Fluxes, Geometries and Non-Geometries

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Outline

I Some Background

II M-theory/IIA Flux Compactifications

III Non-Geometries

arXiv:1004.5447, J. McOrist, D. R. Morrison & S. S.

arXiv:1004.XXXX, J. McOrist & S. S.

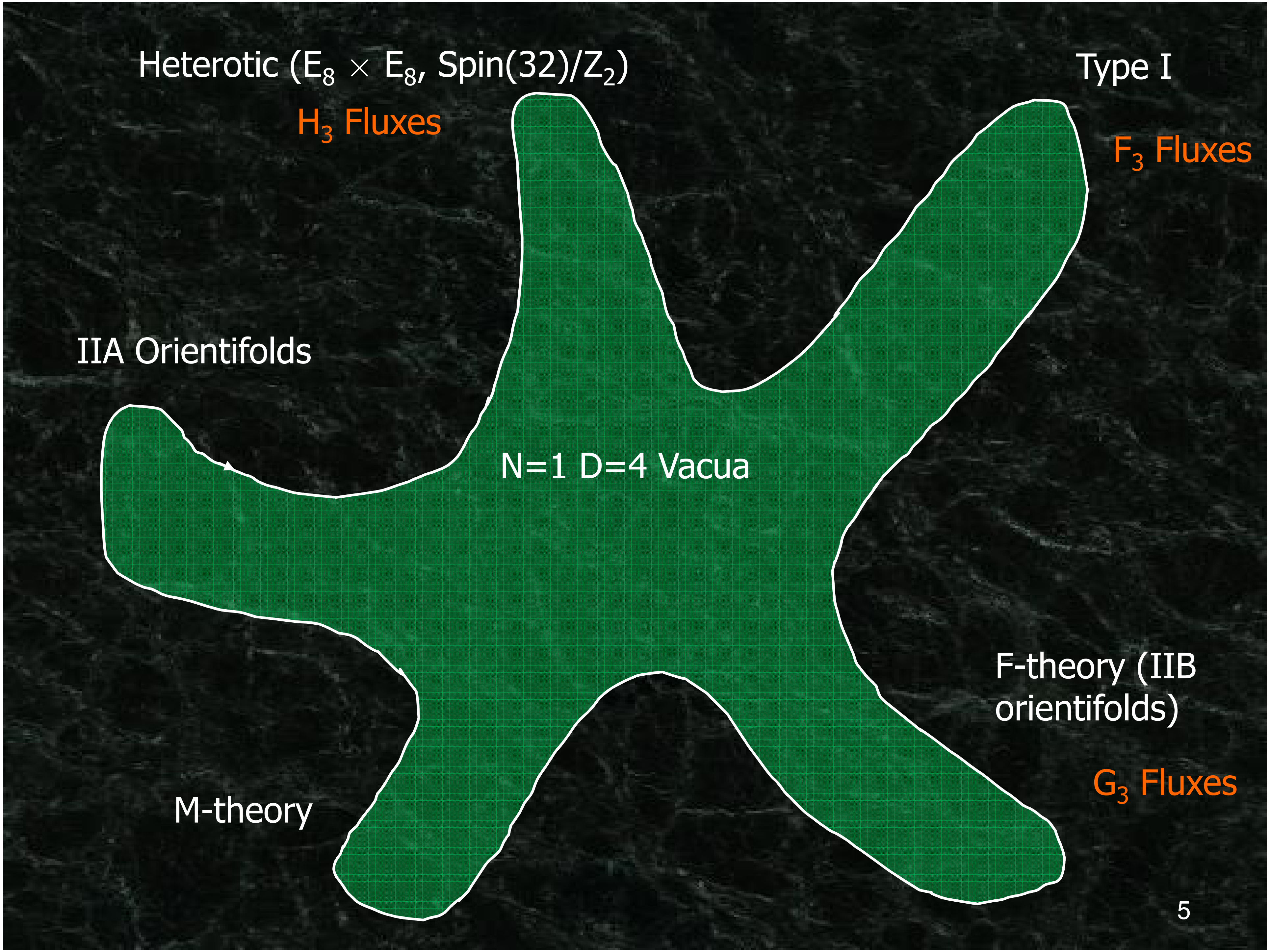
Works in progress ….

I Some Background

There are many ways to construct N=1 D=4 string vacua.

The large degeneracy of type IIB and heterotic vacua has led to the idea of a landscape of metastable vacua in string theory.

Little, including the existence, phenomenology and cosmology is understood about this landscape.


Heterotic ($E_8 \times E_8$, Spin(32)/Z_2)
H_3 Fluxes
Type I
F_3 Fluxes
IIA Orientifolds
N=1 D=4 Vacua
F-theory (IIB orientifolds)
G_3 Fluxes
M-theory

In addition to geometric compactifications, there is a potentially vast space of non-geometric compactifications.

The arena where we will be most likely to understand quantum aspects of string compactifications is the heterotic string.

In this talk, I want to focus on two issues:

(1) The first is a puzzle about turning on fluxes in M-theory and type IIA string backgrounds.

(2) The second involves the construction of non-geometric heterotic compactifications.

These compactifications will be SUSY. The issues one meets in non-SUSY backgrounds are much more challenging.

Are there flux compactifications of M-theory to D=4 with N=1?

The answer is clearly yes since $AdS_4 \times S^7$ Freund-Rubin type compactifications exist.

What we would really like are backgrounds without cosmological constants of order the compactification scale i.e. actual compactifications.

So we should ask: are N=1 compactifications to Minkowski space possible?

In the absence of flux, the gravitino variation under SUSY:

$$\delta\Psi_M \sim \nabla_M \epsilon$$

implies a 7-dimensional space with G_2 holonomy.

How about with flux?

In this case, there is a familiar no-go theorem which is essentially the same for all flux compactifications.

$$ds^2 = g_{MN} dx^M dx^N = e^{-\phi} \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n$$

$$G_{mnpq},\ G_{\mu\nu\rho\tau} = m\epsilon_{\mu\nu\rho\tau}$$

Einstein equations give:

$$\nabla_m (e^{-3\phi} \partial^m e^\phi) \propto e^{-2\phi} \left(|G_4|^2 + 2e^{4\phi} m^2 \right)$$

Now in other cases like the type IIB string or heterotic string, higher derivative corrections to SUGRA allow us to evade similar no-go results.

There are sources like orientifold planes and D-branes that support couplings that modify this constraint.

In the heterotic string:

$$d\mathcal{H}_3 = \frac{\alpha'}{4}\left(\mathrm{Tr}(R \wedge R) - \mathrm{Tr}(F \wedge F)\right)$$

This modified Bianchi identity means there is a gravitational source for NS5-brane charge.

In type IIB string theory, 7-branes support a coupling

$$\int C_4 \wedge p_1, \qquad p_1 = \frac{1}{8\pi^2} \mathrm{Tr} R \wedge R$$

which induces a background D3-brane charge.

Typically the size of this induced background charge controls the degeneracy of solutions which correspond to the number of ways of satisfying the tadpole condition.

Without these additional beyond SUGRA ingredients, no compact flux backgrounds are possible.

One might think that orientifold 6-planes and D6-branes in type IIA string theory might be sufficient.

Yet D6-branes and “most” O6-planes lift to smooth metric in M-theory.

For example, a D6-brane lifts to M-theory compactified on a Taub-NUT space which is a smooth 4-dimensional hyperkahler metric:

$$ds^2 = V(dr^2 + r^2 d\Omega^2) + V^{-1}(dy + A)^2$$

$$V = 1 + \frac{1}{r},\ A = \cos\theta d\psi$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\psi^2, \qquad y \sim y + 2\pi$$

There are 4 possible flavors of O6-plane in IIA string theory:

$O6^-$	SO(2N)	$\rightarrow$	Atiyah-Hitchin manifold (even c.c.)
$O6^{-\prime}$	SO(2N+1)	$\rightarrow$	Massive IIA (odd c.c.)
$O6^+$	Sp(N)	$\rightarrow$	"Frozen D_4" singularity
$O6^{+\prime}$	Sp(N)	$\rightarrow$	Massive IIA (odd c.c.)
D6		$\rightarrow$	Taub-NUT space

The correlation of even/odd cosmological constant was nicely discussed by Hyakutake, Imamura & Sugimoto.

Let's stick to vanilla ingredients that can be understood in conventional string theory/supergravity.

Vanilla D6/O6 branes/planes in a pure metric background cannot be the resolution of this puzzle unlike the case of type IIB or the heterotic/type I strings.

That concludes the background.

II M-theory/IIA Flux Vacua

To understand the origin of the needed tadpole, let's return to type IIB where on D7-branes:

$$\int C_4 \wedge p_1, \qquad p_1 = \frac{1}{8\pi^2}\mathrm{Tr}\, R \wedge R$$

Suppose we wrap the D7-brane on a Taub-NUT space. This is a space with a good circle isometry and

$$p_1(TN) = 2$$

so there is an induced (negative) D3-brane charge.

This charge cannot disappear under T-duality!

There is also a source for D3-brane charge from the coupling:

$$\int C_4 \wedge \mathrm{Tr}\,(F \wedge F)$$

This is a 2-derivative coupling that will be responsible for brane creation when branes cross after T-duality.

(Hanany-Witten)

We will want to separate these two quite different effects.

Taub-NUT metric:

$$ds^2 = V(dr^2 + r^2 d\Omega^2) + V^{-1}(dy + A)^2$$

$$V = 1 + \frac{1}{r},\ A = \cos\theta d\psi$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\psi^2, \qquad y \sim y + 2\pi$$

In a T-dual picture, we will find a D6-brane transverse to the y-circle while the Taub-NUT space turns into a smeared NS5-brane described by the background:

$$\widetilde{ds}^2 = V(dr^2 + r^2 d\Omega^2 + dy^2)$$

$$B_{\psi y} = \cos\theta, \quad e^{2\phi} = V = 1 + \frac{1}{r}.$$

$$(C_4)_{\mu_0 \ldots \mu_3} \to (C_5)_{\mu_0 \ldots \mu_3 y}.$$

On a space with a U(1) isometry, we can integrate out the circle direction:

$$\int_y R \wedge R = X_3^y$$

This is a 3-form charged under the $\mathrm{Spin}(3)_R$ symmetry of the D6-brane.

This leads to a coupling on the D6-brane:

$$\int (C_5)_y \wedge X_3^y$$

Let's take a general metric with a U(1) isometry. The topology of the twist encoded in A will turn into flux.

$$ds^2 = e^m e^m + e^y e^y, \quad e^y = f(x)(dy + A)$$

$$de^n + \omega^{nm} e^m = 0,$$

$$g_n e^n = d(\log f), \quad h_{mn} e^m e^n = f dA.$$

$$X_3^y = -(R_{mn} + H^y_{mn} H_{qpy} e^q \wedge e^p - H^y_{qm} H_{pny} e^q \wedge e^p)\alpha^y_{mn}$$

$$+(\tilde{g}_n H^y_{qp} e^q \wedge e^p + \tilde{g}_q H^y_{np} e^q \wedge e^p$$

$$-dH^y_{np} e^p - \omega_{np} H^y_{pq} e^q - H^y_{np} \omega_{pq} e^q)\beta_n.$$

The 1-forms α and β take the form:

$$\alpha_{mny} = (dH_{mny} + 2H_{mny} g^p e^p + H_{mpy}\omega_{pn} - H_{npy}\omega_{pm}$$

$$+g_m H_{npy} e^p - g_n H_{mpy}),$$

$$\beta_n = (dg_n - g_p \omega_{pn} + f^2 H_{npy} H_{pqy} e^q + g_n g_p e^p).$$

These couplings don't look particularly elegant but they do modify the physical charge!

They should fit into a framework that describes D-brane charge in the presence of H-flux consistent with T-duality.

They can be massaged into a nicer form and that's a current undertaking.

The upshot is that D-branes in the presence of transverse H-fluxes and curvatures can lead to induced charges.

This is the missing ingredient.

Very similar couplings on branes have been found by scattering and duality arguments (Garousi; Becker, Guo & Robbins).

To recap: in IIA, we see that D6-branes and O6-planes in transverse H-fluxes can give rise to negative D4-brane charge.

We started our discussion in M-theory and it's easy to see that these 6-brane flux couplings will lift to M-theory.

For example, let's recall how the D6-brane coupling which induces D2-brane charge

$$\int C_3 \wedge p_1$$

arises from M-theory.

In the M-theory effective action, there is a coupling central to flux compactifications:

$$\int C_3 \wedge X_8, \qquad X_8 = \frac{1}{192}\left(p_1^2 - 4p_2\right)$$

The D6-brane is just a gravitational background for which:

$$\int_{TN} X_8 \sim p_1$$

$$\int C_3 \wedge X_8 \quad \rightarrow \quad \int C_3 \wedge p_1$$

So we can see that a coupling like the one we found lifts

$$\int (C_5)_n \wedge X_3^n \quad \rightarrow \quad \int C_6 \wedge X_5$$

where X_5 is constructed from G_4, connections and curvatures.

The lift of the 4-derivative couplings to M-theory therefore leads to 8-derivative couplings which are quadratic, quartic etc. in the fluxes.

These couplings appear in the SUSY completion of:

$$S_{\text{SUGRA}} + \int (R^4 + E_8 + C_3 \wedge X_8$$

$$+O(\partial^6 G^2) + \ldots O(G^8))$$

Some of these terms have been studied by Hyakutake.

These 8-derivative flux couplings give an induced M5-brane charge.

These charge tadpoles control the degeneracy of IIA and M-theory flux backgrounds.

There is also a beautiful classical contribution to the charge which involves “geometric flux”.

This will force the spaces in type IIA to be non-CY and, in fact, non-Kahler. Something that can be confirmed from studying local SUSY constraints in type IIA.

We can again understand how the desired structure comes about by starting in type IIB where fluxes generate D3-brane charge via:

$$*F_5 = F_5, \qquad dF_5 = H_3 \wedge F_3$$

We need only consider $T^6 \sim T^3 \times T^3$ to see what is happening.

(Dasgupta, Rajesh, S.S.)

The NS flux H_3 threads the space. Imagine the first T^3 factor supporting this flux:

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2, \qquad x_i \sim x_i + 1$$

N units of flux $H_3 = dB_2 = N\, dx_1 \wedge dx_2 \wedge dx_3$

Pick a trivialization $B_2 = N\, x_1\, dx_2 \wedge dx_3$ and T-dualize along x_3:

$$\widetilde{ds}^2 = dx_1^2 + dx_2^2 + (dx_3 - N x_1 dx_2)^2$$

$$x_1 \sim x_1 + 1, \qquad x_3 \sim x_3 + N x_2$$

The topology of the space has changed in an interesting way. The interesting Betti numbers of the torus:

$b_1=3$, $b_2=3$ $\Rightarrow$ $b_1=2$, $b_2=2$.

In particular:

$\omega = dx_3 - N\, x_1\, dx_2$ is globally defined so $d\omega = dx_1 \wedge dx_2$ is trivial.

$$dF_5 \Rightarrow dF_4 \sim dx_1 \wedge dx_2 \wedge dV_3$$

$$Q_{D3} = \int_{X_6} dF_5 \quad \Rightarrow \quad Q_{D4} = \int_{X_5} dF_4 = \int_{\partial X_5} F_4$$

III Non-Geometries

So far we've discussed how to turn on fluxes and avoid classic no-go theorems.

Now let's turn our attention to intrinsically quantum string compactifications and see what we can learn from duality.

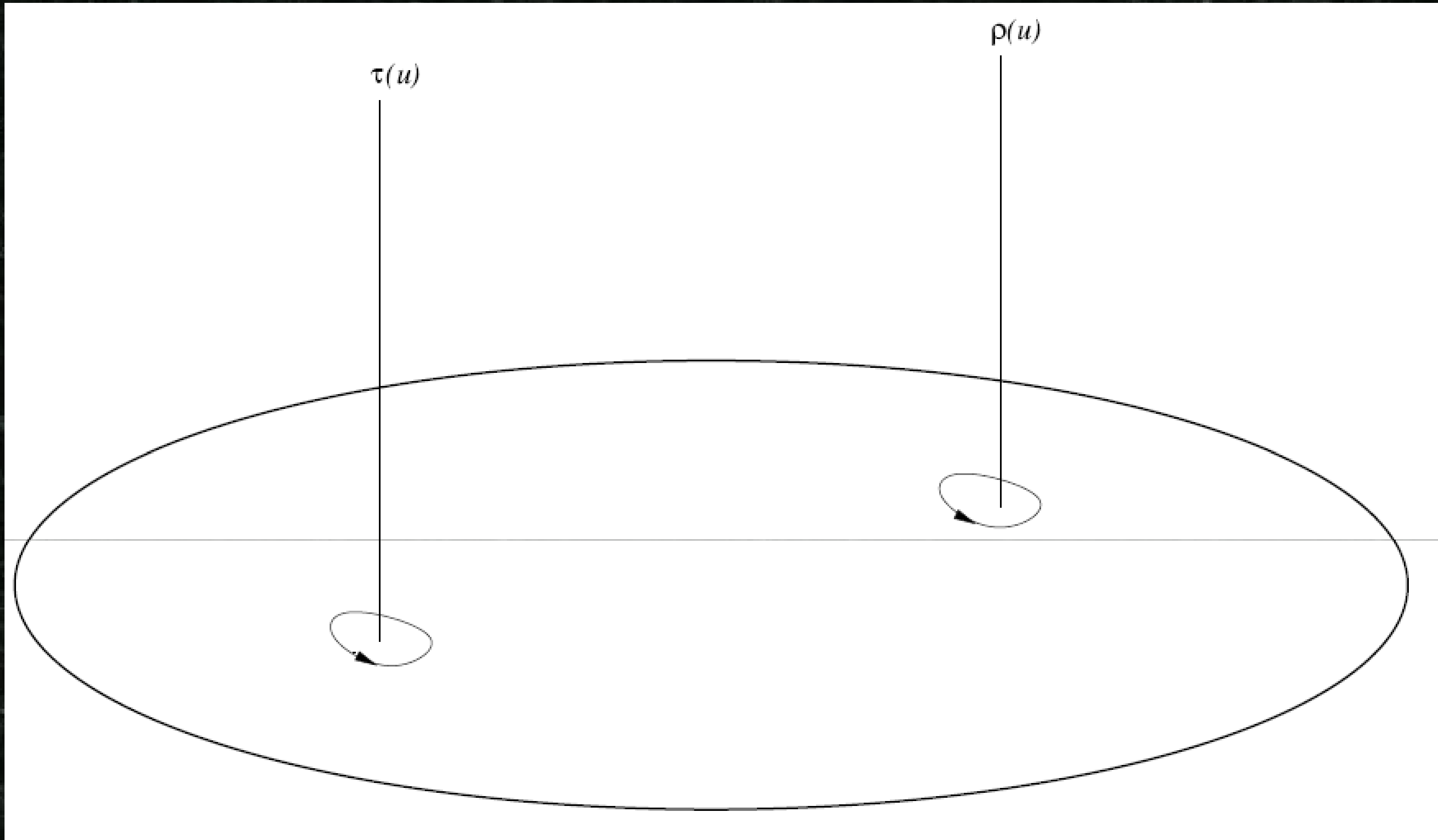
String theory on T^2 has two moduli: τ and ρ = B+iV.

In principle, we should consider fibrations of both moduli allowing monodromies valued in PSL(2,Z).

Since this includes V $\rightarrow$ 1/V, the compactifications are intrinsically quantum.

These are sometimes called "T-folds."

(Hellerman, McGreevy, Williams; Hull; Shelton, Taylor, Wecht, ….)



From an 8-dimensional perspective, these models have two families of (p,q) 5-branes where τ and ρ degenerate.

So we would like to be able to describe the fibration data for such a compactification that will satisfy all anomaly cancelation constraints.

We would also like to understand how to present a world-sheet description.

We are going to use the duality between the F-theory and the heterotic string to extract defining data for non-geometric heterotic compactifications satisfying a generalized Bianchi condition.

The basic equivalence:

F-theory on K3 = Heterotic on T^2

We would like to extract τ and ρ from the K3 geometry and fiber this picture.

Consider an elliptic space with section $Z \to S$ with a Weierstrass model:

$y^2 = x^3 + f(s)\, x + g(s)$

where (f,g) are sections of O(4L) and O(6L) respectively.

We choose $O(L) = O(-K_S)$ to ensure Z is CY.

The fibers degenerate at the zeroes of the discriminant

$\Delta(s) = 4\, f(s)^3 + 27\, g(s)^2$.

Lastly, the j-function which is the SL(2,Z) invariant characterization of τ is given by:

$$j(s) = 1728\, \frac{4 f(s)^3}{\Delta(s)}.$$

We will want to freeze all Wilson line moduli to isolate ρ and τ.

To do so, we need Weierstrass models for maximal unbroken gauge symmetry.

Let's consider a model for $G=E_8 \times E_8$:

$$Y^2 = X^3 + a\,\sigma^4 X + b\,\sigma^5 + c\,\sigma^6 + d\,\sigma^7$$

where σ is a coordinate on P^1 and (a,b,c,d) are constants.

This has the right singularity structure with type II^* fibers at $\sigma=0, \infty$.

There is a similar model for $Spin(32)/Z_2$ with the form

$$y^2 = x^3 + (p_0 s^3 + p_1 s^2 + p_2 s + p_3) x^2 + \epsilon x$$

and discriminant:

$$\Delta = - \epsilon^2 (p(s)^2 - 4 \epsilon)$$

where $p(s) = p_0 s^3 + p_1 s^2 + p_2 s + p_3$.

Again there is a fiber of type I_{12}^* at $s=\infty$. This ensures the right gauge symmetry.

For both models, one can extract data about the heterotic τ and ρ.

For example, for the $E_8 \times E_8$ case, one finds:

$$j(\tau)j(\rho) = -1728^2 \frac{a^3}{27bd}$$

$$(j(\tau) - 1728)\,(j(\rho) - 1728) = 1728^2 \frac{c^2}{4bd}.$$

(Cardoso, Curio, Lust & Mohaupt)

We require more data to specify the heterotic fibration.

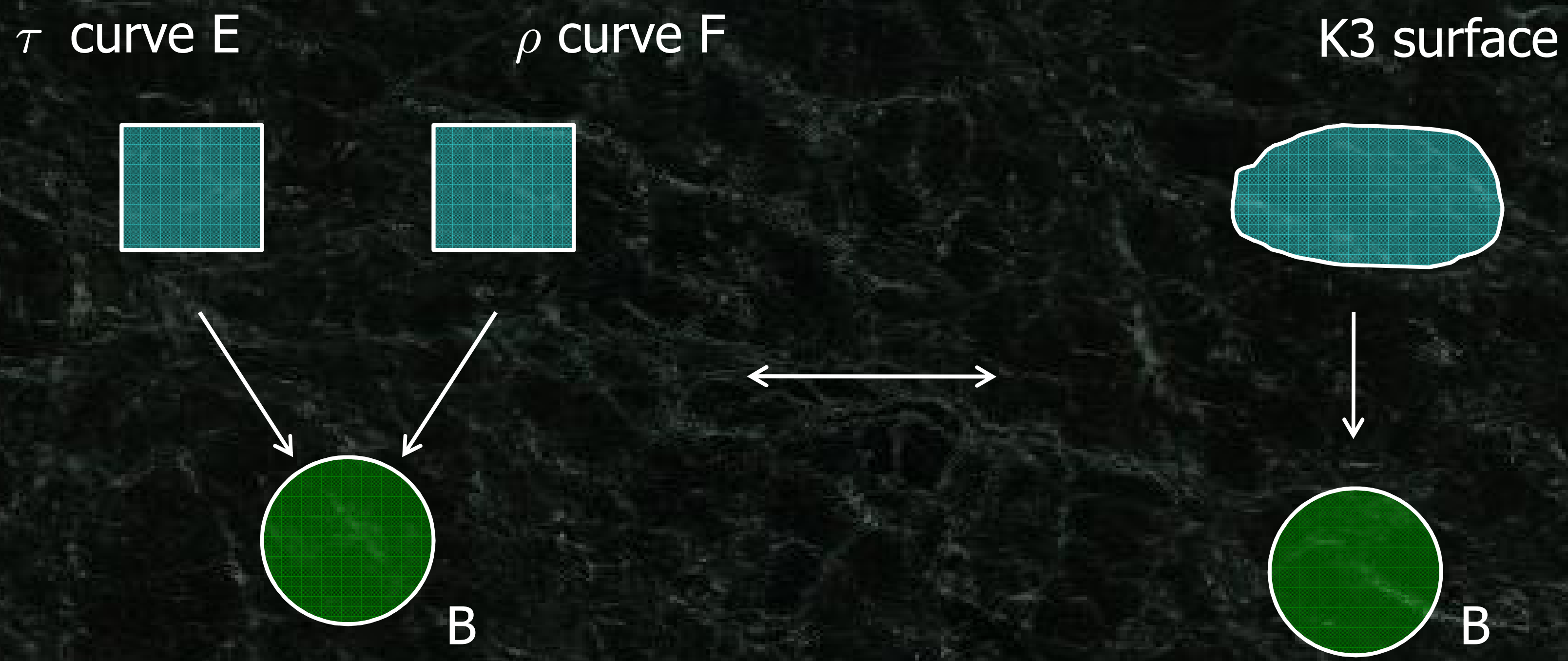
What we ideally want are the f's and g's associated to the heterotic τ and ρ parameters.

The geometry we'll employ are K3 surfaces Z which admit a Shioda-Inose structure.

This implies the existence of an automorphism, I, of order 2 preserving the holomorphic 2-form such that:

Z/I ~ Kummer surface $(E \times F)/Z_2$

Clingher & Doran '06 worked out the structure for the Spin$(32)/Z_2$ case.


τ curve E
ρ curve F
K3 surface
B
B

It’s convenient to state the map as follows: suppose we have two curves E and F with Weierstrass equations:

$v^2 = u^3 + \lambda_2 u + \lambda_3$

$w^2 = z^3 + \mu_2 z + \mu_3$

These encode the heterotic (τ, ρ) fibration data in the parameters (λ_2, λ_3) and (μ_2, μ_3).

For the Spin(32)/Z_2 case, we find the F-theory dual fibration given by the Weierstrass form:

$$y^2 = x^3 + (s^3 - 3\lambda_2\mu_2 s - \frac{27}{2}\lambda_3\mu_3)x^2$$

$$+\frac{1}{16}(4\lambda_2^3 + 27\lambda_3^2)(4\mu_2^3 + 27\mu_3^2)x.$$

We can then check the conditions for which this is a good F-theory model.

There is a similar map for the $E_8 \times E_8$ case.

$$\frac{1}{4}(4\lambda_2^3 + 27\lambda_3^2) = b(\lambda)d(\lambda),$$

$$\frac{1}{4}(4\mu_2^3 + 27\mu_3^2) = b(\mu)d(\mu)$$

$$Y^2 = X^3 - 3\lambda_2\mu_2\sigma^4 X + b(\lambda)b(\mu)\sigma^5$$

$$-\frac{27}{2}\lambda_3\mu_3\sigma^6 + d(\lambda)d(\mu)\sigma^7$$

That’s the strategy and the components of the moduli space including ρ variations can be now be studied.

For example, there appear to be no components among the D=6 compactifications that have purely non-geometric moduli spaces.

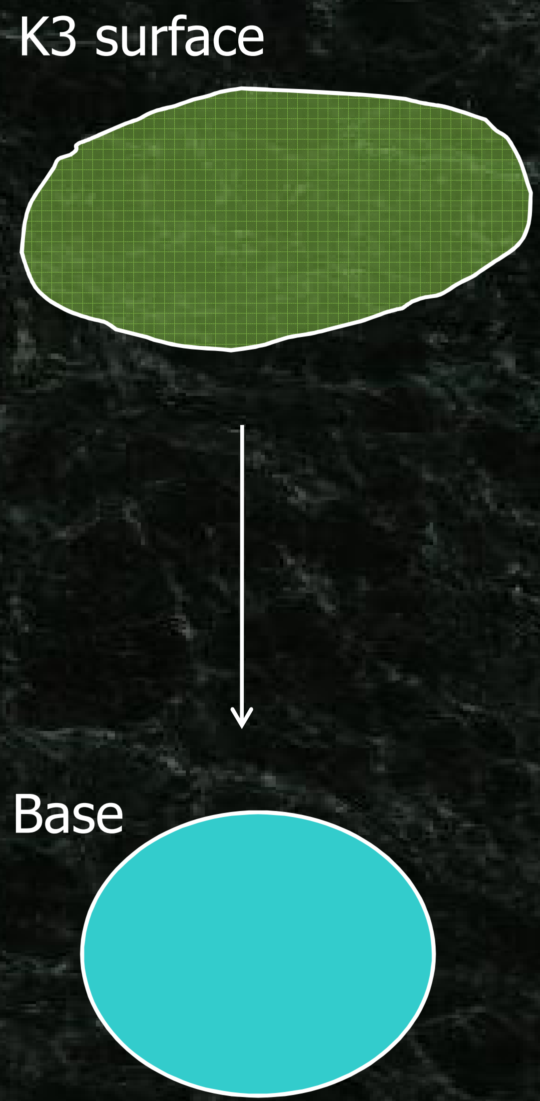
On the other hand, once one considers D=4, there do appear to be purely non-geometric components.

Further fluxes in F-theory will typically freeze us at loci for which the heterotic string is non-geometric.

This clearly enlarges the class of heterotic vacua that we should consider and it strongly suggests that notions of quantum bundles and quantum compactifications are critical for understanding the landscape of heterotic compactifications.

It also suggests that Kahler, non-Kahler and non-geometric heterotic compactifications are on the same footing and should be treated uniformly.

There is much to develop along these lines but let's end with the question of how we should define world-sheet theories for non-geometries.



The heterotic string is an M5-brane wrapping the K3 surface of the fibration.

We have heard a number of talks about M5-branes wrapping $M_4 \times C_2$ but duality suggests this fibered case (with flux) is going to be interesting.

A single M5-brane supports:

$h_3 = db_2 \quad h_3 = * h_3 \qquad \phi^i \quad i=1, \ldots 5.$

The symmetry group is $Spin(5,1) \times Spin(5)_R$

Wrapping on the fibered K3 surface means that the world-volume theory is topologically twisted.

In the presence of background G_4 flux,

$h \rightarrow db_2 + C_3$

The moduli space of this compactified tensor theory captures non-geometry!

This provides an interesting natural way of defining the worldsheet theory.