String Theory and
The Velo-Zwanziger Problem

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- M. Porrati, RR and A. Sagnotti, arXiv:1011.6411 [hep-th], *Nucl. Phys. B* **846**, 250 (2011).
- G. Velo and D. Zwanziger, *Phys. Rev.* **186**, 1337 (1969); *Phys. Rev.* **188**, 2218 (1969). G. Velo, *Nucl. Phys. B* **43**, 389 (1972).
- P. C. Argyres and C. R. Nappi, *Phys. Lett. B* **224**, 89 (1989); *Nucl. Phys. B* **330**, 151 (1990).
- S. M. Klishevich, *Int. J. Mod. Phys. A* **15**, 395 (2000).

Motivation

- Massive charged high-spin particles do exist in nature in the form of hadronic resonances. The dynamics of these composite particles should be described by local actions at least in the quasi-collinear regime, when the exchanged momenta are small compared to the particle mass.

- Naïve attempts of coupling such a particle to EM generally give rise to pathologies, e.g. propagation of unphysical modes & loss of (causal) propagation (the Velo-Zwanziger problem).

- String theory is a valuable laboratory to investigate these type of EM interactions. Does string theory provide a consistent description at all? If so, how does it achieve this feat?

A Few Comments and Caveats

- The Velo-Zwanziger problem is generic to any spin $s > 1$.
- It persists for a wide class of non-minimal extensions, so that constructing consistent EM interactions for massive high-spin fields is quite challenging from a field-theoretic vantage point.
- We will consider constant EM backgrounds. This is nontrivial as the Velo-Zwanziger problem arises for such backgrounds.
- We will consider EM field invariants much smaller than m^2/e. For large values of the invariants instabilities appear in the form of Schwinger pair production or Nielsen-Olesen instability, so that the concept of long-lived propagating particle makes little sense. The Velo-Zwanziger problem appears for small invariants.
- We set the Regge slope, $\alpha' = \frac{1}{2}$.

Methodology & Some Basics of SFT

- We consider the Sigma Model for open bosonic string in a constant EM background. This is exactly solvable, and a careful definition of the mode expansion gives smooth limits for neutral and free strings (contrary to existing claims in the literature).

- In parallel with the free string case, there is an infinite set of creation and annihilation operators, which are well-defined in physically interesting situations, away from instabilities.

- We have commuting center-of-mass coordinates, which obey canonical commutation relations with the covariant momenta. The latter are, of course, non-commuting in an EM background.

- The mode $\alpha_0{}^{\mu}$ is covariant momentum up to a rotation:

$$\alpha_0^{\mu} = Q^{\mu}{}_{\nu}\, p^{\nu}_{\rm cov}, \qquad\qquad Q = \sqrt{\frac{G}{eF}}\ .$$

$$G \ = \ \frac{1}{\pi}\left[\,\tanh^{-1}(\pi e_0 F) + \tanh^{-1}(\pi e_\pi F)\,\right]$$

$$e \equiv e_0 + e_\pi \qquad\qquad [p^{\mu}_{\rm cov}, p^{\nu}_{\rm cov}] \ = \ ieF^{\mu\nu}.$$

- Virasoro algebra is that of the free theory. But L_0 has a shift.

$$[L_m, L_n] \ = \ (m-n)L_{m+n} + \tfrac{1}{12}\, d(m^3 - m)\delta_{m,-n}\ ,$$

$$L_0 \ \rightarrow \ L_0 + \tfrac{1}{4}{\rm Tr}G^2$$

- A generic string state is a certain number of creation operators applied on the string ground state:

$$|\Psi\rangle = \sum_{s=1}^{\infty} \sum_{m_i=1}^{\infty} \psi^{(m_1 m_2 \ldots m_s)}_{\mu_1 \mu_2 \ldots \mu_s} \, a^{\dagger \mu_1}_{m_1} a^{\dagger \mu_2}_{m_2} \ldots a^{\dagger \mu_s}_{m_s} |0\rangle$$

- Given ***(m₁ , m₂ ,…, mₛ)***, the coefficient $\psi^{(m_1 m_2 \ldots m_s)}_{\mu_1 \mu_2 \ldots \mu_s}$ is a rank-***s*** Lorentz tensor, interpreted as a string field, and as such is a function of the string center-of-mass coordinates.

- The Virasoro constraints result in physical state conditions:

$$\begin{aligned} (L_0 - 1)|\Phi\rangle &= 0 \\ L_1|\Phi\rangle &= 0 \\ L_2|\Phi\rangle &= 0 \end{aligned}$$

- We are interested in what these conditions translate into in the of language of string fields.

- Explicit expressions for ***L_0***, ***L_1***, and ***L_2*** are needed. They are:

$$
\begin{aligned}
L_0 &= -\tfrac{1}{2}\mathcal{D}^2 + \sum_{m=1}^{\infty}(m+iG)_{\mu\nu}\, a_m^{\dagger\mu} a_m^{\nu} + \tfrac{1}{4}\,\mathrm{Tr}G^2 \\
&\equiv -\tfrac{1}{2}\mathcal{D}^2 + \left(\mathcal{N} + \tfrac{1}{4}\,\mathrm{Tr}G^2\right) + i\sum_{m=1}^{\infty} G_{\mu\nu}\, a_m^{\dagger\mu} a_m^{\nu}\ , \\
L_1 &= -i\left[\sqrt{1+iG}\right]_{\mu\nu}\mathcal{D}^{\mu} a_1^{\nu} + \sum_{m=2}^{\infty}\left[\sqrt{(m+iG)(m-1+iG)}\right]_{\mu\nu} a_{m-1}^{\dagger\mu} a_m^{\nu}\ , \\
L_2 &= -i\left[\sqrt{2+iG}\right]_{\mu\nu}\mathcal{D}^{\mu} a_2^{\nu} \ + \ \tfrac{1}{2}\left[\sqrt{1+G^2}\right]_{\mu\nu} a_1^{\mu} a_1^{\nu} \\
&\qquad + \sum_{m=3}^{\infty}\left[\sqrt{(m+iG)(m-2+iG)}\right]_{\mu\nu} a_{m-2}^{\dagger\mu} a_m^{\nu}
\end{aligned}
$$

- Number operator, $\mathcal{N} \equiv \sum_{n=1}^{\infty} n\, a_n^{\dagger}\cdot a_n$, has integer eigenvalues.
- And $\mathcal{D}^{\mu} \equiv \left(\sqrt{G/eF}\right)^{\mu\nu} D_{\nu} = i\,\alpha_0^{\mu}\ , \qquad [\mathcal{D}^{\mu},\mathcal{D}^{\nu}] = -i\,G^{\mu\nu}\ .$

Level $N = 2$: The Massive Spin 2

- A generic state at this level is given by:

$$|\Phi\rangle \;=\; h_{\mu\nu}(x)\, a_1^{\dagger\mu} a_1^{\dagger\nu}\,|0\rangle \;+\; \sqrt{2}\, i\, B_\mu(x)\, a_2^{\dagger\mu}\,|0\rangle$$

- We do a couple of field redefinitions:

$$\mathcal{H}_{\mu\nu} \equiv \left(\sqrt{1+iG}\cdot h\cdot\sqrt{1-iG}\right)_{\mu\nu}\,, \quad \mathcal{B}_\mu \equiv \left(\sqrt{1+\tfrac{i}{2}G}\cdot B\right)_\mu$$

- The physical state conditions then give rise to

$$\begin{aligned}
\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\,\mathrm{Tr}G^2\right)\mathcal{H}_{\mu\nu} - 2\,i\,(G\mathcal{H} - \mathcal{H}G)_{\mu\nu} &= 0\,,\\
\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\,\mathrm{Tr}G^2\right)\mathcal{B}_\mu - 2\,i\,G_\mu{}^\nu\mathcal{B}_\nu &= 0\,,\\
\mathcal{D}^\mu\mathcal{H}_{\mu\nu} - (1+iG)_\nu{}^\rho\mathcal{B}_\rho &= 0\,,\\
\mathcal{H}^\mu{}_\mu + 2\mathcal{D}^\mu\mathcal{B}_\mu &= 0\,.
\end{aligned}$$

- The most general on-shell gauge transformation is

$$
\begin{aligned}
\delta\mathcal{H}_{\mu\nu} &= [J\cdot\mathcal{D}]_\mu\,\xi_\nu + [J\cdot\mathcal{D}]_\nu\,\xi_\mu + \tfrac{1}{2}(K_{\mu\nu}+K_{\nu\mu})(\mathcal{D}\cdot\xi)\ ,\\
\delta\mathcal{B}_\mu &= (L\cdot\xi)_\mu + [M\cdot\mathcal{D}]_\mu(\mathcal{D}\cdot\xi)\ ,
\end{aligned}
$$

$$
\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\,\mathrm{Tr}G^2\right)\xi_\mu - 2\,i\,G_\mu{}^\nu\xi_\nu \;=\; 0\ .
$$

- ***J***, ***K***, ***L*** and ***M*** are functions of ***G***.

- The gauge transformation is a symmetry only in ***d = 26***, and if

$$
\begin{aligned}
J &= \mathbf{1} + \tfrac{3}{2}\left(\tfrac{d-6}{d+4}\right)\,i\,G\ ,\\
K &= -\left(\tfrac{10}{d+4}\right)\,\left[\mathbf{1} + \tfrac{1}{20}\,(d-6)G^2\right]\ ,\\
L &= 2\cdot\mathbf{1} + \tfrac{3}{2}\left(\tfrac{d-6}{d+4}\right)\,i\,G\ ,\\
M &= \left(\tfrac{d-6}{d+4}\right)\left[\mathbf{1} + \tfrac{1}{2}\,i\,G\right]\ .
\end{aligned}
$$

- The vector field $\mathcal{B}_\mu$ can be gauged away only in ***d = 26***, in contrast with the case with no background. Gauge fixing gives

$$\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\,\mathrm{Tr}G^2\right)\mathcal{H}_{\mu\nu} - 2\,i\,(G\mathcal{H} - \mathcal{H}G)_{\mu\nu} \;=\; 0\;,$$

$$\mathcal{D}^\mu \mathcal{H}_{\mu\nu} \;=\; 0\;, \qquad \mathcal{H}^\mu{}_\mu \;=\; 0$$

- The above system is algebraically consistent, and gives the correct DoF count for a massive spin 2 particle, with

$$(\mathrm{mass})^2 \;=\; \frac{1}{\alpha'}\,\left(1 + \tfrac{1}{4}\,\mathrm{Tr}G^2\right)$$

- The above describes a hyperbolic system that admits causal propagation only, in physically interesting situations. We will present the proof of causal propagation a bit later.

Level $N = 3$: The Massive Spin 3

- This level has two physical fields: a symmetric rank 3 in the 1st Regge trajectory and an antisymmetric rank 2 in the 2nd Regge trajectory. Consistency requires an auxiliary scalar from the 2nd Regge trajectory. For the free theory, the latter vanishes on shell, and the two physical fields are independently consistent.

- In the presence of a background, the resulting equations with all three fields are algebraically consistent.

- The antisymmetric rank 2 cannot be consistently described if any of the other two fields is set to zero (unwarranted constraint):

$$G^{\mu\nu}\mathcal{A}_{\mu\nu} + \mathcal{O}(G^2) \ = \ 0$$

- The Lagrangian is bound to mix the two Regge trajectories.

- Setting the antisymmetric rank 2 to zero is OK for algebraic consistency. In this case, the auxiliary scalar vanishes on shell, and one is left with the Fierz-Pauli system for a massive spin 3, i.e. Klein-Gordon-like equation, divergence and trace constraints

$$\left(\mathcal{D}^2 - 4 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\Phi_{\mu\nu\rho} + 2\,i\,G^{\alpha}{}_{(\mu}\Phi_{\nu\rho)\alpha} \;=\; 0\;,$$

$$\mathcal{D}^{\mu}\Phi_{\mu\nu\rho} \;=\; 0\;,$$

$$\Phi^{\mu}{}_{\mu\nu} \;=\; 0\;.$$

- The 1st Regge trajectory field is consistent in isolation, while the subleading Regge trajectory field is not.
- The spin-3 Fierz-Pauli system gives the correct DoF count and ensures causal propagation.

Causal Propagation for Any Integer Spin *s* at Level *N* = *s*

- At any arbitrary level, the 1st Regge trajectory field, which is a symmetric rank-***s*** tensor, can by itself be consistently described.

- We have a Fierz-Pauli system for a massive spin ***s*** field:

$$\left[\mathcal{D}^2 - 2(s-1) - \tfrac{1}{2}\,\mathrm{Tr}G^2\right]\Phi_{\mu_1\ldots\mu_s} + 2\,i\,G^{\alpha}{}_{(\mu_1}\Phi_{\mu_2\ldots\mu_s)\alpha} \;=\; 0\;,$$

$$\mathcal{D}^{\mu}\Phi_{\mu\mu_2\ldots\mu_s} \;=\; 0\;,$$

$$\Phi^{\mu}{}_{\mu\mu_3\ldots\mu_s} \;=\; 0\;.$$

- It gives the correct DoF count for a massive spin ***s*** particle.

- Causal propagation of the physical modes can be proved by using the characteristic determinant method (Argyres-Nappi).

- The vanishing of the characteristic determinant requires

$$(G/eF)^{\mu}{}_{\nu}\; n_{\mu}\, n^{\nu} \;=\; 0$$

- The EM field strength ***F*** can be rendered block skew-diagonal,

$\boldsymbol{F^{\mu}{}_{\nu}}$ **= diag (*F*$_1$, *F*$_2$, *F*$_3$, … …)** , by a Lorentz transformation:

$$F_1 \;=\; a\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad F_{i\neq 1} \;=\; b_i\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- ***a*** and ***b$_i$***'s are real-valued functions of the EM field invariants.

They are much smaller than unity in physically interesting cases.

- The same transformation also makes ***G*** block skew-diagonal,

$\boldsymbol{G^{\mu}{}_{\nu}}$ **= diag (*G*$_1$, *G*$_2$, *G*$_3$, … …)** , because

$$G = (1/\pi)[\,\tanh^{-1}(\pi e_0 F) + \tanh^{-1}(\pi e_\pi F)\,]$$

$$
G_1 = f(a)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad G_{i\neq 1} = g(b_i)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
$$

$$
\begin{aligned}
f(a) &\equiv \frac{1}{\pi}\left[\tanh^{-1}(\pi e_0 a) + \tanh^{-1}(\pi e_\pi a)\right], \\
g(b_i) &\equiv \frac{1}{\pi}\left[\tan^{-1}(\pi e_0 b_i) + \tan^{-1}(\pi e_\pi b_i)\right].
\end{aligned}
$$

- With small EM field invariants, ***f(a)*** and ***g(b_i)*** are well-defined with absolute values much smaller than unity.

- Thus ***G/eF*** becomes a diagonal matrix:

$$
\left(\frac{G}{eF}\right)^{\mu}_{\ \nu} = \mathrm{diag}\left[\frac{f(a)}{ea}, \frac{f(a)}{ea}, \frac{g(b_2)}{eb_2}, \frac{g(b_2)}{eb_2}, \frac{g(b_3)}{eb_3}, \frac{g(b_3)}{eb_3}, \dots \dots\right]
$$

- But we have the following identities:

$$\frac{f(a)}{ea} \geq 1 \,, \qquad 0 < \frac{g(b_i)}{eb_i} \leq 1$$

- This implies that any solution $\boldsymbol{n_\mu}$ of the characteristic equation must be space-like:

$$n^2 \geq 0$$

- This is a Lorentz invariant statement. Therefore, propagation will be causal in any Lorentz frame.

- This is true for any single charged massive particle of arbitrary integer spin coupled to a constant EM background. String theory cures the Velo-Zwanziger problem through non-minimal terms.

Lagrangian Formulation

- Requires extending the Fock space by including world-sheet (anti)ghosts, and thereby using the BRST technique.
- BRST invariance helps remove the pure gauge modes. This is possible only in ***d* = 26**, in which the BRST charge is nilpotent.
- Writing down a Lagrangian is extremely cumbersome, and is not particularly illuminating.
- String theory guarantees a Lagrangian in ***d* = 26**.
- The spin-2 Lagrangian was written down by Argyres-Nappi.
- The spin-3 Lagrangian was given by Klishevich.

Critical Dimension & Gyromagnetic Ratio

- The string-theoretic Lagrangian, even after a complete gauge fixing gives a Fierz-Pauli system only in ***d* = 26**. Otherwise the trace constraint is lost.

- This is in contrast with the case when there is no background. The backgrounds "knows" about space-time dimensionality.

- Consider the Argyres-Nappi spin-2 Lagrangian:

$$L_{\mathrm{AN}} = \mathcal{H}^*_{\mu\nu}\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h}^{\mu\nu} - 2i\mathcal{H}^*_{\mu\nu}(G\mathfrak{h} - \mathfrak{h}G)^{\mu\nu} - \mathcal{H}^*\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathcal{H}$$

$$- \mathcal{H}^*_{\mu\nu}\left\{\mathcal{D}^\mu\mathcal{D}^\rho[(1+iG)\mathfrak{h}]_\rho{}^\nu - \tfrac{1}{2}\mathcal{D}^\mu\mathcal{D}^\nu\mathcal{H} + (\mu \leftrightarrow \nu)\right\} + \mathcal{H}^*\mathcal{D}^\mu\mathcal{D}^\nu\mathcal{H}_{\mu\nu}$$

$$\mathcal{H}_{\mu\nu} \equiv (1+iG)_\mu{}^\alpha(1+iG)_\nu{}^\beta\,\mathfrak{h}_{\alpha\beta}$$

- Manipulate with the trace, divergence and double-divergence of the resulting EoMs to obtain the would-be trace condition.

$$\left[2(d-1)+\tfrac{1}{4}\,\mathrm{Tr}G^2(d+14+2\,\mathrm{Tr}G^2)+\tfrac{1}{2}(d-26)\,\mathrm{Tr}\left(\frac{G^2}{4+G^2}\right)\right]\mathcal{H}$$
$$-\tfrac{1}{2}(d-26)\left(\frac{G^2}{4+G^2}\right)^{\mu\nu}\mathcal{D}_\mu\mathcal{D}_\nu\mathcal{H} \;=\; 0\;,$$

- It gives a non-dynamical trace only in ***d=26***.
- Spacetime dimensionality enters only at ***O(F²)***.
- There are kinetic deformations at ***O(F)***.

$$\begin{aligned}\delta L_{\rm kin} \;=\;& -i(e/m^2)(F\mathfrak{h}^*-\mathfrak{h}^*F)_{\mu\nu}\left[D^2\mathfrak{h}^{\mu\nu}-(D^\mu D^\rho\mathfrak{h}_\rho{}^\nu+D^\nu D^\rho\mathfrak{h}_\rho{}^\mu)+\tfrac{1}{2}D^{(\mu}D^{\nu)}\mathfrak{h}\right]\\ & -i(e/m^2)D^\mu(F\mathfrak{h}^*-\mathfrak{h}^*F)_{\mu\nu}(D_\rho\mathfrak{h}^{\rho\nu}-D^\nu\mathfrak{h})+\mathrm{h.c.}\;.\end{aligned}$$

- ***O(F)***-kinetic deformations are removed by field redefinitions.

But it gives a dipole contribution:

$$\delta L_{\text{kin}} \;=\; -4\,i\,e\,\mathrm{Tr}(\mathfrak{h}\cdot F\cdot\mathfrak{h}^{*}) + \mathcal{O}(F^2)$$

so that the gyromagnetic ratio, ***g = 2***.

- ***g = 2*** is the unique value a consistent Lagrangian must have in any number of spacetime dimensions.

- While ***g = ½*** is implied with minimal kinetic terms, already at ***O(F²)*** non-minimal kinetic terms are indispensible.

- For any (integer) spin, ***g = 2***, as also seen from the EoMs. This is in accordance with results using tree-level unitarity argument.

No-Ghost Theorem

- As consistency calls for non-standard kinetic terms, one may wonder whether the Hilbert space contains negative norm states.
- However, standard arguments (Polchinski) continue to hold with minor modifications, in the presence of a generic constant EM background in the regime of physical interest.
- The kinetic deformations present are judicious ones as there are no negative norm states.
- Again this is guaranteed only in the critical dimension.

Consistent Dimensional Reduction

- Take the Argyres-Nappi Lagrangian, consistent in ***D = 26***, to reduce it to spacetime dimensions ***d < D***, keeping only singlets of the ***(D-d)*** internal coordinates.

$$h_{\mu\nu} = \left(\begin{array}{c|c} h_{mn} & 0 \\ \hline 0 & \frac{1}{D-d}\,\delta_{MN}\,\phi \end{array}\right), \qquad G_{\mu\nu} = \left(\begin{array}{c|c} G_{mn} & 0 \\ \hline 0 & 0 \end{array}\right)$$

$$\mathcal{H}_{\mu\nu} = \left(\begin{array}{c|c} [1+iG]_{ma}[1+iG]_{nb}\, h^{ab} & 0 \\ \hline 0 & \frac{1}{D-d}\,\delta_{MN}\,\phi \end{array}\right) \equiv \left(\begin{array}{c|c} \mathfrak{h}_{mn} & 0 \\ \hline 0 & \frac{1}{D-d}\,\delta_{MN}\,\phi \end{array}\right)$$

$$\mathcal{D}^{\mu} = \left(\begin{array}{c} \mathcal{D}^{m} \\ \hline 0 \end{array}\right)$$

- The ***d***-dimensional model obtained thereby is:

$$
\begin{aligned}
L_d \;=\; & \mathfrak{h}^*_{mn}\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)h^{mn} - 2\,i\,\mathfrak{h}^*_{mn}(G\cdot h - h\cdot G)^{mn} - \mathfrak{h}^*\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h} \\
& - \mathfrak{h}^*_{mn}\left\{\mathcal{D}^m\mathcal{D}^p\big[(1+iG)\cdot h\big]_p{}^n - \tfrac{1}{2}\mathcal{D}^m\mathcal{D}^n\mathfrak{h} + (m \leftrightarrow n)\right\} + \mathfrak{h}^*\mathcal{D}^m\mathcal{D}^n\mathfrak{h}_{mn} \\
& + \left[\mathfrak{h}^*_{mn}\mathcal{D}^m\mathcal{D}^n\phi - \left\{\mathfrak{h}^* + \tfrac{1}{2}\left(\tfrac{D-d-1}{D-d}\right)\phi^*\right\}\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi \;+\; \mathrm{h.c.}\right]
\end{aligned}
$$

- Minimal kinetic mixing can be removed by field redefinition

$$
\mathfrak{h}_{mn} \;\to\; \mathfrak{h}_{mn} - \left(\tfrac{1}{d-1}\right)\left[\eta_{mn}\phi - \tfrac{1}{4}\mathcal{D}_{(m}\mathcal{D}_{n)}\phi\right]
$$

- Manipulations with the EoMs give for any dimension ***d*** :

$$
\left[2(D-1) + \tfrac{1}{4}\mathrm{Tr}G^2(D + 14 + 2\,\mathrm{Tr}G^2) - \tfrac{1}{2}(D-26)\left(\tfrac{iG}{2+iG}\right)^{mn}\mathcal{D}_m\mathcal{D}_n\right]\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi \;=\; 0
$$

- The scalar has Klein-Gordon-like EoM, since ***D = 26***.

- We obtain a consistent set of EoMs in ***d* < *26***.

$$\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h}_{mn} - 2\,i\,(G\cdot\mathfrak{h} - \mathfrak{h}\cdot G)_{mn} \;=\; 0$$

$$\mathcal{D}^m\mathfrak{h}_{mn} \;=\; 0 \qquad\qquad \mathfrak{h} \;=\; -\phi$$

$$\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi \;=\; 0$$

- For higher spins, the 26-dimensional Lagrangian exists in principle. We have the explicit EoMs, that can be dimensionally reduced to have consistent description of a system of spin ***s*** and spin (***s*-2**) coupled in the presence of an EM background.

- The ***d***-dimensional model is guaranteed to exist.

Conclusion & Future Perspectives

- String theory cures the Velo-Zwanziger problem for massive charged particles with integer spin, at least in $\boldsymbol{d = 26}$.
- Only the 1st Regge trajectory fields are consistent in isolation.
- Non-constant backgrounds are very difficult (non-linear string Sigma Model). It may inevitably involve non-locality.
- An analysis for fermionic fields requires superstring theory (much harder), and is currently being done.
- Dimensional reduction (from $\boldsymbol{d = 26}$) of the string-theoretic Lagrangian, keeping only the singlets of the internal coordinates, can give in $\boldsymbol{d < 26}$ consistent EM interactions of fields with spin $\boldsymbol{s}$ and ($\boldsymbol{s-2}$). The (im)possibility of decoupling the fields by field redefinitions is under investigation.