Universal Phenomena at Strong Coupling and Gravity

Ayan Mukhopadhyay

Harish-Chandra Research Institute,
Chhatnag Road, Jhusi
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ayan@mri.ernet.in

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Main Reference

Ramakrishnan Iyer and AM, “An AdS/CFT Connection between Boltzmann and Einstein,” [arxiv:0907.1156[hep-th]]

Preliminaries and Problems A

Preliminaries

- Gauge/gravity duality at
  (a) strong coupling
  (b) large rank of the gauge group (N)
  defines a "universal sector" of dynamics in gauge theories as dual of pure classical gravity in five dimensions. This is so because the theory of classical gravity always admits a consistent truncation to Einstein's equation in five dimensions with a negative cosmological constant. The embedding of the universal sector in the full theory depends on the details of the theory but not the dynamics within the sector.

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- In this sector, all observables can be determined by the energy-momentum tensor *alone*. This is so because the metric which solves Einstein's equation with negative cosmological constant is uniquely determined by the boundary stress tensor [Henningson, Skenderis, Balasubramanian, Krauss] which is identified with the energy-momentum tensor of the gauge theory.

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- (a) State in the field theory $\leftrightarrow$ Solution in gravity with a smooth final horizon
  (b)Temperature of the final equilibrium $\leftrightarrow$ Final temperature of the horizon
- Regularity/irregularity in the five dimensional solution of Einstein's solution implies regularity/irregularity in the full solution of gravity as the lift to the full solution is trivial (without involving any warping). The transport coefficients in hydrodynamics can be systematically determined by the regularity of the future horizon.

Preliminaries and Problems B

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- A field-theoretic understanding of how all observables get determined by the energy-momentum tensor alone.
- To solve for the condition on the energy-momentum tensor which gives solutions of Einstein's equation with smooth future horizons.
- A precise way to decode phenomena in the gauge theory from the metric.

Our Results

- We show that the relativistic semiclassical Boltzmann equation has "conservative solutions" which could be determined by the energy-momentum tensor alone. We can justify our study of Boltzmann equation at weak coupling because previous work of Arnold, Yaffe and others have demonstrated that an effective Boltzmann equation is as good as perturbative gauge theory to study, for example, transport phenomena in high temperature QCD.

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- We naturally identify the *conservative solutions* with the *universal sector* at strong coupling and large N.
- We find the right method of extrapolating the *conservative* condition at weak coupling to *regularity* condition in gravity at strong coupling.

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- Discussion : Open Issues in how Irreversibility (decoherence/thermalization) emerges at Long Time Scales

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- Boltzmann's H-theorem states that the phase space integral of $f \; ln \; f$ monotonically decreases with time and further the final state where the time derivative vanishes is that of global equilibrium. We will call irreversibility of this type "irreversibility at all time-scales."

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- The origin of irreversibility is chiefly due to the assumption made in the Boltzmann equation that the two particle velocity distribution locally factorises. This is called the *ergodic hypothesis*. There is a very rigorous modern understanding of how the ergodic hypothesis emerges for "good" multiparticle phase space distributions which do not distinguish between precollisional and postcollisional configurations [Lanford, Cercignani, Spohn, etc] so that the Boltzmann equation can be *rigorously* derived from reversible Hamiltonian mechanics in large classical systems.

The Boltzmann Equation : Brief Description 2/3

- The Boltzmann equation for the one particle phase space distribution $f(\mathbf{x},\xi)$ is

$$\left(\frac{\partial}{\partial t} + \xi . \frac{\partial}{\partial \mathbf{x}}\right) f(\mathbf{x},\xi) = J(f,f)(\mathbf{x},\xi) \quad (1)$$

where,

$$J(f,g) = \int (f(\mathbf{x},\xi^{'})g(\mathbf{x},\xi^{*'}) - f(\mathbf{x},\xi)g(\mathbf{x},\xi^{*}))B(\theta,V)d\xi^{*}d\epsilon d\theta \quad (2)$$

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- The collison variables are explained in the figure below:

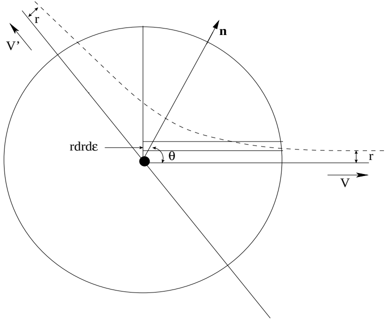

Figure: The collision variables

The Boltzmann Equation : Brief Description 3/3

- The collisional kernel $B(\theta, V)$ is defined as below

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- The velocities of the target and bullet molecule are related to the initial velocities of the target and bullet molecule as below

$$\begin{aligned} \xi_i^{'} &= \xi_i - n_i(\mathbf{n}.\mathbf{V}) \\ \xi_i^{*'} &= \xi_i^* + n_i(\mathbf{n}.\mathbf{V}) \end{aligned} \tag{4}$$

so that $\mathbf{V}^{'}.\mathbf{n} = \mathbf{V}.\mathbf{n}$.

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- Let $\phi(\xi)$ be a function of ξ. We will call it a collisional invariant if $\phi(\xi) + \phi(\xi^*) - \phi(\xi^{'}) - \phi(\xi^{*'}) = 0$. Clearly the collisional invariants are five in number and they are $1, \xi_i, \xi^2$. We will collectively denote them as ψ_α.

Hydrodynamic Equations from the Boltzmann Equation (1/2)

- Using symmetry one can easily prove that:

$$\int \phi(\xi)(J(f,g)+J(g,f))d\xi = \frac{1}{4}\int(\phi(\xi)+\phi(\xi^{*})-\phi(\xi^{'})-\phi(\xi^{*'}))(J(f,g)+J(g,f))d\xi \quad (5)$$

Therefore if $\phi(\xi)$ is a collisional invariant, i.e if $\phi(\xi)=\psi_\alpha(\xi)$,

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- So, the Boltzmann equation implies

$$\frac{\partial\rho^\alpha}{\partial t}+\frac{\partial}{\partial x_i}(\int \xi_i\psi_\alpha f d\xi)=0 \tag{7}$$

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where $\rho_\alpha = \int \psi_\alpha f d\xi$ are the locally conserved quantities. Instead of using ρ_α, we will use the hydrodynamic variables:

$$\rho = \int f d\xi, \quad u_i = \frac{1}{\rho}\int \xi_i f d\xi, \quad p = \frac{1}{3}\int \xi^2 f d\xi \tag{8}$$

Hydrodynamic Equations from the Boltzmann Equation (2/2)

- Now, our hydrodynamic equations are as below:

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\frac{\partial u_i}{\partial t} + u_r \frac{\partial u_i}{\partial x_r} + \frac{1}{\rho}\frac{\partial (p\delta_{ir} + p_{ir})}{\partial x_r} = 0 \\
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- The shear stress tensor, p_{ij}, is defined as follows

$$p_{ij} = \int (c_i c_j - RT\delta_{ij}) f d\xi \tag{10}$$

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- Also we define, S_{ijk} as below:

$$S_{ijk} = \int c_i c_j c_k f d\xi \tag{11}$$

and the heat flow vector S_i through $S_i = S_{ijk}\delta_{ij}$.

The Moment Equations

- Let us now define the n-th moment of f to be

$$f^{(n)} = \int \mathbf{c}^n f d\xi \qquad (12)$$

We note that $f_{ij}^{(2)} = p\delta_{ij} + p_{ij}$, $f_{ijk}^{(3)} = S_{ijk}$, etc

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- The equation satisfied by the moments $f^{(n)}$'s for $n \geq 2$ are as follows:

$$\frac{\partial f^{(n)}}{\partial t} + \frac{\partial}{\partial x_i}(u_i f^{(n)} + f_i^{(n+1)}) + \frac{\partial \mathbf{u}}{\partial x_i} f_i^{(n)} - \frac{1}{\rho} f^{(n-1)} \frac{\partial f_i^{(2)}}{\partial x_i} = J^{(n)} \tag{13}$$

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- It can be shown that $B^{(2,2,0)}_{ijkl}(\rho, T) = B^{(2)}(\rho, T)\delta_{ik}\delta_{jl}$.

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- Let $f^{(n)}$, a tensor of rank n be the n-th velocity moment of $f(\mathbf{x},\xi)$ so that $f^{(n)} = \int \mathbf{c}^n f d\xi$, where $c_i = \xi_i - u_i$. At equilibrium all these $f^{(n)}$'s vanish. However in conservative solutions these do not vanish and in fact can be very large. These $f^{(n)}$'s are determined functionally in terms of the ten independent variables of the energy-momentum tensor.

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$$S_i = \frac{15pR}{2B^{(2)}}\frac{\partial T}{\partial x_i} + \frac{3}{2B^{(2)}}(2RT\frac{\partial p_{ir}}{\partial x_r} + 7Rp_{ir}\frac{\partial T}{\partial x_r} - \frac{2p_{ir}}{\rho}\frac{\partial p}{\partial x_r}) + \ldots \qquad (16)$$

where $B^{(2)}$ is a specific function of the molecular mass, radius, local density and temperature and can be determined from the Boltzmann equation.

Conservative Solutions (1/4)

- We will first describe the *conservative* solutions of the Boltzmann equation for **nonrelativistic monoatomic gases**.
- The energy momentum tensor can always be parametrised by (a) the five hydrodynamic variables (ρ, u_i, p) and (b) the shear stress tensor (p_{ij}) in a comoving locally inertial frame. We have seen how these can be defined through the first ten velocity moments of f.
- Let $f^{(n)}$, a tensor of rank n be the n-th velocity moment of $f(\mathbf{x},\xi)$ so that $f^{(n)} = \int \mathbf{c}^n f d\xi$, where $c_i = \xi_i - u_i$. At equilibrium all these $f^{(n)}$'s vanish. However in conservative solutions these do not vanish and in fact can be very large. These $f^{(n)}$'s are determined functionally in terms of the ten independent variables of the energy-momentum tensor. For instance, the heat-flow vector S_i is given in terms of the ten variables as below:

$$S_i = \frac{15pR}{2B^{(2)}}\frac{\partial T}{\partial x_i} + \frac{3}{2B^{(2)}}(2RT\frac{\partial p_{ir}}{\partial x_r} + 7Rp_{ir}\frac{\partial T}{\partial x_r} - \frac{2p_{ir}}{\rho}\frac{\partial p}{\partial x_r}) + ... \tag{16}$$

where $B^{(2)}$ is a specific function of the molecular mass, radius, local density and temperature and can be determined from the Boltzmann equation.

All the higher moments similarly can be systematically determined for such solutions in unique functional forms of the ten independent variables. These functional forms have systematic expansions in two parameters, the derivative expansion parameter which is (typical scale of variation/ mean free path) and amplitude expansion parameter (typical value of non-hydrodynamic shear stress/ hydrostatic pressure). Only spatial derivatives and no time derivative appear in the functional forms of $f^{(n)}$.

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- Since all the velocity moments of f are unique local functions of the ten variables and their spatial derivatives, it follows that f is also uniquely determined by the ten variables. Once f is determined, any observable can also be determined through f.

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- The ten variables satisfy the following equations of motion closed amongst themselves

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_r}(\rho u_r) = 0 \\
\frac{\partial u_i}{\partial t} + u_r \frac{\partial u_i}{\partial x_r} + \frac{1}{\rho}\frac{\partial (p\delta_{ir} + p_{ir})}{\partial x_r} = 0 \\
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x_r}(u_r p) + \frac{2}{3}(p\delta_{ir} + p_{ir})\frac{\partial u_i}{\partial x_r} + \frac{1}{3}\frac{\partial S_r}{\partial x_r} = 0 \\
\frac{\partial p_{ij}}{\partial t} + \frac{\partial}{\partial x_r}(u_r p_{ij}) + \frac{\partial S_{ijr}}{\partial x_r} - \frac{1}{3}\delta_{ij}\frac{\partial S_r}{\partial x_r} \\
+ \frac{\partial u_j}{\partial x_r} p_{ir} + \frac{\partial u_i}{\partial x_r} p_{jr} - \frac{2}{3}\delta_{ij} p_{rs}\frac{\partial u_r}{\partial x_s} \\
+ p(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_r}{\partial x_r}) = B^{(2)}(\rho, T) p_{ij} \\
+ \sum_{p,q=0;\ p\geq q;\ (p,q)\neq(2,0)}^{\infty} B^{(2,p,q)}_{ij\nu\rho}(\rho, T) f^{(p)}_{\nu} f^{(q)}_{\rho}
\end{aligned} \tag{17}$$

Above all the higher moments, including S_i, has been determined in terms of the hydrodynamic variables, the shear stress tensor and their *spatial* derivatives. Since spatial derivatives of arbitrary orders are present in these functional forms, we need analytic data as initial conditions for these equations

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  Above all the higher moments, including S_i, has been determined in terms of the hydrodynamic variables, the shear stress tensor and their *spatial* derivatives. Since spatial derivatives of arbitrary orders are present in these functional forms, we need analytic data as initial conditions for these equations
- Any solution of the above equations of motion of the ten variables can be *uniquely* lifted to a full solution of the Boltzmann equation for f through the functional forms for $f^{(n)}$ already determined.

Conservative Solutions (3/4)

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  (a) *normal* or purely-hydrodynamic solutions [Enskog(1917), Burnett(1935), Chapman(1939)] where f is determined as functional of the five hydrodynamic variables and their spatial derivatives only

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- The normal solutions can be found by noting that the equation for p_{ij} has a special algebraic solution given in terms of hydrodynamic variables only. This solution is unique. Upto two derivatives this solution is as below:

$$\begin{aligned} p_{ij} &= \eta\sigma_{ij} + \lambda_1\frac{\eta^2}{p}(\partial.u)\sigma_{ij} + \lambda_2\frac{\eta^2}{p}(\frac{D}{Dt}\sigma_{ij} - 2(\sigma_{ik}\sigma_{kj} - \frac{1}{3}\delta_{ij}\sigma_{lm}\sigma_{lm})) \\ &+\lambda_3\frac{\eta^2}{\rho T}(\partial_i\partial_j T - \frac{1}{3}\delta_{ij}\Box T) + \lambda_4\frac{\eta^2}{p\rho T}(\partial_i p\partial_j T + \partial_j p\partial_i T - \frac{2}{3}\delta_{ij}\partial_l p\partial_l T) \\ &+\lambda_5\frac{\eta^2}{p\rho T}(\partial_i T\partial_j T - \frac{1}{3}\delta_{ij}\partial_l T\partial_l T) + ... \end{aligned} \tag{18}$$

where $\sigma_{ij} = \partial_i u_j + \partial_j u_i - (2/3)\delta_{ij}\partial_l u_l$, $\eta = (p/B^2)$ and the λ's which are pure numbers can be determined from the Boltzmann equation. Note all time-derivatives can be replaced by spatial derivatives through hydrodynamic equations of motion. This matches with the second order expression for p_{ij} for normal solutions [Chapman and Cowling, Chapter 15]

Conservative Solutions (4/4)

- Interestingly, the homogenous non-hydrodynamic solutions has singularities. For instance $f^{(4)}_{ijkl} = (2B^{(2)}\delta_{(klmn)(ijtu)} - B^{(4,4,0)}_{(klmn)(ijtu)})^{-1} B^{(4,2,2)}_{(ijtu)(pqrs)} p_{pq} p_{rs} + \ldots..$ and this becomes indeterminate when $(2B^{(2)}\delta_{(klmn)(ijtu)} - B^{(4,4,0)}_{(klmn)(ijtu)})$ regarded as an 81×81 matrix fails to be invertible. Such singularities also appear in normal solutions in kinetic theory of liquids (to be discussed later) and has the interpretation of local nucleation of solid phase and so here the singularities probably signal local condensation of the liquid phase.

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- Any generic solution of the Boltzmann Equation at sufficiently late times is approximated by an appropriate conservative solution. Since the maximum of the propagation speeds of the linear modes increases as more and more moments are included [Boillat, Muller], we can argue that, at a sufficiently late time, the part of the higher moments functionally independent of the hydrodynamic variables and the shear stress tensor becomes irrelevant, so that the dynamics is well approximated by an appropriate conservative solution. Thus ten variables suffice to capture systematically a whole range of phenomena which includes hydrodynamics and relaxation.

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- It can be shown that the relativistic semiclassical Boltzmann equation has conservative solutions as well.

Multi-Component Systems

In order to generalize conservative solutions of Boltzmann equation to relativistic gauge theories we also need to understand how to construct such solutions for multi-component systems.

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- In $\mathcal{N} = 4$ SYM theory, all the particles form a multiplet whose internal degrees of freedom are spin and $(SO(6)_R)$ charge along with the color indices. From the point of view of gravity, since in the universal sector we have pure gravity on the dual side, not only local and global charges and currents, but also the higher multipole moments of these charge distrubutions are absent at the boundary. So, the most natural reflection of this on the conservative solutions is that there is equipartition at every point in phase space over the internal, i.e the spin, charge and color degrees of freedom. Then we can easily construct an effective single component Boltzmann equation by summing over interactions in all spin, charge and color channels.

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- For other conformal gauge theories with gravity duals, we may also do the same even though all particles do not form a multiplet. This is possible because of mass degeneracy.

Definitions of the Nine Parameters

- The equilibrium energy momentum tensor for a conformal theory is $t_{(0)\mu\nu} = (\pi T)^4 (4u^\mu u^\nu + \eta_{\mu\nu})$. Going to the comoving inertial frame where $t_{(0)\mu\nu}$ is $diag(\epsilon, p, p, p)$, we find that the energy density ϵ and the pressure p are $\frac{\epsilon}{3} = p = (\pi T)^4$. Let $\pi_{\mu\nu}$ be the non-equilibrium part of the energy momentum tensor so that the total energy-momentum tensor is $t_{\mu\nu} = t_{(0)\mu\nu} + \pi_{\mu\nu}$.

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- Thus the total energy-momentum tensor can be parametrised by nine variables u^μ, T and the five independent components of $\pi_{\mu\nu}$.
- The conservation of the energy-momentum tensor $\partial^\mu t_{\mu\nu} = 0$, gives us the forced relativistic Euler equation which shows that $\pi_{\mu\nu}$ is the relativistic shear stress tensor.

The Conformal Hydrodynamic Energy-Momentum Tensor (1/2)

The most general form of the traceless hydrodynamic conformal shear stress tensor upto two orders in the derivative expansion, when all conserved currents vanish and with our definitions of u^μ and T is as below [Baier, Romatschke, Son, Starinets, Stephanov (2007)]

$$\begin{aligned}\pi^{\mu\nu} \quad = \quad & -\eta\sigma^{\mu\nu} \\ & +\alpha_1\left[(u\cdot\partial)\sigma^{\mu\nu}+\frac{1}{3}\sigma^{\mu\nu}(\partial\cdot u)-(u^\nu\sigma^{\mu\beta}+u^\mu\sigma^{\nu\beta})(u\cdot\partial)u_\beta\right] \\ & +\alpha_2\left(\sigma^{\alpha\mu}\sigma_\alpha{}^\nu-\frac{1}{3}P^{\mu\nu}\sigma_{\alpha\beta}\sigma^{\alpha\beta}\right)+\alpha_3(\sigma^{\alpha\mu}\omega_\alpha{}^\nu+\sigma^{\alpha\mu}\omega_\alpha{}^\nu) \\ & +\alpha_4\left(\omega^{\alpha\mu}\omega_\alpha{}^\nu-\frac{1}{3}P^{\mu\nu}\omega_{\alpha\beta}\omega^{\alpha\beta}\right)+O(\partial^3u)\end{aligned} \tag{19}$$

where $P_{\mu\nu}$ is the projection tensor orthogonal to u^μ

$$P_{\mu\nu}=u_\mu u_\nu+\eta_{\mu\nu} \tag{20}$$

$\sigma_{\mu\nu}$ is the hydrodynamic strain rate

$$\sigma^{\mu\nu}=\frac{1}{2}P^{\mu\alpha}P^{\nu\beta}(\partial_\alpha u_\beta+\partial_\beta u_\alpha)-\frac{1}{3}P^{\mu\nu}(\partial.u) \tag{21}$$

$\omega^{\mu\nu}$ is the hydrodynamic vorticity tensor

$$\omega^{\mu\nu}=\frac{1}{2}P^{\mu\alpha}P^{\nu\beta}(\partial_\alpha u_\beta-\partial_\alpha u_\beta) \tag{22}$$

The Conformal Hydrodynamic Energy-Momentum Tensor (2/2)

The semiclassical Boltzmann equation can be used to determine hydrodynamic transport coefficients in high temperature QCD and it is as good as the full perturbative description [Arnold and Yaffe]. In fact, we only need to use only tree level S-matrices and ignoring bare quark masses the hydrodynamic energy-momentum tensor at high temperature is conformal. At weak coupling, η/s is parametrically $O(1/(g^4\ ln(1/g)))$, in fact all $ln(1/g)$ terms can be resummed [Arnold, Moore, Yaffe].

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The Boltzmann equation has been used to find out the higher order hydrodynamic transport coefficients at the leading order and as mentioned this is as good as the full perturbative description [Moore and York (2009)]. The results for 3-quark QCD at leading order are $\frac{Ts\alpha_1}{\eta^2} = 5.9\ to\ 5.0$ (varies with g), $\frac{Ts\alpha_2}{\eta^2} = 5.2\ to\ 4.1$ (varies with g), $\frac{Ts\alpha_3}{\eta^2} = 2\frac{Ts\alpha_1}{\eta^2}$ and $\frac{Ts\alpha_4}{\eta^2} = 0$. Here, the effective coupling constant is defined as $g = \frac{1}{6\pi}(\frac{m_D}{T})^2$.

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At strong coupling for conformal gauge theories with gravity duals the hydrodynamic transport coefficients in absence of charged currents are universal as they can be obtained by looking at regular linear or non-linear perturbations (of Einstein's equation in five dimensions with a negative cosmological constant) about the black brane solution which are slowly varying both spatially and temporally with respect to the temperature. The famous result is that $\eta/s = 1/4\pi$ [Kovtun, Son, Starinets]. The results [Baier, Romatschke, Son, Starinets, Stephanov; Bhattacharya, Hubeny, Minwalla, Rangamani (2007)] for higher order transport coefficients are : $\frac{\alpha_1}{\eta} = \frac{2 - ln\ 2}{2\pi T}, \frac{\alpha_2}{\eta} = \frac{1}{2\pi T}, \frac{\alpha_3}{\eta} = \frac{ln\ 2}{2\pi T}$ and $\alpha_4 = 0$.

Do Conservative Solutions exist in the Exact Microscopic Theory?

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- The viscosity, for instance receives corrections as in $\eta = \frac{1}{15}\int \nu(r)\phi^{'}(r)r^{3}dr$ + the gaseous part ($\nu(r)$ is given by the two-body phase space distribution function where r is the relative separation), and increases with temperature unlike gases.

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- So, it is likely that normal and conservative solutions exist in the exact relativistic quantum guage theories like QCD such that a generic state at sufficient late times can be approximated by an appropriate conservative solution.
- The higher order transport coefficients could be exactly defined (at least implicitly) if we can construct normal solutions of the exact relativistic quantum gauge theories. We are investigating this currently as well.

The Israel-Stewart-Muller Formalism and Gauge/Gravity duality

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- The "philosophy" of the ISM formalism is to do phenomenology of irreversible transient phenomena using kinetic moments as independent variables even beyond the weak coupling regime where any kinetic theory can be constructed.
- The formalism employs the restriction that one can construct an entropy current of the form su^{μ} whose local divergence is positive definite. This formalism also restricts second order hydrodynamics, as the higher moments have solutions which are "purely hydrodynamic" in nature.

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- Interestingly, Ilya Prigogine also made an attempt to rewrite exact microscopic Hamiltonian dynamics in a "proto-thermodynamic" language. So our approach also conforms with his vision. In fact, it could be the first instance, where his vision could be concretely formulated and understood.

A Proposal for the Regularity Condition in Gravity (1/2)

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- When $\pi_{\mu\nu}$ is given in terms of hydrodynamic variables only, we will have "normal" solutions of the microscopic theory and the gravity duals at strong coupling could be easily identified with the "tubewise black brane solutions" found by Bhattachaya, et al. In a radial tube from every point at the boundary these solutions can be parametrised by the local hydrodynamic variables which from the gravity viewpoint are the Goldstone-like fields corresponding to boost and scale invariance, the maximally commuting broken symmetries present in the asymptotic geometry.

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- We propose the regularity condition as the most general equation for $\pi_{\mu\nu}$ which can reproduce the correct purely hydrodynamic energy-momentum tensor known exactly upto second order in derivatives as a special solution.

A Proposal for the Regularity Condition in Gravity (2/2)

Therefore, our regularity condition for pure gravity in AdS_5 is:

$$
\begin{aligned}
(1-\lambda_3)\left[(u\cdot\partial)\pi^{\mu\nu}+\frac{4}{3}\pi^{\mu\nu}(\partial\cdot u)-\left(\pi^{\mu\beta}u^{\nu}+\pi^{\nu\beta}u^{\mu}\right)(u\cdot\partial)u_{\beta}\right] \\
=-\frac{2\pi T}{(2-\ln 2)}[\pi^{\mu\nu}+2(\pi T)^3\sigma^{\mu\nu} \\
-\lambda_3(2-\ln 2)(\pi T)^2\left((u\cdot\partial)\sigma^{\mu\nu}+\frac{1}{3}\sigma^{\mu\nu}(\partial\cdot u)-\left(u^{\nu}\sigma^{\mu\beta}+u^{\mu}\sigma^{\nu\beta}\right)(u\cdot\partial)u_{\beta}\right) \\
-\lambda_4(\ln 2)(\pi T)^2(\sigma^{\alpha\mu}\omega_{\alpha}{}^{\nu}+\sigma^{\alpha\mu}\omega_{\alpha}{}^{\nu}) \\
-2\lambda_1(\pi T)^2\left(\sigma^{\alpha\mu}\sigma^{\nu}{}_{\alpha}-\frac{1}{3}P^{\mu\nu}\sigma^{\alpha\beta}\sigma_{\alpha\beta}\right)] \\
-(1-\lambda_4)\frac{ln2}{(2-\ln 2)}(\pi^{\mu}{}_{\alpha}\omega^{\alpha\nu}+\pi^{\nu}{}_{\alpha}\omega^{\alpha\mu}) \\
-\frac{2\lambda_2}{(2-\ln 2)}\left[\frac{1}{2}(\pi^{\mu\alpha}\sigma^{\nu}{}_{\alpha}+\pi^{\nu\alpha}\sigma^{\mu}{}_{\alpha})-\frac{1}{3}P^{\mu\nu}\pi^{\alpha\beta}\sigma_{\alpha\beta}\right] \\
+\frac{1-\lambda_1-\lambda_2}{(2-\ln 2)(\pi T)^3}\left(\pi^{\mu\alpha}\pi^{\nu}{}_{\alpha}-\frac{1}{3}P^{\mu\nu}\pi^{\alpha\beta}\pi_{\alpha\beta}\right)+ \\
O\left(\pi^3,\pi\partial\pi,\partial^2\pi,\pi^2\partial u,\pi\partial^2 u,\partial^2\pi,\partial^3 u,(\partial u)(\partial^2 u),(\partial u)^3\right)
\end{aligned}
\tag{23}
$$

where the $O(\pi^3,\pi\partial\pi,...)$ term indicates that the corrections to our proposal can include terms of the structures displayed or those with more derivatives or containing more powers of $\pi_{\mu\nu}$ or both only. Also, the four λ_i's $(i=1,2,3,4)$ are pure numbers.

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- The conservation of the energy-momentum tensor can be achieved if we put $\pi_{00} = \pi_{0i} = 0$ and π_{ij}'s are functions of time t only. The linearized equation $\pi_{\mu\nu}$ is solved if $\pi_{ij} = \mathcal{A}_{ij} exp(-t/\tau_\pi)$ with $\tau_\pi = \frac{(1-\lambda_3)(2-ln\ 2)}{2\pi T}$ and $\mathcal{A}_{ij}$ is a spatio-temporal constant and traceless matrix.

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- We note that the third branch is the branch that contains the above mode $\omega = -i\tau_\pi^{-1}, \mathbf{k} = 0$. This mode at weak coupling was associated with relaxation or local equilibriation in the quasiparticle-velocity space, so we will call this branch as the relaxation branch. Such a branch is not present in the quasinormal mode spectrum.

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- So, our proposal predicts that we should have such a linearized regular perturbation of the black brane unless $\lambda_3 = 1$. If $\lambda_3 = 1$, to the orders that we have written our equation describing regular non-linear perturbations about global equilibrium we have two possibilities, firstly there is a strange kind of gapless zero mode or there is no third branch at all and the non-hydrodynamic solutions arise only when the full non-linear equation is considered.

Consistency Checks of Our Proposal (1/2)

The internal consistency of our proposal can be checked, by determining λ's by various independent means. Here we will first look at two independent means of determining λ_3 and $\lambda_1 + \lambda_2$. The two independent means will be considering two different kinds of fluctuations about the linearized homogenous non-hydrodynamic solution we discussed before.

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Configration 1

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- π_{ij} obeys the following equation of motion which is exact upto third order terms.

$$\begin{aligned}(1-\lambda_3)\frac{d\pi_{ij}}{dt} &= -\frac{2\pi T}{(2-ln2)}\pi_{ij} \\ &+\frac{1-\lambda_1-\lambda_2}{(2-ln2)(\pi T)^3}(\pi_{ik}\pi_{kj}-\frac{1}{3}\delta_{ij}\pi_{lm}\pi_{lm}) \\ &+O(\frac{d^2\pi}{dt^2},\frac{d\pi}{dt}\pi,\pi^3)\end{aligned} \tag{24}$$

- The expansion parameters are ϵ' defined as $(\tau/\mathcal{T})$, where τ is the relaxation time and $\mathcal{T}$ is the typical time scale of variation of the solution; and δ defined as $|\pi_{ij}|/(\pi T)^4$.

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- The expansion parameters are $\epsilon^{'}$ defined as $(\tau/\mathcal{T})$, where τ is the relaxation time and $\mathcal{T}$ is the typical time scale of variation of the solution; and δ defined as $|\pi_{ij}|/(\pi T)^4$.

Incidentally, we also find how in this case gravity may reproduce quantum coherent behaviour as opposed to the explicitly irreversible case of the Boltzmann equation as a $d^2\pi/dt^2$ may appear at $\epsilon^{'2}\delta$ order.

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- The linearized equations exact at orders ϵ, δ and $\epsilon\delta$ are

$$(u_0.\partial)\delta u_\nu + u_{0\nu}(\partial.\delta u) = \left(\frac{\partial^\mu \delta\pi_{\mu\nu}}{4(\pi T)^4}\right) \quad (25)$$

$$\begin{aligned}(1-\lambda_3)(u_0.\partial)\delta\pi^{\mu\nu} = &-(1-\lambda_3)\left((\delta u.\partial)\pi_0^{\mu\nu} + \frac{4}{3}\pi_0^{\mu\nu}(\partial.\delta u) - (\pi_0^{\mu\beta}u_0^\nu + \pi_0^{\mu\beta}u_0^\nu)(u_0.\partial)\delta u_\beta\right)\\ &-\frac{2\pi T}{(2-ln2)}\left(\delta\pi^{\mu\nu} + 2(\pi T)^3\sigma^{\mu\nu}\right)\\ &-(1-\lambda_4)\frac{ln2}{(2-ln2)}\left(\pi_0^{\mu\alpha}\omega_\alpha{}^\nu + \pi_0^{\nu\alpha}\omega_\alpha{}^\mu\right)\\ &-\frac{2\lambda_2}{(2-ln2)}\left(\frac{1}{2}\left(\pi_0^{\mu\alpha}\sigma_\alpha{}^\nu + \pi_0^{\mu\alpha}\sigma_\alpha{}^\nu\right) - \frac{1}{3}P_0^{\mu\nu}\pi_0^{\alpha\beta}\sigma_{\alpha\beta}\right)\\ &+\frac{1-\lambda_1-\lambda_2}{(2-ln2)(\pi T)^3}\left(\pi_0^{\mu\alpha}\delta\pi^\nu{}_\alpha + \pi_0^{\nu\alpha}\delta\pi^\mu{}_\alpha - \frac{1}{3}(u_0^\mu\delta u^\nu + u_0^\nu\delta u^\mu)\pi_0^{\alpha\beta}\pi_{0\alpha\beta} - \frac{2}{3}P_0^{\mu\nu}\pi_0^{\alpha\beta}\delta\pi_{\alpha\beta}\right)\end{aligned}$$

where, $\pi_0^{\mu\nu}$ is as in the basic configuration and

$$\sigma^{\mu\nu} = \frac{1}{2}P_0^{\mu\alpha}P_0^{\nu\beta}(\partial_\alpha\delta u_\beta + \partial_\beta\delta u_\alpha) - \frac{1}{3}P_0^{\mu\nu}(\partial.\delta u) \quad (26)$$

$$l_\mu = \epsilon_{\alpha\beta\gamma\mu}u_0^\alpha\partial^\beta\delta u^\gamma$$

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- One can determine the other λ's through coefficients of various non-linear terms as well and further check consistency.

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- It is also more plausible that the universal sector should contain only finite number of branches which could be blind to the particle content and other microscopic details of the theory.

Issues in Irreversibility, etc

How does gravity see loss of quantum coherence?

To understand this, we may go back to the conservative solutions, but now construct them in quantum kinetic theories. It will be interesting if we can study the universal part of the phenomenology of pure to mixed state transition and at least some aspects of decoherence in general in terms of only ten variables parametrising the energy-momentum tensor.

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In gravity, it has been shown that the hydrodynamic solutions, possess an entropy current whose divergence is positive definite. One can try to understand if it also so for the normal solutions of the infinite BBGKY heirarchy. We may thus check whether the purely hydrodynamic behaviour in the exact microscopic theory is generically irreversible. We should also understand why the physical entropy current is not of the form su^{μ}.

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How do we connect to experiment?

Our proposal implies that universal phenomena at strong coupling consists of the dynamics of three branches in the spectrum, namely the two hydrodynamic branches and the relaxation branch. It will be important to understand how we can connect this observation with actual experiments. The spectrum in the case of cold atoms tuned at Feshback resonance which is independent of all possible dimensionless parameters may give us support for our proposal.

Thank You