + r^2 d\Omega_{3,k}^2\right) \qquad H(r) = 1 + \frac{Q}{r^2},$$

$$A = \sqrt{\frac{3(kQ+\mu)}{Q}}\left(1 - \frac{1}{H}\right)dt, \qquad f(r) = k - \frac{\mu}{r^2} + g^2 r^2 H^3$$

μ → non-extremality

Q R-charge

k=1, μ=0 : BPS solution, naked singularity (superstar)

- Einstein GR: entropy → area of event horizon
- Higher derivative terms → Wald's formula →

$$S = 2\pi \int_\Sigma d^{D-2}x \sqrt{-h} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma}$$

→ Entropy in terms of dual CFT central charges

$$s = \frac{2a(r_0^2 + Q)^{3/2}}{\pi L^6} \left(1 + \frac{c-a}{a} \frac{3Q^2 - 14Qr_0^2 - 21r_0^4}{8r_0^2(Q - 2r_0^2)}\right)$$

Hydrodynamics

0903.3244 S.C. et al.
& 0903.2834 Myers et al.

Our original motivation: dynamics of system (transport coefficients)

- **<u>Long-distance</u>, <u>low-frequency</u> behavior of any interacting theory at finite temperature is described by hydrodynamics**

***effective description* of dynamics of the system**
at large wavelengths and long time scales

Relativistic Hydrodynamics:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

$$\sigma_{ij} = \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u^k\right) + \zeta\delta_{ij}\partial_k u^k$$

Shear Viscosity

- η can be extracted from certain correlators of the boundary $T_{\mu\nu}$:
  (Kubo's formula: retarded Green's fn of stress tensor)

$$G^R_{xy,xy}(\omega,\mathbf{0}) = \int dt\, d\mathbf{x}\, e^{i\omega t}\theta(t)\langle[T_{xy}(t,\mathbf{x}),\ T_{xy}(0,\mathbf{0})]\rangle = -i\eta\omega + O(\omega^2)$$

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,G^R_{xy,xy}(\omega,\mathbf{0})$$

- Use Minkowski modification of standard AdS/CFT recipe (Son & Starinets):

$$\left\langle \exp\left(\int d^{D-1}x\,\phi_0(x)\,T_{12}(x)\right)\right\rangle_{\mathrm{CFT}} = e^{-S_{\mathrm{SG}}[\phi]}\Big|_{\mathrm{AdS}}$$

- AdS/CFT dictionary: source for $T_{\mu\nu}$ is the metric

→ Set up appropriate metric perturbations

$$g_{xy} \to g_{xy} + h_{xy}$$

Bound Violation

$$s = \frac{2a(r_0^2+Q)^{3/2}}{\pi L^6}\left(1+\frac{c-a}{a}\frac{3Q^2-14Qr_0^2-21r_0^4}{8r_0^2(Q-2r_0^2)}\right)$$

$$\eta \sim \frac{(1+Q)^{3/2}}{16\pi}\left[1-\frac{c-a}{a}\frac{5Q^2+6Q+5}{8(Q-2)}\right]$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1-\frac{c-a}{a}(1+Q)\right]$$

Suprisingly simple dependence on R-charge: some form of universality?

- **<u>Bound violated</u> for c - a > 0**
- **R-charge makes violation worse**
- **Violation is small !**

$$\frac{1}{4\pi}\left(1-3\frac{c-a}{a}\right) \leq \frac{\eta}{s} \leq \frac{1}{4\pi}\left(1-\frac{c-a}{a}\right)$$

Violation is 1/N correction

- For N = 4 SYM $a = c$ → no R^2 corrections
- In general $a = c = \mathcal{O}(N^2)$ only, and $\frac{c-a}{a} \sim \frac{1}{N}$

- Correction is 1/N

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{c-a}{a}(1+Q)\right]}$$

  - These are not 1-loop corrections in the bulk → $\mathcal{O}(1)$
  - Due to presence of fundamental matter

- Contrast to IIB on $AdS_5 \times S^5$

Can we see 1/N dependence more explictly?

Simple example: Kats & Petrov (R^2 corrections in Type IIB on $\mathbf{AdS_5 \times S^5/\mathbb{Z}_2}$)

- Decoupling limit of N D3's sitting inside 8 D7's coincident on O7 plane

R^2 terms arise from world-volume action of D7-branes
(matter in fundamental representation)

Alternatively, if matter content of theory is known, (c-a) can be determined precisely (central charges are a measure of number of degrees of freedom)

Main point:
If the CFT central charges are known, we can use the AdS/CFT dictionary to fix the gravitational couplings -- even if we lack a detailed understanding of the microscopic origin of the couplings

Which higher derivative terms matter?

$$\mathcal{L} = -R - \frac{1}{4}F^2 + \ldots + \frac{c_2}{g^2}\Big[\alpha_1 R^2_{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \alpha_3 \nabla^\mu F^{\nu\rho}\nabla_\mu F_{\nu\rho} + \alpha_4 \nabla^\mu F^{\nu\rho}\nabla_\nu F_{\rho\mu} + \cdots\Big]$$

$$\Rightarrow \quad \boxed{\frac{\eta}{s} = \frac{1}{4\pi}\Big[1 - 4\bar{c}_2\big(2\alpha_1 - q(\alpha_1 + 6\tilde{\alpha}_2)\big)\Big]}$$

Only terms with <u>explicit dependence on Riemann tensor</u>

Remarkable simplification:

- **Having SUSY completion of higher derivative terms <u>naively</u> did not play a role (but SUSY governs structure of couplings)**

Sign of c-a ?

- **Bound is <u>always</u> violated if c-a > 0**

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{c-a}{a}(1+Q)\right]}$$

- **CFTs give <u>both</u> c-a < 0 and c-a > 0**

  (more *free* vectors than hypers will give c-a<0)

- **Until recently all CFT examples <u>with SUGRA duals</u> have c-a > 0**

  → <u>violation of the bound is the rule rather than the exception</u>

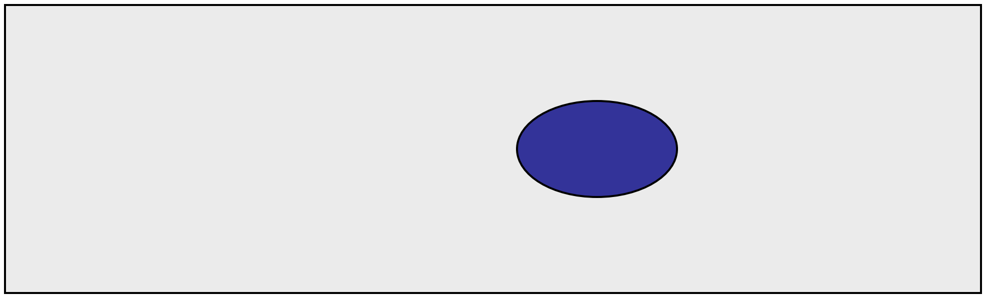
Caveat: new CFTs (Gaiotto/Maldacena 0904.4466) with SUGRA duals with c-a taking either sign - but different class of theories (quiver gauge theories from M5s wrapping 2d Riemann surface)

- **Is gravity somehow <u>constraining</u> the sign of c-a to be positive ?**

The sign of c-a and the weak gravity conjecture

- Is string theory constraining the sign of (c-a) and allowed dual CFTs ?

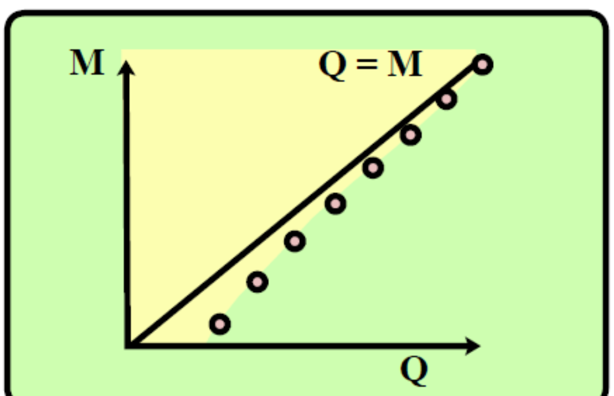
  → swampland vs. landscape ideas



- "Gravity is the weakest force" conjecture (Vafa et al., AH et al.)

  → there must be particles with smaller M/Q than extremal b.h.

  decay channel for extremal b.h. (don't want infinite # of stable particles)

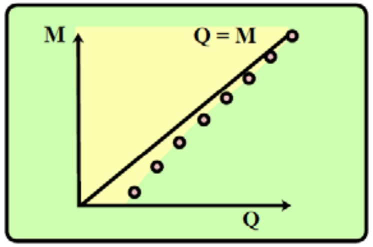


M=Q relation cannot be exact

must receive corrections as Q decreases

The sign of c-a and the weak gravity conjecture

SC et al., arXiv:0910.5159



- Higher derivatives modify the linear m=q relation for extremal b.h.
- According to weak GR conjecture, M/Q must decrease as Q decreases
  → verified in some ST setups where sign of correction could be checked (hep-th/0606100, 0909.5264)
- For IIB R-charged black holes with <u>higher derivative terms constrained by SUSY</u>, we find correlation between correction to m/q and correction to η/s

$$\left(\frac{M}{Q}\right)_{d=5} = \left(\frac{M}{Q}\right)_0 \left[1 - \frac{c-a}{c} f(r_h)\right] \qquad \frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{c-a}{c} g(Q)\right]$$

Behavior required by weak GR conjecture (c-a)>0
is correlated with violation of viscosity bound

Shear viscosity bound violation seemingly related to *constraints* on GR side

Final Remarks - I

AdS/CFT : playground to explore strongly coupled field theories (new set of tools)

- **Applications to QGP:**
  - **development of universal relations useful for providing inputs into realistic simulations of the RHIC fireball**
  - **Bulk viscosity, thermalization time …**

- **GR higher derivative corrections associated with finite N and λ corrections**

- **"Phenomenological" approach: scan CFT landscape by dialing corrections and tuning parameters to better agree with data**
  - **R-charged chemical potential – one more parameter we can tune**

Final Remarks - II

Is shear viscosity bound related to fundamental <u>constraints</u> on GR side?

- **For 5D R-charged black holes, correlation between behavior of η/s and correction to M/Q**
  - **hints at close connection between sign of (c-a) and possible restrictions imposed by requirement of quantum gravity – swampland vs. landscape**
  - **theories with c-a < 0 naively in conflict with weak GR conjecture and may possess unusual features**
- **Need better geometrical understanding of origin of higher derivative couplings**
  - **e.g. work on relating geometric data of generic SUSY AdS_5 solutions of IIB to central charges, but only in leading SUGRA approximation (Gauntlett et al.)**

The End