M2-branes, non-relativistic CFT & Schrodinger geometries

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talk based on:

Hee-Cheol Kim, S.K., Ki-Myeong Lee & Jaemo Park, in preparation

"Emergent Schrodinger geometries from mass-deformed CFT"

related refs:

D. T. Son, Phys. Rev. D**78**, 046003 (2008), arXiv:0804.3972.

K. Balasubramanian & J. McGreevy, Phys. Rev. Lett. **101**, 061601 (2008), arXiv:0804.4053.

J. Gauntlett, S.K., O. Varela & D. Waldram, JHEP **0904**, 102 (2009), arXiv:0901.0676.

E. 'O Colgain, O. Varela & H. Yavartanoo, JHEP 0907, 081 (2009), arXiv:0906.0261.

J. Gauntlett, J. Sonner & T. Wiseman, JHEP **1002**, 060 (2010), arXiv.0912.0512.

Hee-Cheol Kim & S.K. Nucl. Phys. B**839**, 96 (2010), arXiv:1001.3153.

Sangmo Cheon, Hee-Cheol Kim & S.K. arXiv:1101.1101.

CFT's with mass deformations

- Interesting QFT's often appear by perturbing well-known CFT's by relevant operators which trigger RG flows.
- Deformations leaving massless fields: often flow to new interacting CFT
- Or more nontrivial vacua (e.g. confinement, spontaeous symmetry breakings, …)

- Very often, mass terms are simplest relevant operators
- Familiar examples in string theory (also in AdS_5/CFT_4):

✓ From C^3/Z_2 to conifold CFT by a mass perturbation [Klebanov-Witten]

✓ N=1 mass to N=4 SYM [Donagi-Witten] [Dorey] [Polchinski-Strassler]: confining/Higgs vacua, etc.

✓ And more… many holographic RG flows known…

Mass-deformed CFT_3 & non-relativistic CFT

- Various mass deformations of CFT_3 (or $SCFT_3$) for M2-branes known
- Rich IR physics: dynamical SUSY breaking, sometimes confinement, spontaneous symmetry breakings & superconductivity …

- When all propagating degrees are massive: flows to non-relativistic CFT
- Shown very concretely with a SUSY mass deformation of ABJM [Nakayama-Sakaguchi-Yoshida] [Lee-Lee-Lee] (2009)
- gravity duals of non-relativistic CFT_3 [Son] [Balasubramanian-McGreevy] (2008)

- This talk: gravity duals of such RG flows?
- examples with string theory embeddings: e.g. M2-brane CFT's

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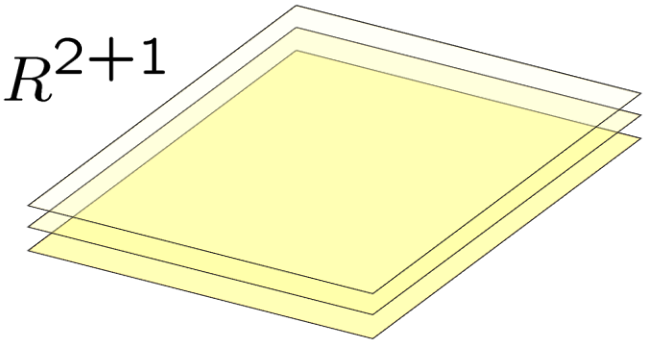
M2-branes in flux backgrounds

- Fermion mass on M2's probing R^8 : self-dual 4-forms

$$\star_8 G_4 = G_4$$

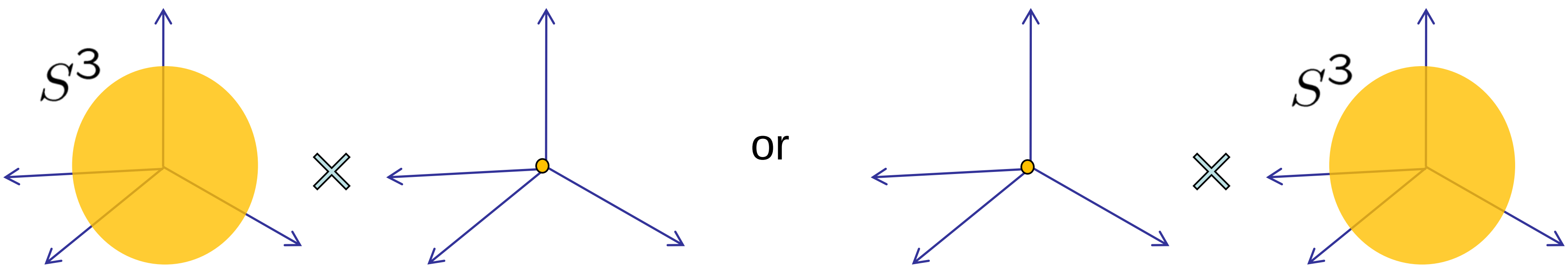


$$\times (R^4 \times R^4)/Z_k$$

$$G_{1234} = G_{5678} \sim \mu$$

fermion mass admitting maximal SUSY completion

- Discrete classical vacua from polarized M5's [Bena] [Myers]
- On the field theory side, nontrivial scalar VEV's



The field theory (perserving all Poincare SUSY)

- N=6 Chern-Simons-matter theory [ABJM 2008]: fields & charges (I=1,2,3,4)

fields	$U(N)\times U(N)$	$SO(6)_R$	$U(1)_b$
Z_I	$(N,\bar{N})$	$\mathbf{4}$	$+\frac{1}{2}$
Ψ^I	$(N,\bar{N})$	$\bar{\mathbf{4}}$	$+\frac{1}{2}$
A_μ	$(\mathrm{adj},1)$	$\mathbf{1}$	0
$\tilde{A}_\mu$	$(1,\mathrm{adj})$	$\mathbf{1}$	0

- Lagrangian with mass deformation: CS level k is inverse coupling const.

$$\mathcal{L} = \frac{k}{4\pi}\mathrm{tr}\left(AdA-\frac{2i}{3}A^3\right)-\frac{k}{4\pi}\mathrm{tr}\left(\tilde{A}d\tilde{A}-\frac{2i}{3}\tilde{A}^3\right)-D_\mu Z_I D^\mu \bar{Z}^I$$
$$-\frac{4\pi^2}{3k^2}\mathrm{tr}\left(Z_I\bar{Z}^I Z_J\bar{Z}^J Z_K\bar{Z}^K+\bar{Z}^I Z_I\bar{Z}^J Z_J\bar{Z}^I Z_K+4Z_I\bar{Z}^J Z_K\bar{Z}^I Z_J\bar{Z}^K\right.$$
$$\left.-6Z_I\bar{Z}^J Z_J\bar{Z}^I Z_K\bar{Z}^K\right)+\text{fermions}$$

$$-\mathrm{tr}\left(m^2\bar{Z}^I Z_I+\frac{4\pi}{k}(Z_I\bar{Z}^I Z_J\bar{Z}^K-\bar{Z}^I Z_I\bar{Z}^K Z_J)\Omega^J{}_K\right)+\text{fermions}$$

$$\Omega^I{}_J\equiv m\,\mathrm{diag}(1,1,-1,-1)$$

- Vacua: scalars Z_1, Z_2, Z_3, Z_4 “spaning” R^8 assume nontrivial VEV

Non-relativistic CFT

- Simplest vacuum: all scalars VEV's =0 ('symmetric' vacuum)
- For this vacuum, we have non-relativistic CFT in IR [Nakayama-Sakaguchi-Yoshida][Lee-Lee-Lee] (2009) (can exist for more general vacua…)
- Use $U(1)_b$ charge as particle number symmetry

$$Z_I = \frac{\hbar}{\sqrt{2m}}\; z_I\; e^{-imc^2 t/\hbar}$$

- Take the limit $c \to \infty$ or $\frac{p^2}{2m} \ll mc^2$ (after restoring c & Plank const.)

$$\mathcal{L} \;=\; \mathrm{tr}\left[i\hbar \bar{z}^I D_t z_I - \frac{\hbar^2}{2m}|D_i z_I|^2 + \frac{\pi\hbar^2}{mk}(z_I\bar{z}^I z_J \bar{z}^K \Omega^J{}_K - \bar{z}^I z_I \bar{z}^J z_K \Omega^K{}_J)\right]$$
$$+\frac{k\hbar}{4\pi}\mathrm{tr}\left[AdA - \frac{2i}{3}A^3 - \tilde{A}d\tilde{A} + \frac{2i}{3}\tilde{A}^3\right] + \text{fermions}$$

- No sextic, just quartic potential (anisotropic scale invariance)

Non-relativistic CFT *(continued)*

- particle number (N) from $U(1)_b$ (only positively charged modes kept)
- translations (H, P_i), rotation (J), Galilean boosts (G_i)
- scale inv. (D) with z=2: (scale dim=1 for bosons/fermions) $\delta t = \alpha^2 t, \ \delta x_i = \alpha x_i$
- special conformal transformation (K): (also, appropriately transform fields)

$$\delta t = \alpha t^2, \ \delta x_i = -\alpha t x_i$$

- Schrodinger algebra:

$$[H, P_\pm] = 0 \ , \quad [P_+, P_-] = 0 \ , \quad [J, H] = 0 \ , \quad [J, H] = 0 \ ,$$
$$[J, P_\pm] = \pm P_\pm \ , \quad [G_+, G_-] = 0 \ , \quad [J, G_\pm] = \pm G_\pm \ ,$$
$$i[H, G_\pm] = P_\pm \ , \quad i[P_+, G_-] = i[P_-, G_+] = 2m\mathcal{N} \ .$$

} Galilean algebra

$$i[D, H] = 2H \ , \quad i[D, K] = -2K \ , \quad i[K, H] = D$$

} SO(2,1)

… and…

$$[D, J] = 0 \ , \quad i[D, P_\pm] = P_\pm \ , \quad i[D, G_\pm] = -G_\pm$$
$$[K, J] = 0 \ , \quad i[K, P_\pm] = -G_\pm \ , \quad [K, G_\pm] = 0 \ ,$$

- Classical action has 2 extra NR conformal SUSY: 14 SUSY in total

A gravity dual of NR CFT

- At least Galilean + scale symmetries are generic in IR for any type of mass deformations (classically), when all dynamical degrees are massive.
- Question: what is the gravity dual of NR CFT in the IR?
- candidate: Schrodinger geometry [D.T. Son] [Balasubramanian,McGreevy] (2008)

$$ds_5^2 = L^2\left[-ar^4(dx^+)^2 + r^2(d\vec{x}^2 + dx^+dx^-) + \frac{dr^2}{r^2}\right]$$

translational symmetry along x^- :
particle number symmetry

- Sch_5 for 2+1d NR CFT obtained by perturbing 3+1d relativistic CFT (i.e. AdS_5) by irrelevant operators breaking 4d symmetry [Maldacena-Martelli-Tachikawa] [Herzog-Rangamani-Ross] [Adams-Balasubramanian-McGreevy] (2008)
- We want flows from AdS_4 to Sch_5: solutions with AdS_4/Sch_5 in UV/IR

Gravity solutions for a holographic flow

- We want to clearly see if Sch_5 is indeed suitable for describing our NR M2-brane system: flow solution for **any** mass deformation of AdS_4 x S^7?

- 4d SUGRA for AdS_4 x SE_7 (skew-whiffed) [Gauntlett, S.K., Varela, Waldram] (2009)

$$
\begin{aligned}
ds^2_{11} &= ds^2_4 + e^{2U} ds^2(KE_6) + e^{2V}\eta^2 \\
G_4 &= 6e^{-6U-V}(-1+h^2)\mathrm{vol}_4 + dh \wedge J \wedge \eta + 2hJ \wedge J \\
ds^2_4 &= e^{2A(r)}(\eta_{\mu\nu}dx^\mu dx^\nu) + dr^2
\end{aligned}
$$

self-dual 4-forms in R^8 or $C(SE_7)$

want 2+1d Poincare invariant flow:
flowing to IR itself does not break it

$$\eta \equiv d\psi + \theta,\ d\theta = 2J$$

- Flow from AdS_4 to Sch_5: possible if U(1) circle of SE_7 participates in Sch_5
- plausible from QFT: used internal U(1) as particle number symmetry.

Gravity solutions for holographic flow *(continued)*

- As will be clear shortly, demand that the U(1) circle suitably shrinks in IR:

  IR solution in small radius expansion is given by

$$
\begin{aligned}
e^V &= \rho\left(1 - \frac{7}{16}\rho^2 + \frac{69}{256}\rho^4 - \frac{2209}{12288}\rho^6 + \cdots\right) \\
e^A &= a_0\rho\left(1 + \frac{3}{16}\rho^2 - \frac{15}{256}\rho^4 + \frac{329}{12288}\rho^6 + \cdots\right)
\end{aligned}
\qquad \text{with } U = 0,\ h^2 = 1 - e^{2V},\ \rho = e^{\sqrt{2}r}
$$

$$
\begin{aligned}
ds^2_{11} &= ds^2_4 + e^{2U} ds^2(KE_6) + e^{2V}\eta^2 \\
G_4 &= 6e^{-6U-V}(-1+h^2)\mathrm{vol}_4 + dh \wedge J \wedge \eta + 2hJ \wedge J \\
ds^2_4 &= e^{2A(r)}(\eta_{\mu\nu}dx^\mu dx^\nu) + dr^2
\end{aligned}
$$

- Exact solution [Gauntlett-Sonner-Wiseman] (2009): "zero T limit of neutral black hole"

$$
\begin{aligned}
ds^2_{4E} &= = r^2(-dt^2 + d\vec{x}^2) + g^{-1}(r)dr^2 \\
h(r) &= \frac{\sqrt{3}\sqrt{\frac{6\sqrt{3}(2r^8+4r^4+1)}{\sqrt{Z}} - Z + 9} - \sqrt{3Z} + 3}{6(r^4+1)} \\
Z(r) &\equiv \frac{2\cdot 6^{2/3}(r^4+1)r^{8/3}}{(\sqrt{48r^4+81}-9)^{1/3}} - 6^{1/3}(r^4+1)\left(\sqrt{48r^4+81}-9\right)^{1/3} r^{4/3} + 3
\end{aligned}
$$

- Checked its IR expansion; also, separately found numerical flow solution.

Gravity dual of the non-relativistic limit

- 1^{st} step of QFT non-relativistic limit: redefine energy
- Gravity: redefine IR coordinate $x^- = \psi - a_0 t$
- Discarding anti-particles with small NR energy: suitably scale to IR $\rho = 0$

$$
\begin{aligned}
ds^2_{11} &= a_0^2\rho^2\left(1+\frac{3}{16}\rho^2+\cdots\right)^2(-dt^2+d\vec{x}^2)+\frac{d\rho^2}{2\rho^2}+\rho^2\left(1-\frac{7}{16}\rho^2+\cdots\right)^2(d\psi+\theta)^2+ds^2(KE_6)\\
G_4 &= -6a_0^3\rho^4\,(1+\cdots)\,\mathrm{vol}_3\wedge dr-\frac{1}{2}d\left(\rho^2+\cdots\right)\wedge J\wedge\eta+(2+\cdots)\,J\wedge J
\end{aligned}
$$

$$
\text{scaling limit } \rho\to\epsilon\rho,\ x^+\to\epsilon^{-2}x^+,\ \vec{x}\to\epsilon^{-1}\vec{x},\ x^-\to x^- \text{ with } \epsilon\to 0^+
$$

$$
\begin{aligned}
ds^2_{11} &= \boxed{-\frac{5a_0^2}{4}\rho^4(dx^+)^2+\rho^2\left(a_0^2d\vec{x}^2+2a_0dx^+(dx^-+\theta)\right)+\frac{d\rho^2}{2\rho^2}}+ds^2(KE_6)\\
G_4 &= -3\sqrt{2}a_0^3\rho^3dx^+\wedge dx^1\wedge dx^2\wedge d\rho-a_0\rho d\rho\wedge J\wedge dx^++2J\wedge J\ .
\end{aligned}
$$

- This Sch_5 is related to a known one. [O. Colgain, Varela, Yavartanoo] (2009)

Comments on the IR solution

- The solution near $\rho = 0$ might look like AdS_5 fibered over KE_6

$$ds^2_{11} = a_0^2\rho^2\left(1+\frac{3}{16}\rho^2+\cdots\right)^2(-dt^2+d\vec{x}^2)+\frac{d\rho^2}{2\rho^2}+\rho^2\left(1+\frac{7}{16}\rho^2+\cdots\right)^2(d\psi+\theta)^2+ds^2(KE_6)$$

$$G_4 = -6a_0^3\rho^4\,(1+\cdots)\,\mathrm{vol}_3\wedge dr-\frac{1}{2}d\left(\rho^2+\cdots\right)\wedge J\wedge\eta+(2+\cdots)\,J\wedge J$$

- However, without sub-leading terms, AdS_5 fibered over KE_6 is not a solution. (Without fibration over KE_6, the direct product **is** an exact solution.)

- Also, one cannot take a consistent scaling limit towards this geometry.
- One might be tempted to try

$$\rho\to\epsilon\rho,\ t\to\epsilon^{-1}t,\ \vec{x}\to\epsilon^{-1}\vec{x},\ \psi\to\epsilon^{-1}\psi\ \text{with}\ \epsilon\to 0$$

- The last angle is periodic, meaningless to take it to be `large'

Some aspects of dual QFT

- flow from $AdS_4 \times S^7$ in skew-whiffed frame: SO(8) broken to $SU(4)\times U(1)_b$

$$\begin{aligned} \text{scalar} &: \mathbf{8}_v \to \mathbf{4}_{-1} + \bar{\mathbf{4}}_1 \\ \text{fermion} &: \mathbf{8}_c \to \mathbf{1}_2 + \mathbf{1}_{-2} + \mathbf{6}_0 \\ \text{supercharge} &: \mathbf{8}_s \to \mathbf{4}_1 + \bar{\mathbf{4}}_{-1} \end{aligned}$$

representation changed without skew-whiffing: 6_0 is the ABJM supercharge

- Dual mass operator: fermion mass

$$M_{ab}\psi^a\psi^b$$

(a,b=1,2, … 8)

$$M_{ab} : \mathbf{35}_+ \to \mathbf{1}_0 + \mathbf{1}_4 + \mathbf{1}_{-4} + \mathbf{6}_2 + \mathbf{6}_{-2} + \mathbf{20}$$

$$m_f(\tfrac{1}{2}\Psi^i\Psi^i - 3\bar{\xi}\xi)$$

(i=1,2, … 6)

- Boson masses: $m_b^2 Z_I \bar{Z}^I$
- mass ratios for bosons/fermions in UV: satisfies a supertrace constraint

$$\mathrm{Str}(M^2) = 8{m_b}^2 - \mathrm{tr}({M_f}^2) = 0$$

Comments on SUSY mass deformations

- So is the story same for maximally supersymmetric mass-deformation?
- SUSY Schrodinger solutions for M2: only found solutions with different properties (without 14 SUSY) [Ooguri-Park] [Jeong-Kim-Lee-O Colgain-Yavartanoo]

- For some vacua, SUSY spontaneously broken (or vacua are absent): completely classified by a Witten index calculation [Hee-Cheol Kim, S.K.]
- In particular, w/ VEV=0, SUSY broken when $N > k$: always broken in the large N limit with $N \gg k$, in which gravity description is reliable.

- Flow with spontaneously broken SUSY and IR Schrodinger symmetry? Open question (perhaps the non-SUSY vacuum is meta-stable…)

Comments on SUSY mass deformations *(continued)*

- For the SUSY vacua, the gravity duals are well understood.

- gravity dual for k=1 (no orbifold): bubbling geometry [Lin-Lunin-Maldacena] (2004)
- # of classical SUSY vacua is more than the LLM gravity solution. [Gomis, Rodriguez-Gomez, v. Raamsdonk, Verlinde] (2008)
- With SUSY breaking, complete agreement [S.K., Hee-Cheol Kim] (2010)

- k>1: Z_k orbifold, fractional M2's [S. Cheon, H.-C. Kim, S.K.] (2011)

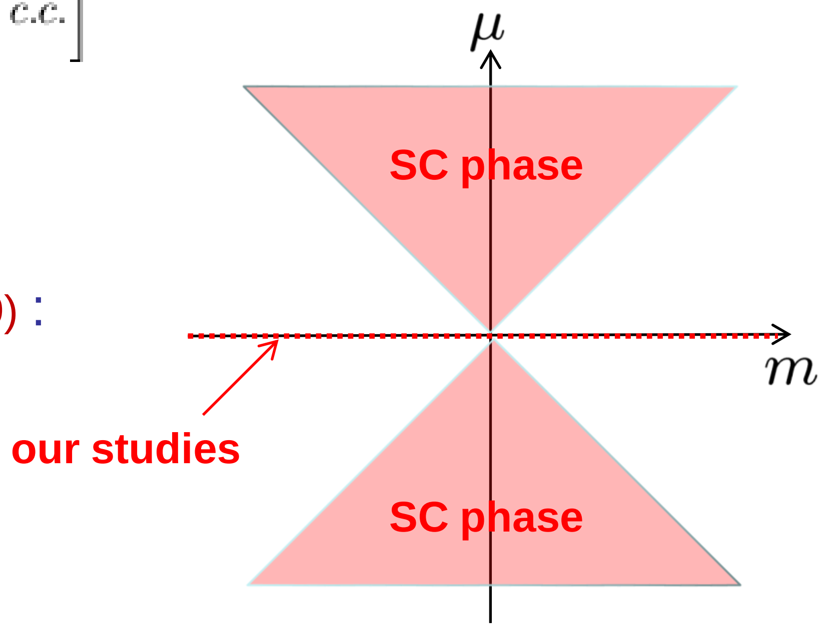
- Generic SUSY vacua partly break U(N)xU(N) gauge symmetry: NR CFT involving massive vectors? Further studies on the gravity dual?

Comments on holographic superconductors

- The SE_7 flows discussed here can be viewed in a broader context, involving holographic superconductors. [Gauntlett-Sonner-Wiseman] (2009)
- One can add a charged scalar & gauge field: (H_2 and dA_2 related)

$$
\begin{aligned}
ds^2_{11} &= ds^2_4 + e^{2U} ds^2(KE_6) + e^{2V}(\eta + A_1)^2 \\
G_4 &= 6e^{-6U-V}(-1 + h^2 + |\chi|^2)\mathrm{vol}_4 + H_2 \wedge J + dh \wedge J \wedge (\eta + A_1) + 2hJ \wedge J \\
&\quad + \sqrt{3}\left[\chi(\eta + A_1) \wedge \Omega - \frac{i}{4} D\chi \wedge \Omega + c.c.\right]
\end{aligned}
$$

- QFT at nonzero chemical potential
- the phase diagram at zero T [GSW] (2009) :



Concluding remarks

- Holography of mass-deformed M2: many vacua & interesting IR physics

- In this talk, I focused on the gravity dual of a non-relativistic CFT obtained by a mass deformation of relativistic CFT.

- More general gravity solutions with Schrodinger symmetry?

✓ If one obtains 2+1d NR CFT from 3+1d CFT with irrelevant operators, presence of x^- is crucial: just extra QFT direction in 3+1d

✓ Exotic property of thermodynamics: presence of infinite particle species, coming from KK tower [Barbon-Fuertes] (2009) [Balasubramanian-McGreevy] (2010)

✓ Our NR limit does not necessarily involve isometry realization.