Mesons as Open Strings
in Holographic QCD

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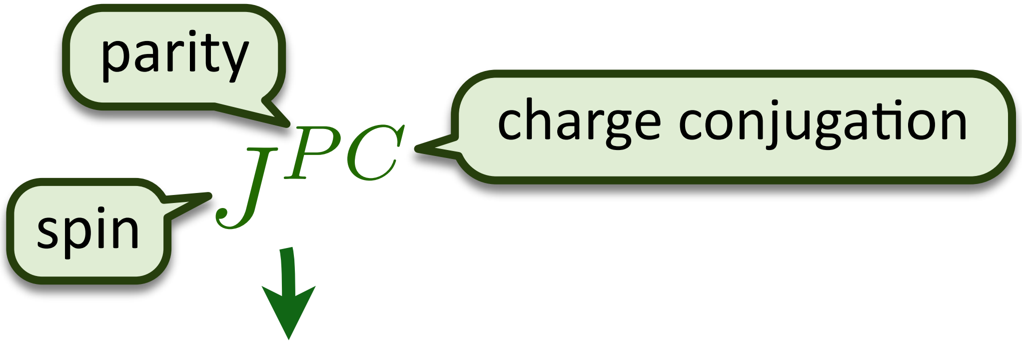
based on: **arXiv:1005.0655**

with **T. Imoto and T. Sakai**

Seminar @ Cambridge 2/17/2011

1 Introduction

mesons (N_f = 2, Isovector)



mass (MeV)

△ ... not established

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	985	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
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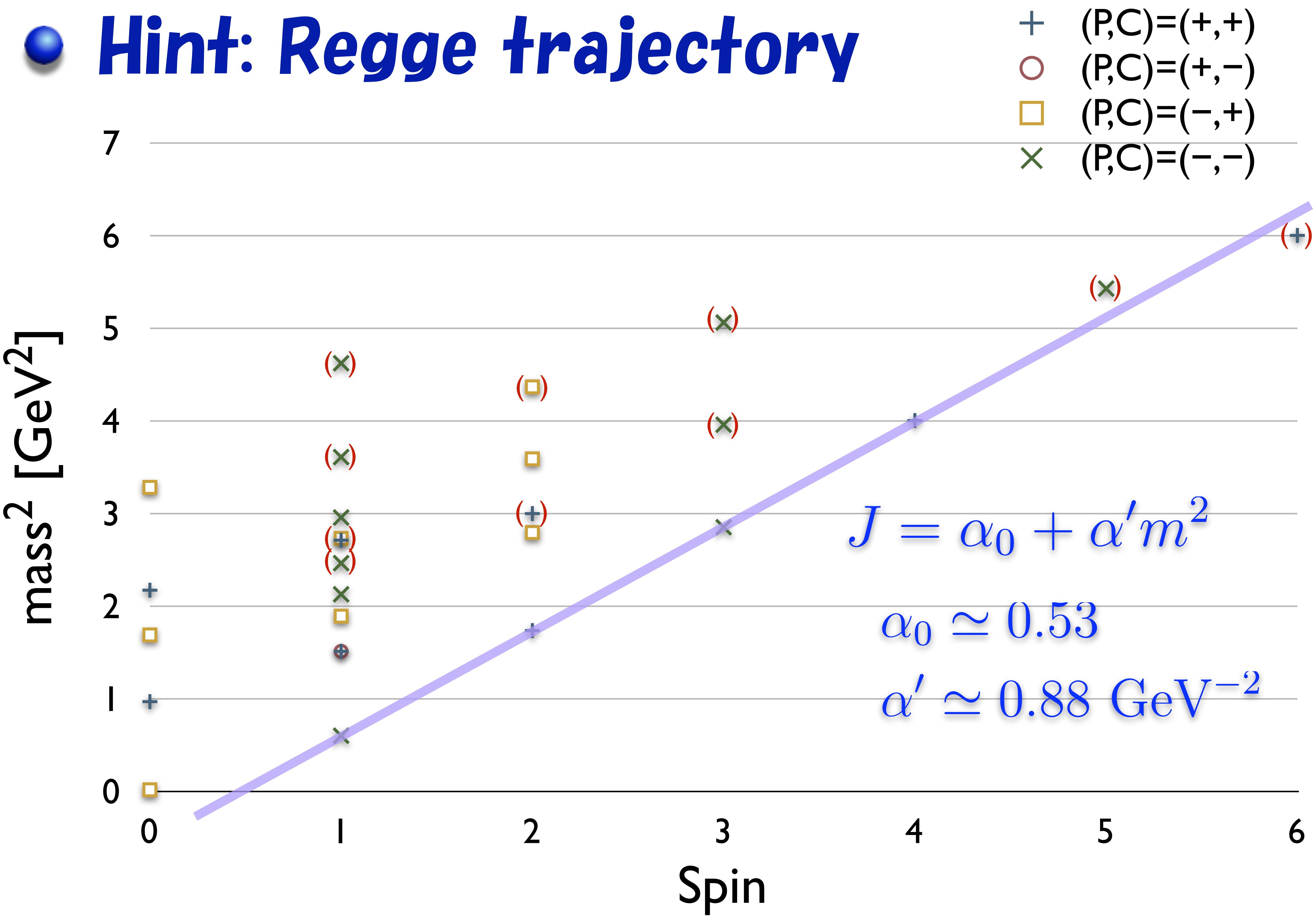
Q: How can we understand these numbers?

Hint: Regge trajectory



$$J = \alpha_0 + \alpha' m^2$$

$$\alpha_0 \simeq 0.53$$

$$\alpha' \simeq 0.88\ \mathrm{GeV}^{-2}$$

Mesons are strings !?

Difficulties in the old days

- Consistent in 10 dim space-time
- ∃ massless particles with $J = 1$ and $J = 2$
  (open) (closed)

$$J = \alpha_0 + \alpha' m^2 \qquad \alpha_0 = 1 \; !$$ (for open string)

Not consistent with meson spectrum !?

Gauge / String duality suggests

- 4 dim gauge theory $\xleftrightarrow[\text{dual}]{}$ 10 dim string theory
  (in a certain curved background)
- massive particles in 4 dim $\xleftrightarrow[\text{dual}]{}$ massless particles in 10 dim

The above difficulties can be solved !!

Holographic QCD

- 4 dim QCD ⟷ 10 dim string theory

  dual

  (in a certain curved background)

  holographic QCD

(in some approximation: low energy , large N_c , large λ , ...)

D4/D8-branes in type IIA string theory

[T.Sakai and S.S. 04]

- mesons are open strings on D8
- $\pi,\ \rho,\ a_1,$ etc. are obtained from the massless mode

0^{-+} 1^{--} 1^{++}

$$m_{a_1}/m_\rho \simeq \begin{cases} 1.53 & \text{(theory)} \\ 1.59 & \text{(exp)} \end{cases}$$

Q: What about the other mesons ?

→ Consider massive modes (excited strings)

Plan

✓ ① Introduction

② Brief review of the model

③ Meson spectrum

④ Comparison with data

⑤ Discussion

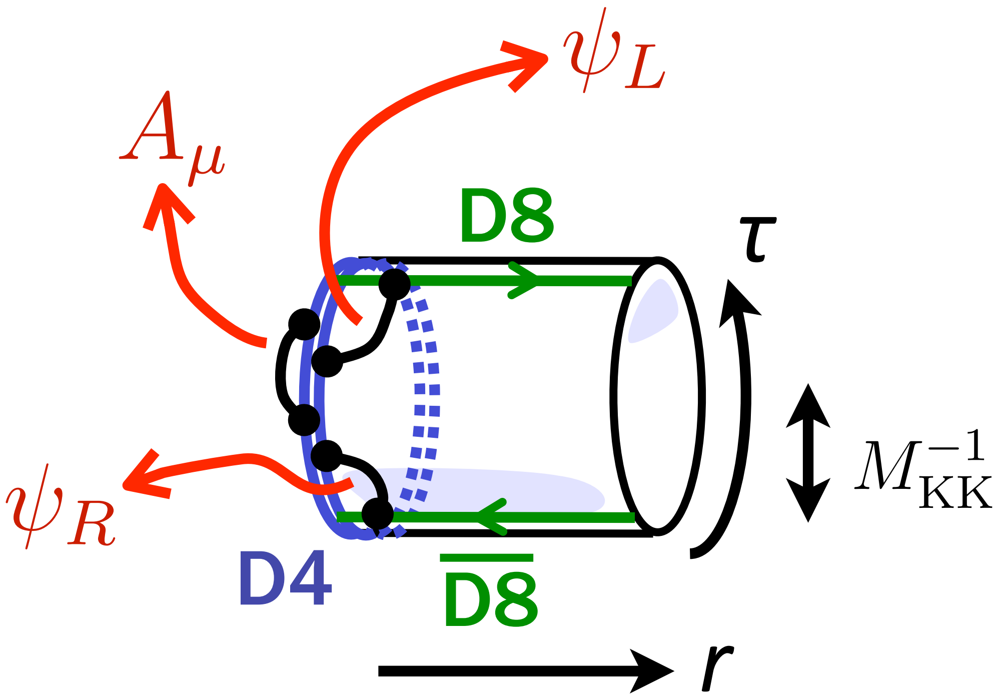
② Brief review of the model

Brane configuration

[T. Sakai and S.S. 04]

	x^0 x^1 x^2 x^3	τ	x^5 x^6 x^7 x^8 x^9
D4 x N_C	○ ○ ○ ○	○	
D8-$\overline{\text{D8}}$ x N_f	○ ○ ○ ○		○ ○ ○ ○ ○

S^1 with ~~SUSY~~ b.c.

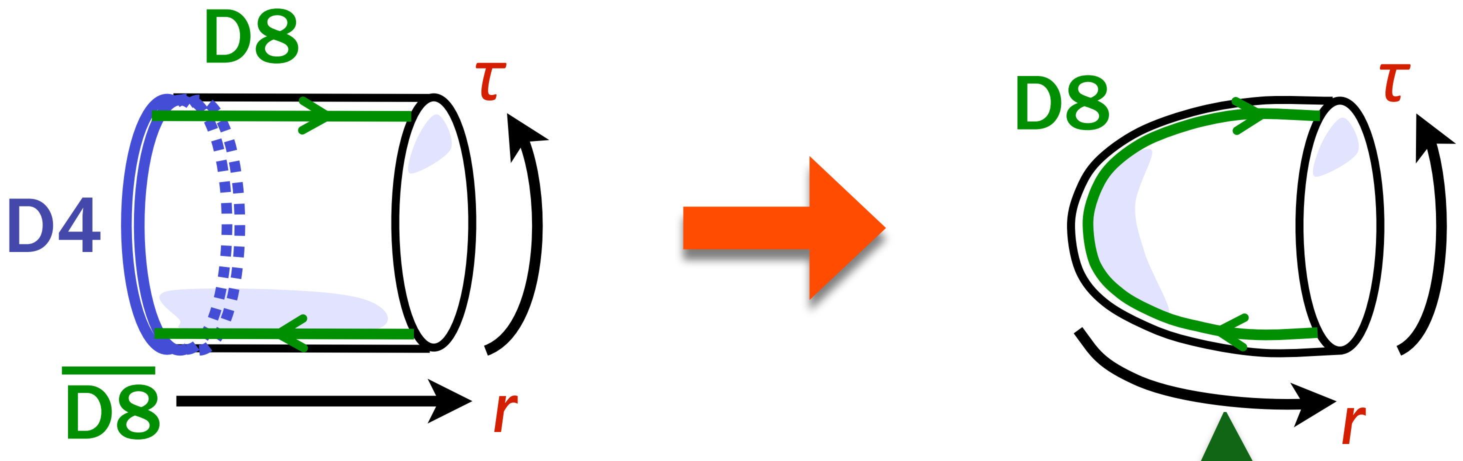


(radial direction of $x^{5\sim9}$)

4 dim $U(N_C)$ QCD
with N_f massless quarks
(at low energy)

Holographic description

- replace D4 with the corresponding curved background
  [Witten 98]
- D8 are treated as probe brane (assuming $N_c \gg N_f$)

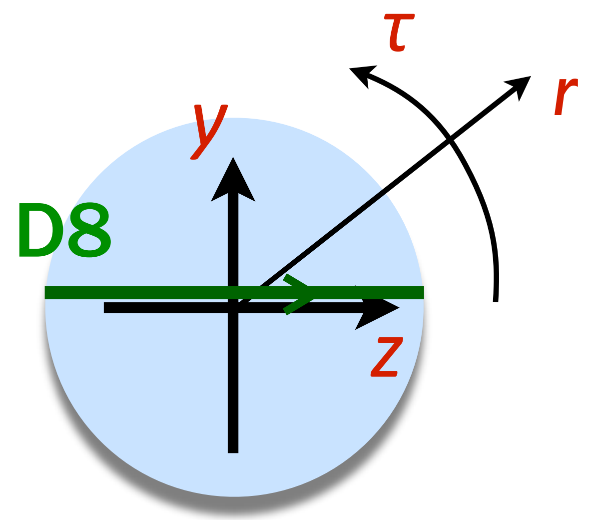


$$\underset{x^{0\sim3}}{\mathbf{R}^{1,3}} \times \underset{\tau}{S^1} \times \underset{(r,\,\theta^{1\sim4})}{\mathbf{R}^5}$$

S^4

$$\underset{x^{0\sim3}}{\mathbf{R}^{1,3}} \times \underset{\underbrace{(r,\tau)}}{\boxed{\mathbf{R}^2}} \times \underset{\theta^{1\sim4}}{S^4} \quad \text{(topology)}$$

$$\begin{cases} y = r\sin\tau \\ z = r\cos\tau \end{cases}$$



Hadrons in the model

$\underset{x^{0\sim 3}}{\mathbf{R}^{1,3}} \times \underset{(z,y)}{\mathbf{R}^2} \times S^4$ with D8 extended along $\underset{(\mu=0\sim 3)}{\overbrace{(x^\mu, z)}^{5\text{dim}}} \times S^4$

particles in $\mathbf{R}^{1,3}$

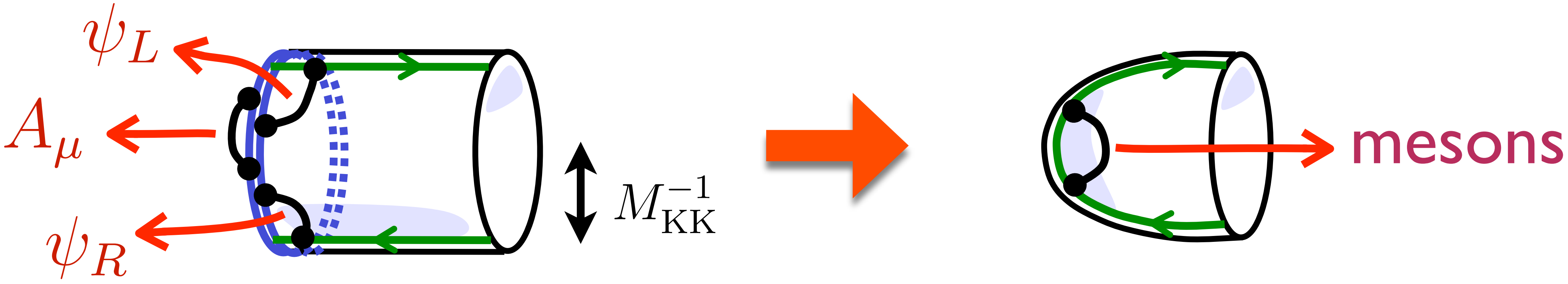
- closed strings ⟷ glueballs
- open strings on D8 ⟷ mesons
- D4 wrapped on S^4 ⟷ baryons

today's topic

QCD mesons vs artifacts

- Our brane config. is invariant under $SO(5) \curvearrowright S^4$
- quarks and gluons are invariant under $SO(5)$

  (non-invariant states are massive KK modes)

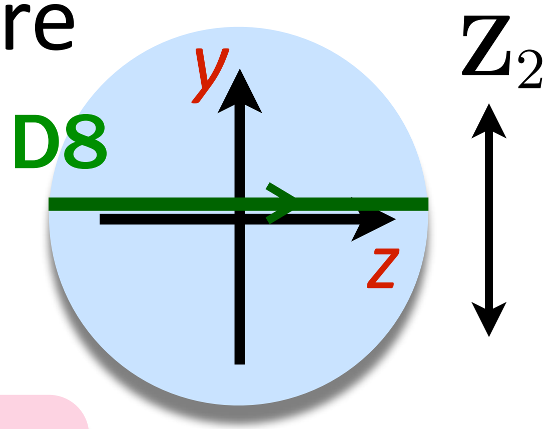


- Bound states of quarks and gluons are $SO(5)$ invariant

  (non-invariant states are artifacts made by unwanted massive modes)

- Similarly, we can show that quarks and gluons are invariant under Z_2 sym generated by $I_{y9}(-1)^{F_L}$

  I_{y9}: $(y, x^9) \rightarrow (-y, -x^9)$ ($\tau \rightarrow -\tau$)



Consider $SO(5) \rtimes \mathbf{Z}_2$ invariant states

③ Meson spectrum

Consider open strings attached on D8

’t Hooft coupling

$\lambda \sim (\text{curvature radius}/l_s)^2$

Strategy

① Consider flat space-time, (justified when $\lambda \gg 1$)
and quantize the open strings attached on D8.

space-time: $\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$ D8-brane: $(x^\mu, z) \times S^4$

(topology) $x^{0\sim 3}$ (z,y) $x^{6\sim 9}$

In the flat space-time limit,

$S^4 \Rightarrow R^4$, $SO(5) \Rightarrow$ rotation and translation of $x^{6\sim 9}$

② Pick up the $SO(5) \rtimes \mathbf{Z}_2$ invariant states.

→ reduced to 5 dim: (x^μ, z)

③ Recover the z dependence of the background (perturbatively).

- General rules for light-cone quantization (NS-sector)

  (light-cone direction $x^{\pm} = x^{0} \pm x^{1}$)

  - Fock vacuum $\quad |0\rangle_{\rm NS}$
  - creation op. $\quad \psi^{i}_{-r}$ fermion $\quad \alpha^{i}_{-n}$ boson $\quad (i = 2, 3, \cdots, 9)$

    $(r = 1/2,\ 3/2, \cdots) \quad (n = 1, 2, 3, \cdots)$
  - physical state $\quad \underbrace{\psi^{i_1}_{-r_1} \cdots \psi^{i_k}_{-r_k}}_{\text{odd}} \alpha^{j_1}_{-n_1} \cdots \alpha^{j_l}_{-n_l} |0\rangle_{\rm NS}$
  - mass $\quad m_0^2 = \dfrac{N}{\alpha'} \qquad N \equiv \sum_{s=1}^{k} r_s + \sum_{t=1}^{l} n_t - \dfrac{1}{2}$

- No $SO(5)$ invariant states in R-sector.

- Parity and Charge conjugation:

  P : $\quad (x^1, x^2, x^3, z) \to (-x^1, -x^2, -x^3, -z)$

  C : $\quad I_z \Omega (-1)^{F_L} \qquad I_z : z \to -z$

Massless mode (N=0)

- $\psi^{I}_{-1/2}|0\rangle_{\rm NS} \quad (I=2,3,z)$ → 5 dim gauge field $A_\mu,\ A_z$

μ=0,1,2,3

- $\psi^{A}_{-1/2}|0\rangle_{\rm NS} \quad (A=y,6,7,8,9)$ → not invariant under $SO(5)\rtimes \mathbf{Z}_2$

KK decomposition along z direction

Recovering the curved background, we obtain
5 dim $U(N_f)$ YM-CS theory in a curved space-time.

$$S_{\rm 5dim}=\kappa\int d^4x dz\,{\rm Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2+K(z)F_{\mu z}^2\right)+\frac{N_c}{24\pi^2}\int_5\omega_5(A) \qquad K(z)=1+z^2$$

$$A_\mu(x^\mu,z)=\sum_{n=1}^{\infty}B_\mu^{(n)}(x^\mu)\underline{\psi_n(z)}$$

(μ=0,1,2,3)

$$A_z(x^\mu,z)=\sum_{n=0}^{\infty}\varphi^{(n)}(x^\mu)\underline{\phi_n(z)}$$

complete sets

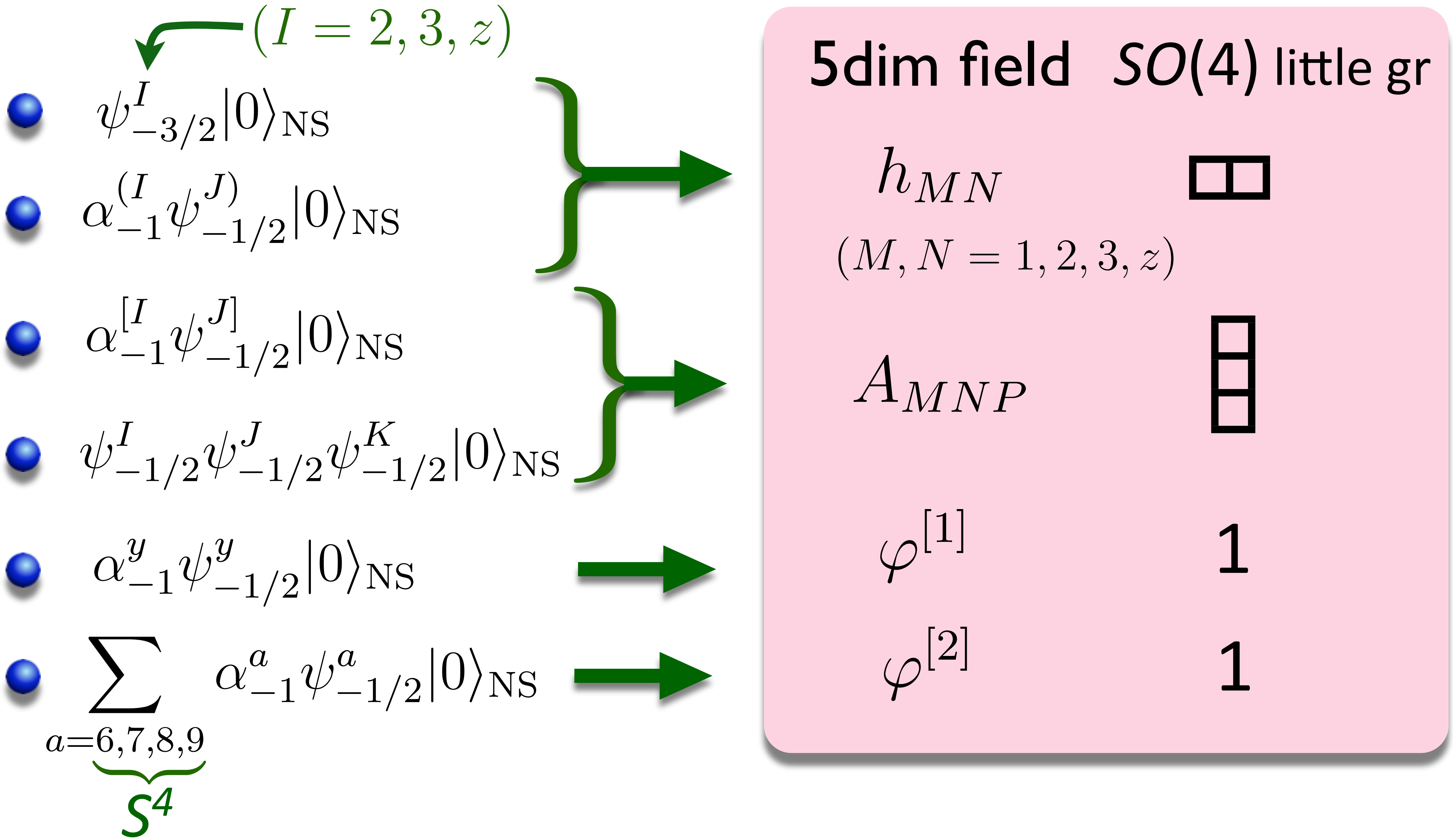
	$B_\mu^{(1)}$	$B_\mu^{(2)}$	$B_\mu^{(3)}$	…	$\varphi^{(0)}$	$\varphi^{(1)}$	…
J^{PC}	1^{--}	1^{++}	1^{--}	…	0^{-+}	eaten	
	ρ	a_1	ρ'	…	π		

[T.Sakai and S.S. 04]

$$-K(z)^{1/3}\partial_z(K(z)\partial_z\psi_n(z))=m_n^2\psi_n(z) \qquad \phi_n(z)=\partial_z\psi_n(z)$$

$\psi_n(z)$: eigenfunction $\quad m_n^2$: eigenvalue ⇒ mass^2 of $B_\mu^{(n)}$

First excited massive modes



$SO(5) \rtimes \mathbf{Z}_2$ invariant states:

$(I = 2, 3, z)$

- $\psi^I_{-3/2}|0\rangle_{\rm NS}$
- $\alpha^{(I}_{-1}\psi^{J)}_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^{[I}_{-1}\psi^{J]}_{-1/2}|0\rangle_{\rm NS}$
- $\psi^I_{-1/2}\psi^J_{-1/2}\psi^K_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^y_{-1}\psi^y_{-1/2}|0\rangle_{\rm NS}$
- $\underbrace{\sum_{a=6,7,8,9}}_{S^4} \alpha^a_{-1}\psi^a_{-1/2}|0\rangle_{\rm NS}$

5dim field	$SO(4)$ little gr
h_{MN} $(M, N = 1, 2, 3, z)$	$\begin{array}{\|c\|c\|}\hline & \\ \hline\end{array}$
A_{MNP}	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \\ \hline\end{array}$
$\varphi^{[1]}$	1
$\varphi^{[2]}$	1

KK decomposition along z direction

$$h_{MN}(x^\mu, z) = \sum_{n=0}^{\infty} h_{MN}^{(n)}(x^\mu)\phi_n(z)$$ etc.

lowest modes: $(i , j , k = 1, 2, 3)$

	$h_{ij}^{(0)}$	$h_{iz}^{(0)}$	$h_{zz}^{(0)}$	$A_{ijk}^{(0)}$	$A_{ijz}^{(0)}$	$\varphi^{[1,2](0)}$
J^{PC}	2^{++}	1^{+-}	0^{++}	0^{-+}	1^{--}	0^{++} x 2

Second excited massive mode

lowest modes:

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

Mass formula (naive shortcut)

- Flat space-time limit:

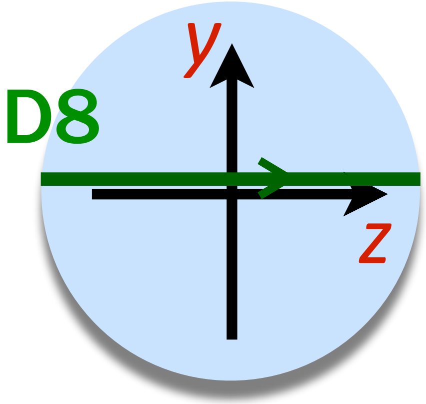
$$\boxed{m_0^2 = \frac{N}{\alpha'}}$$

$$\left(\alpha'^{-1} = \frac{4}{27}\lambda M_{\rm KK}^2\right)$$

$N = 0, 1, 2, \cdots$: excitation level

- Massive particle in curved space-time

$$S = -m_0 \int \sqrt{g_{tt}}\, dt = -m_0(1+z^2)^{1/4} \int dt$$



→ particle in potential: $V(z) = m_0(1+z^2)^{1/4}$

→ $$\boxed{M_n \simeq \underbrace{m_0 + \frac{1}{\sqrt{2}}\left(n+\frac{1}{2}\right) M_{\rm KK}}_{\text{harmonic oscillator approx.}} + \mathcal{O}(\lambda^{-1/2})}$$

$n = 0, 1, 2, \cdots$

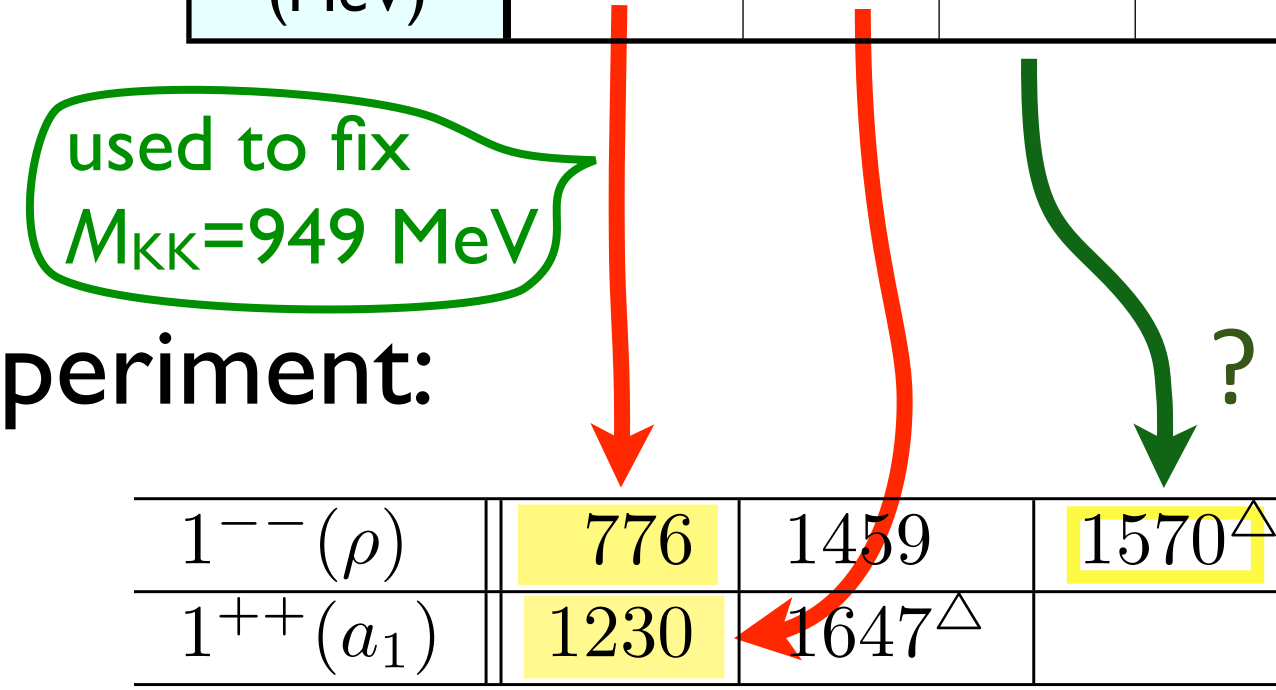
- More careful analysis shows that the *O*(1) term is not affected by the RR-flux, α' correction, etc.

4 Comparison with data

Massless mode

[T.Sakai and S.S. 04]

	$B_\mu{}^{(1)}$	$B_\mu{}^{(2)}$	$B_\mu{}^{(3)}$	$B_\mu{}^{(3)}$	…	$\varphi^{(0)}$
J^{PC}	1^{--}	1^{++}	1^{--}	1^{++}	…	0^{-+}
	ρ	a_1	ρ'	a'_1	…	π
mass (MeV)	[776]	1189	1607	2024	…	0



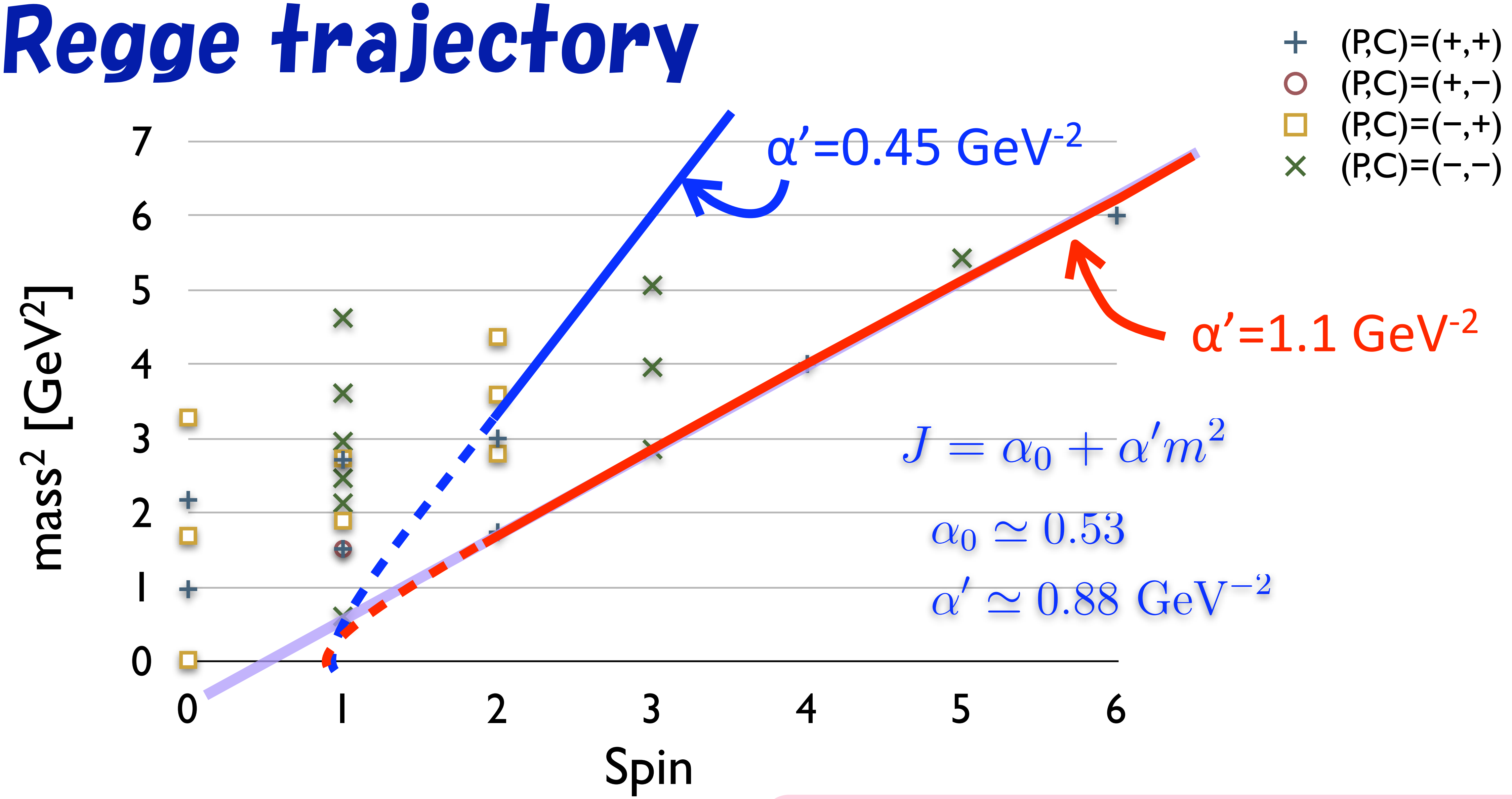
used to fix M_{KK}=949 MeV

experiment:

?

$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				

Regge trajectory



$$J = \alpha_0 + \alpha' m^2$$

$$\alpha_0 \simeq 0.53$$

$$\alpha' \simeq 0.88\ \mathrm{GeV}^{-2}$$

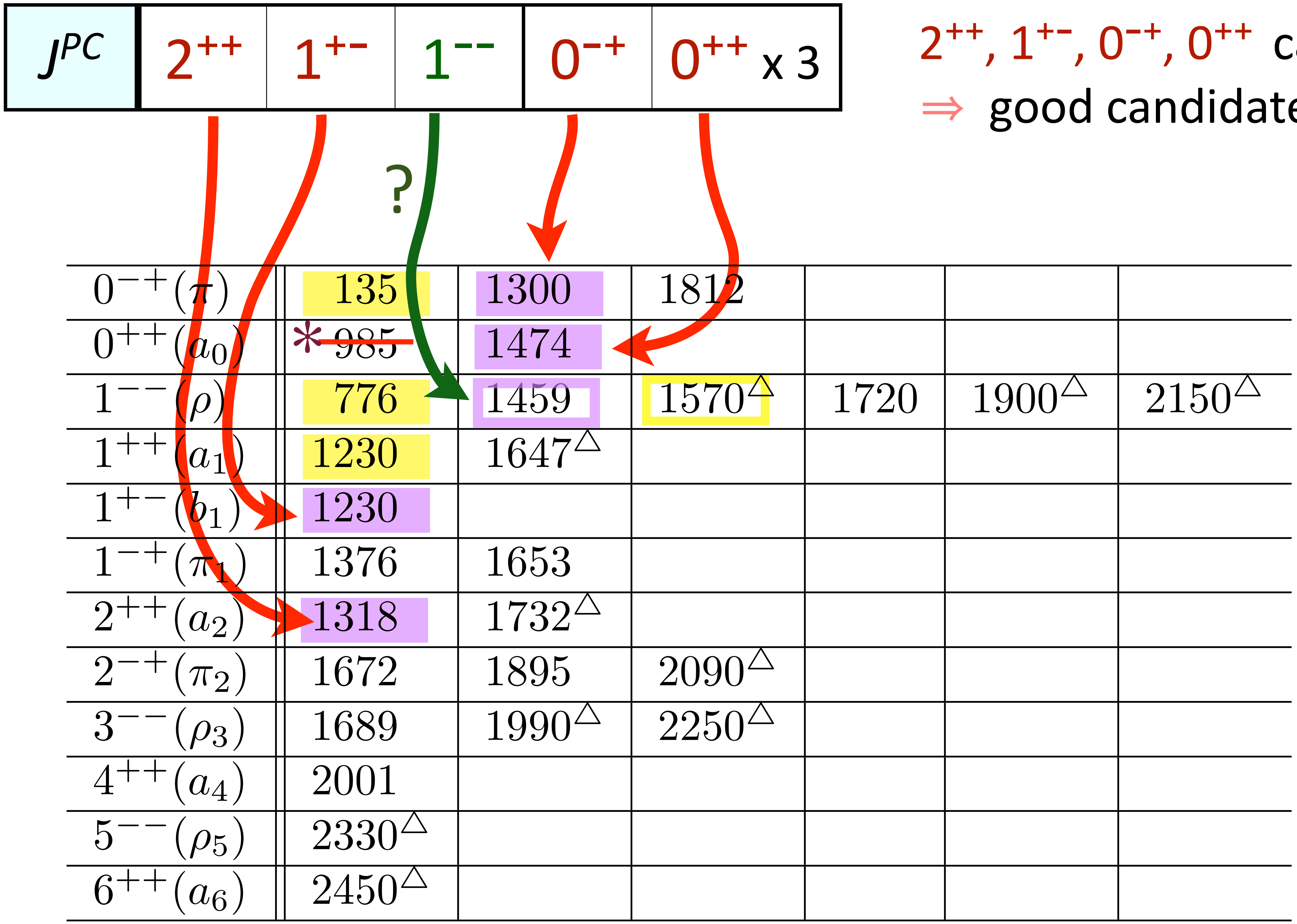
$$M_n \simeq \sqrt{\frac{N}{\alpha'}} + \frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right) M_{\mathrm{KK}} \Rightarrow \boxed{J \simeq 1 + \alpha' M^2 - \frac{\alpha'}{\sqrt{2}} M_{\mathrm{KK}} M + \mathcal{O}(\lambda^{-1})}$$

(for $N \geq 1$)

$N = J - 1,\ \ n = 0$

- If we use f_π to fix α', we obtain α' = 0.45 GeV^{-2}. This is unfortunately too small.
- If we set α'=1.1 GeV^{-2} we get very good fit.

First excited states (*N*=1, *n*=0)



J^{PC}	2^{++}	1^{+-}	1^{--}	0^{-+}	0^{++} x 3

2^{++}, 1^{+-}, 0^{-+}, 0^{++} cannot be N=0

⇒ good candidates for N=1

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	*~~985~~	1474				
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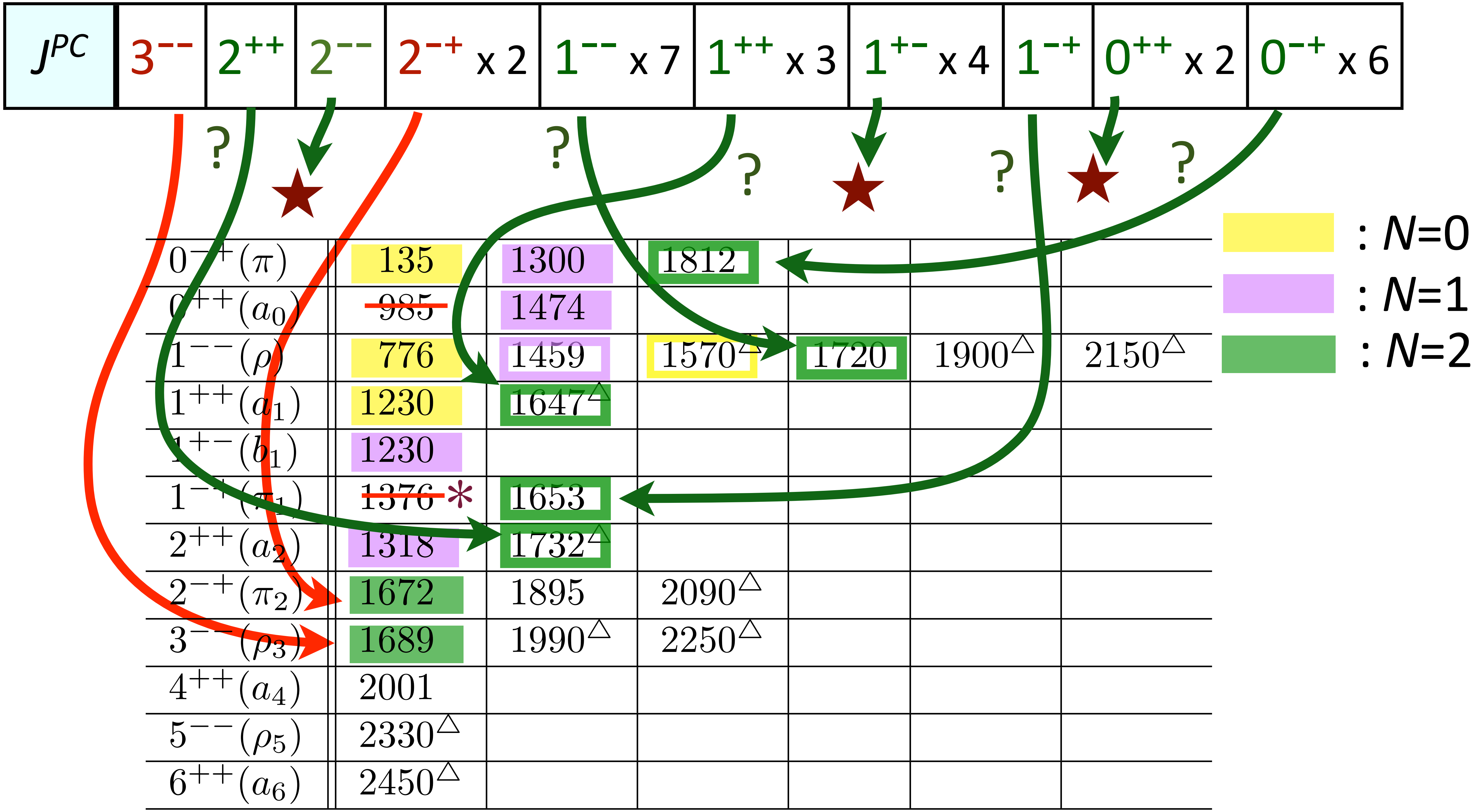
🟨 : N=0

🟪 : N=1

- degenerate around 1300 MeV
- *a_0 (980) is considered to be a four quark state.

Second excited states (*N*=2, *n*=0)



J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

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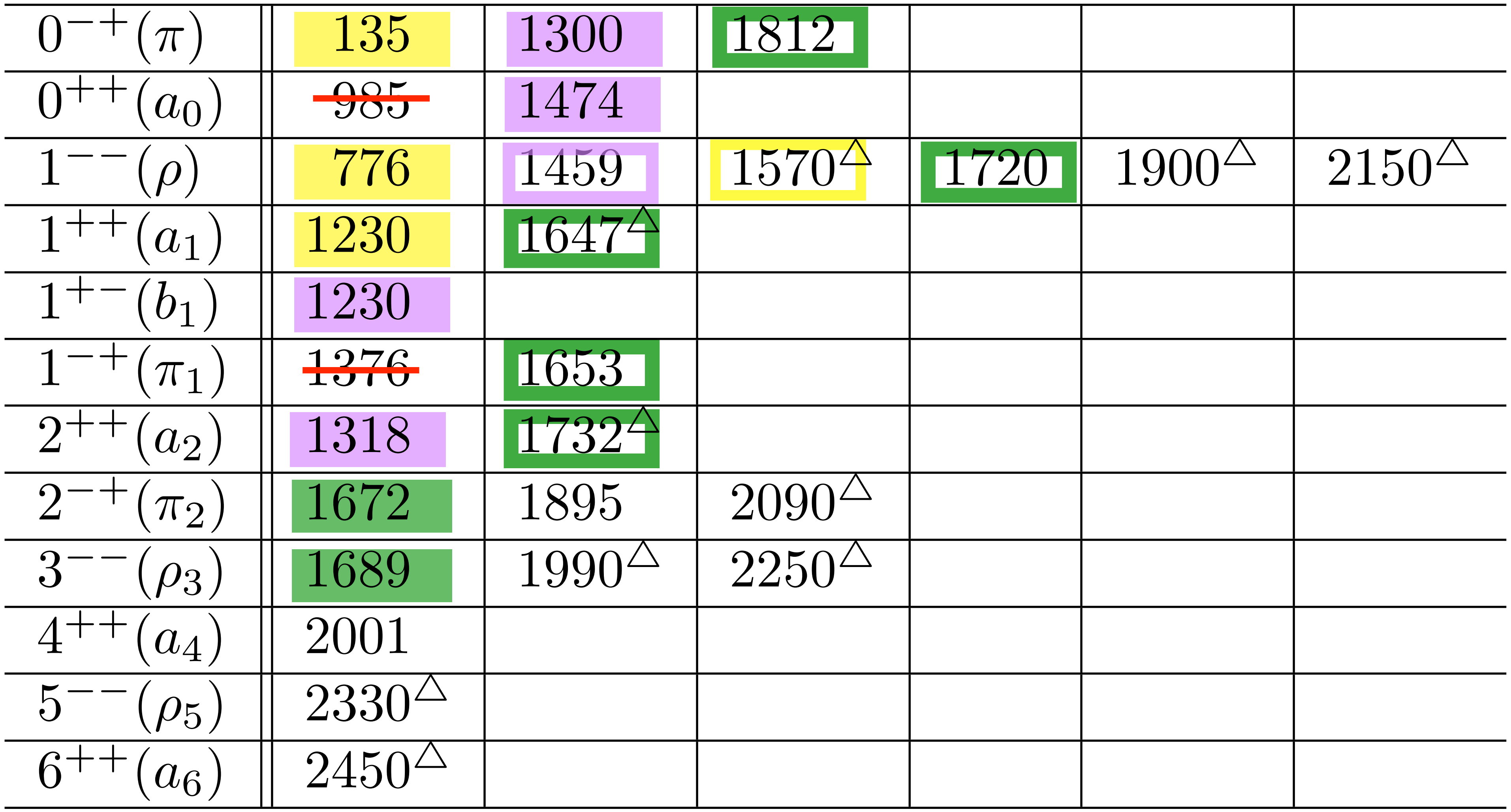
🟨 : *N*=0
🟪 : *N*=1
🟩 : *N*=2

- degenerate around 1700 MeV
- ★: prediction ?
- * π_1 (1400) is claimed to be a four quark state. (could be hybrid)

Summary



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🟨 : *N*=0

🟪 : *N*=1

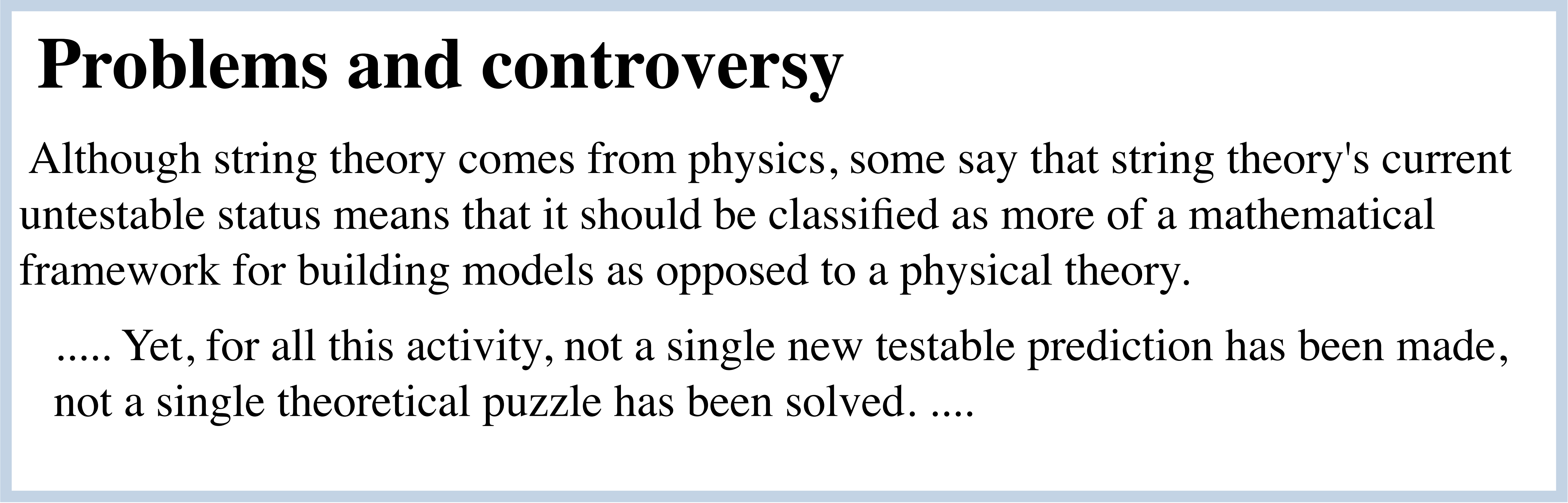
🟩 : *N*=2

— : 4 quarks

I think this is non-trivial.
What do you think?

5 Discussion

- Mesons are Strings
- Wikipedia says:



Problems and controversy

Although string theory comes from physics, some say that string theory's current untestable status means that it should be classified as more of a mathematical framework for building models as opposed to a physical theory.

..... Yet, for all this activity, not a single new testable prediction has been made, not a single theoretical puzzle has been solved. ....

Don’t criticize string theory
in this way anymore !