Inching Towards Strange Metallic Holography

David Tong

UNIVERSITY OF CAMBRIDGE

Based on “Towards Strange Metallic Holography”, 0912.1061
with Sean Hartnoll, Joe Polchinski and Eva Silverstein

Cambridge, Jan 2010

Plan of the Talk

- Motivation
  - Remedial Introduction to Conductivity
  - Anomalous Properties of Strange Metals

- Conductivity from Lifshitz Geometry
  - Dimensional Analysis
  - DC and Hall Conductivity
  - Optical Conductivity

Drude Model: DC Conductivity

- Ohm's Law: $\vec{E} = \rho \vec{j}$

Resistivity: ρ
Conductivity: $\sigma = 1/\rho$

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Scattering Time

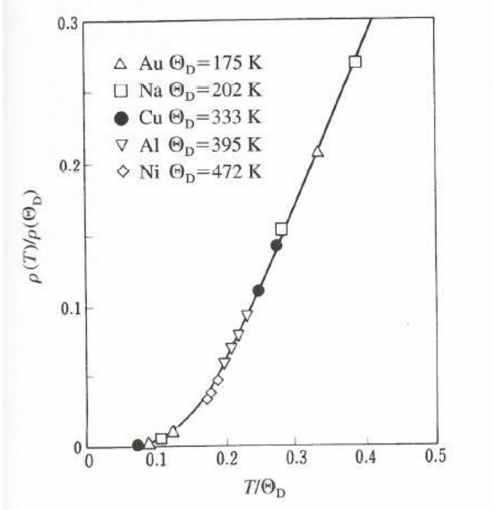
Scattering Mechanisms

- The temperature dependence of the resistivity sits in the scattering time:

Impurities: $\rho \sim T^0$

Phonons: $\begin{cases} \rho \sim T & (T > \Theta_D) \\ \rho \sim T^5 & (T < \Theta_D) \end{cases}$

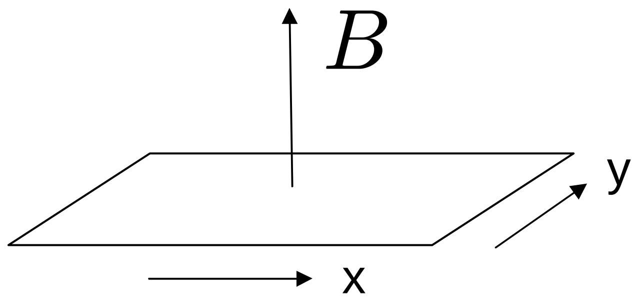
Electrons: $\rho \sim T^2$



Bardeen, 1940

- Resistivity in metals is typically due to phonons and impurities.

Drude Model: Hall Conductivity



$$\vec{j} = \sigma \vec{E}$$

Now a 2x2 matrix

$$\sigma_{xx} = \frac{1}{\rho}\frac{1}{1+\omega_c^2\tau^2} \qquad \sigma_{xy} = \frac{1}{\rho}\frac{\omega_c\tau}{1+\omega_c^2\tau^2} \qquad \left(\omega_c = \frac{eB}{mc}\right)$$

$$\boxed{\frac{\sigma_{xx}}{\sigma_{xy}} = \frac{1}{\omega_c\tau}}$$

Drude Model: AC Conductivity

- Fourier Transform: $\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$

  $\vec{E} = \vec{E}(\omega)e^{-i\omega t}$

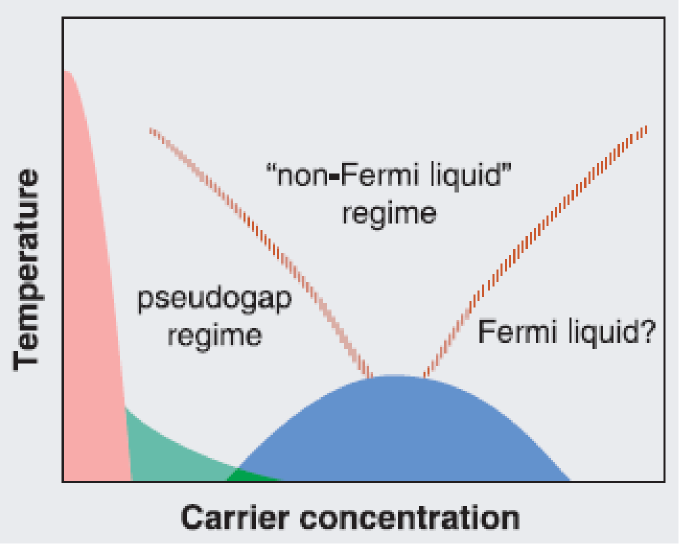
  $\vec{j} = \vec{j}(\omega)e^{-i\omega t}$

$$\boxed{\sigma(\omega) = \frac{1}{\rho}\frac{1}{1-i\omega\tau}}$$

- Note: $\sigma(\omega) \to i/\omega$ as $\omega \to \infty$

Strange Metals

- High Tc Superconductors at optimal doping
  - e.g, cuprates
- Suggestions that behaviour is governed by quantum critical point.
- Heavy Fermion materials have similar properties



Anomalous Properties

- Resistivity:

$$\rho \sim T$$

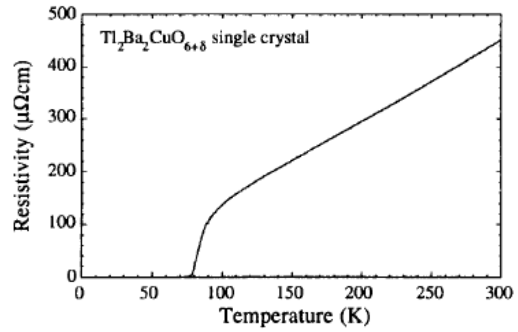
- Hall Conductivity

$$\sigma_{xx}/\sigma_{xy} \sim T^2$$

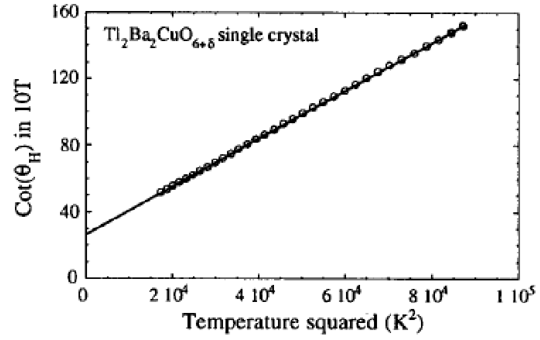
- AC Conductivity

$$\sigma(\omega) \to (i/\omega)^\nu$$

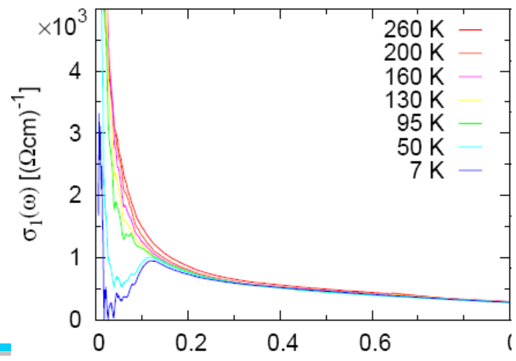
$$\nu \approx 0.65$$



Mackenzie, 1997



Mackenzie, 1997



Van der Marel et al., 2003

Challenge

- Reproduce this anomalous behaviour in conductivity from a (presumably strongly coupled) field theory.

Strategy

- Use AdS/CFT
  - Pros: Simple to compute transport properties in strongly interacting theories
  - Cons: Don't know how the theory is related to microscopic degrees of freedom
    - Low energy physics: meV not MeV
    - The theory is *defined* by the gravitational dual
    - Necessarily "large N"

The Set-Up

- Charged particles moving through a strongly coupled soup

Probe Brane

Gravitational Background

- We will take the strongly coupled soup to obey *Lifshitz Scaling*

Lifshitz Scaling

- Non-Relativistic Conformal Invariance

z: dynamical critical exponent

$$\vec{x} \to \lambda \vec{x} \qquad\qquad t \to \lambda^z t$$

$\vec{x} = (x, y)$: d=2 spatial dimensions

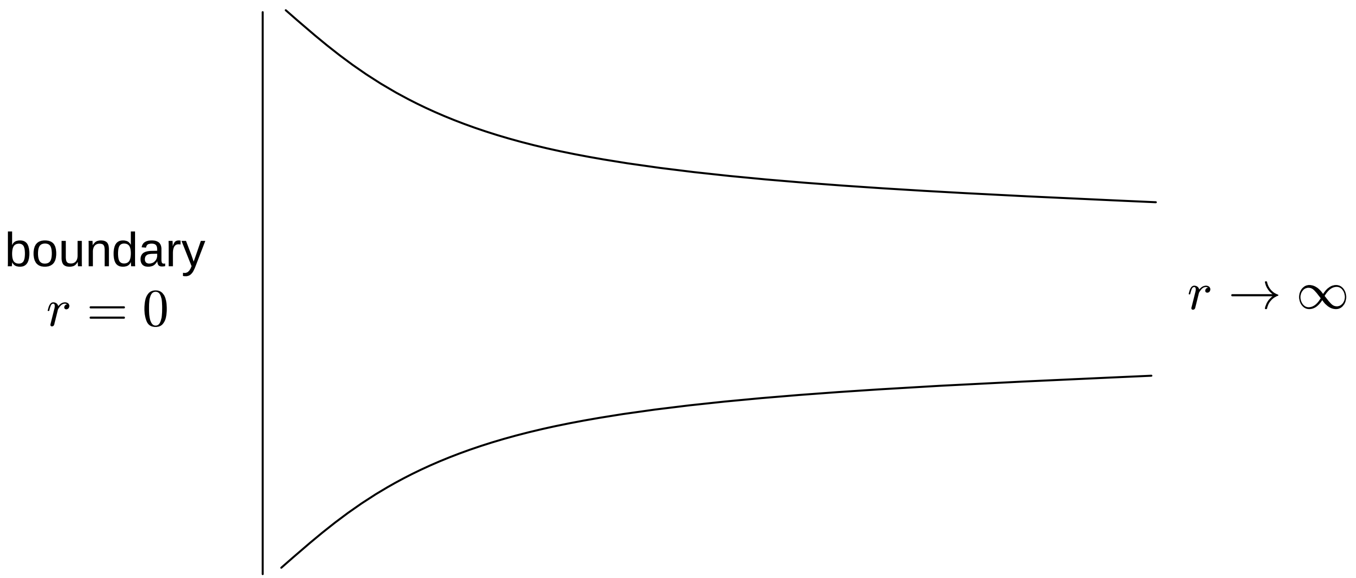
- A weakly coupled example with z=2:

$$S = \int dt d^d x \;\left[\dot{\phi}^2 - (\nabla^2 \phi)^2\right]$$

Lifshitz Geometry

Kachru, Liu, Mulligan '08

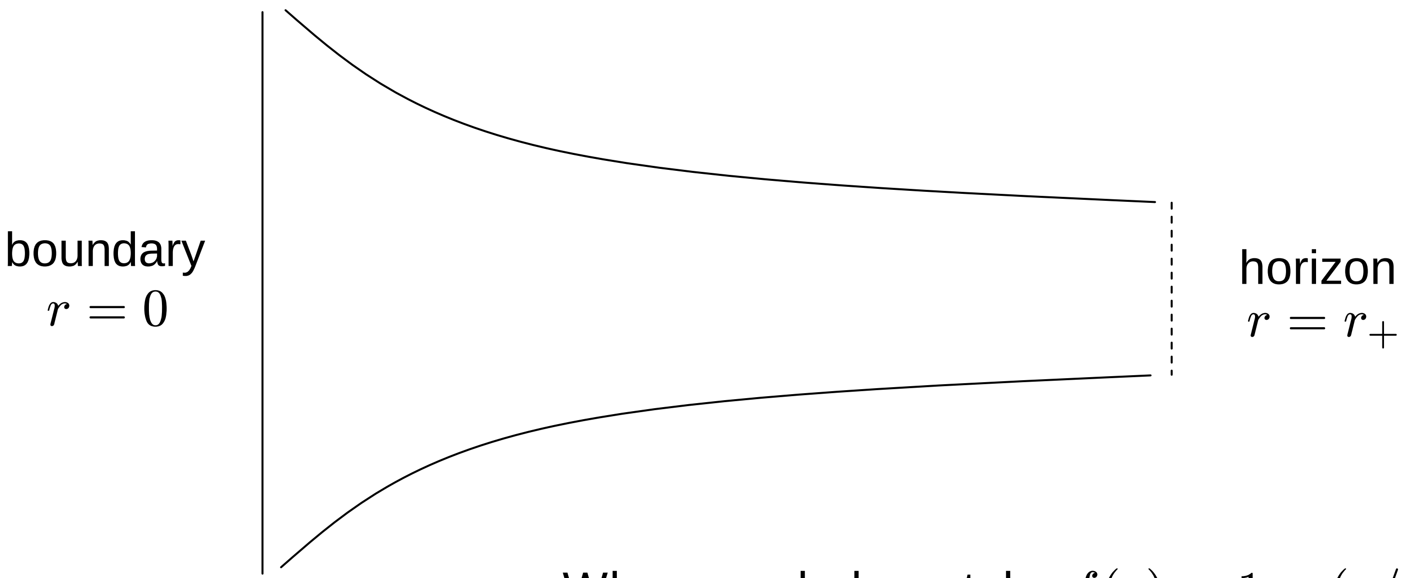
$$ds^2 = L^2 \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2+dy^2}{r^2} \right)$$



Hot Lifshitz Geometry

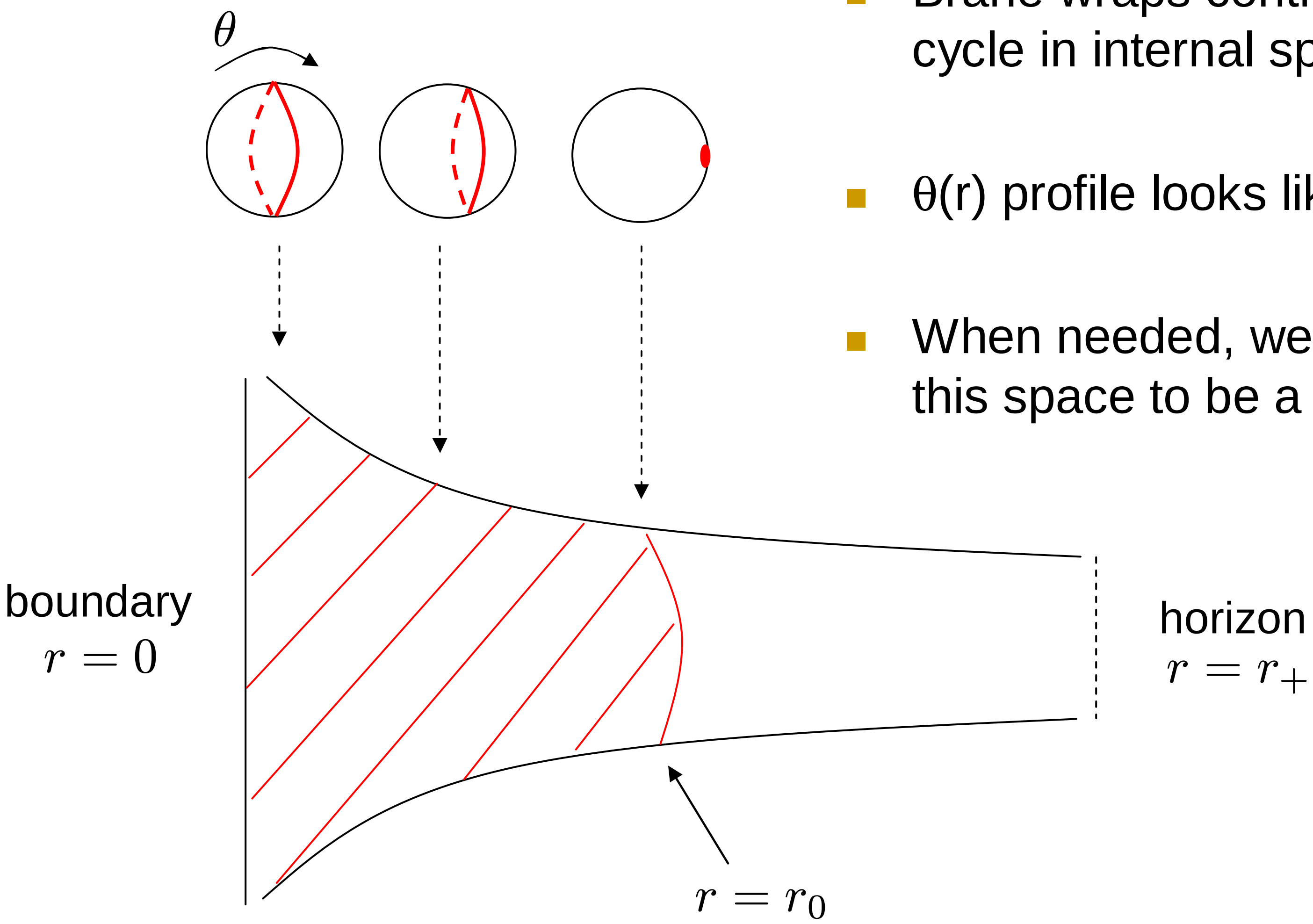
$$ds^2 = L^2 \left(-\frac{f(r)}{r^{2z}} dt^2 + \frac{dr^2}{f(r) r^2} + \frac{dx^2 + dy^2}{r^2} \right)$$

$$f(r_+) = 0 \quad \Rightarrow \quad T \sim f'(r_+)/r_+^{z-1} \sim 1/r_+^z$$



When needed, we take $f(r) = 1 - (r/r_+)^{z+2}$

Adding Probe Branes



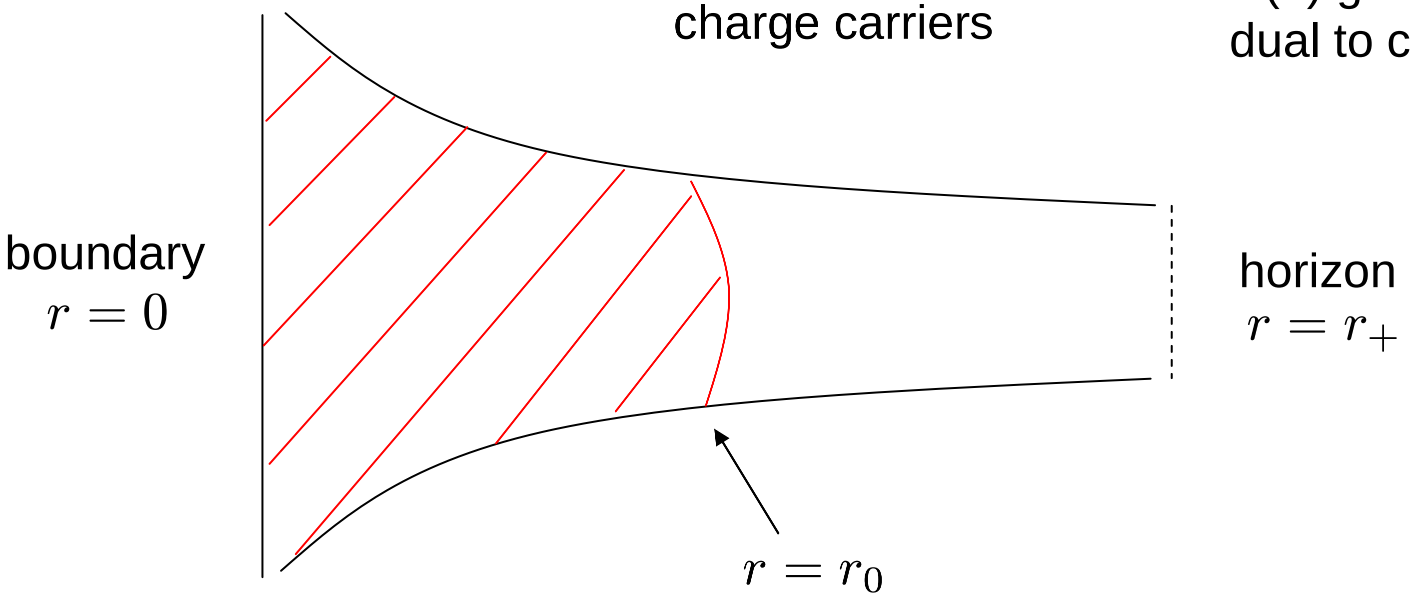
- Brane wraps contractible cycle in internal space
- $\theta(r)$ profile looks like cigar
- When needed, we take this space to be a sphere

Adding Probe Branes

$$S_{\mathrm{DBI}} = -\tau \int dt d^3\sigma \sqrt{-det\left[g_{ab} + \partial_a\theta\partial_b\theta + F_{ab}\right]}$$

Dual to mass of charge carriers

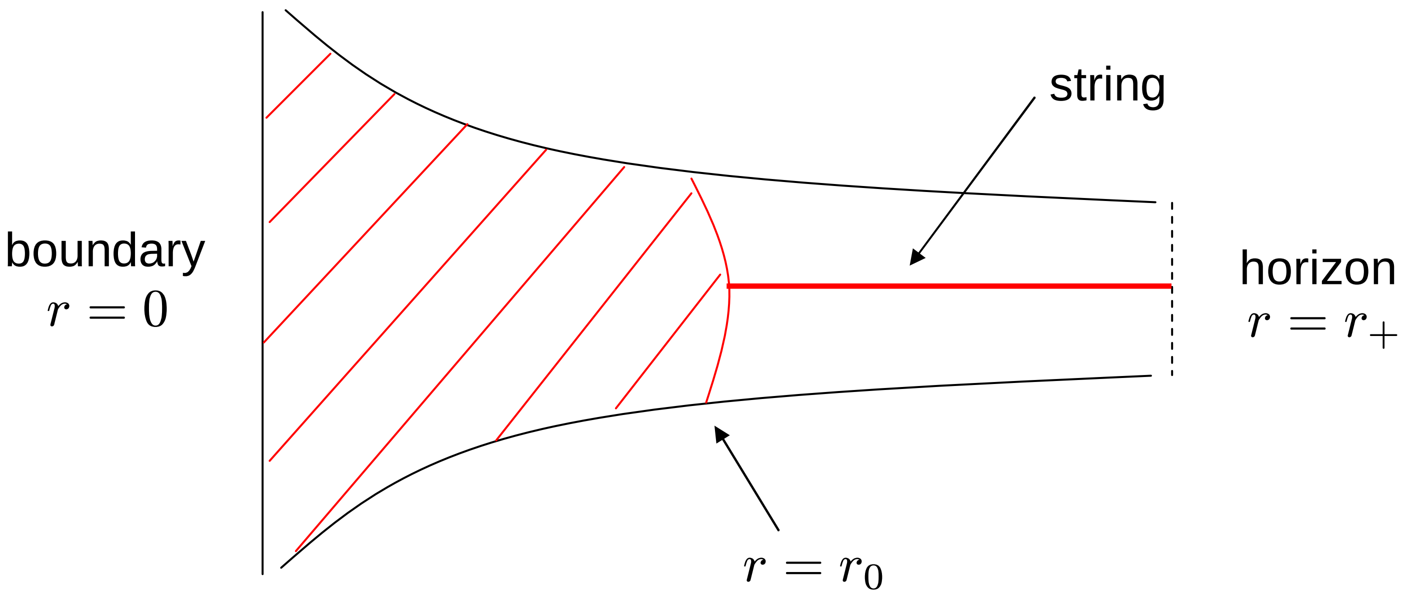
U(1) gauge field, dual to current, J



In this talk, we'll drop powers of τ , L^2 , α'

Charge Carriers

- Charge carrier = string: $m = E_{\text{gap}} \sim 1/r_0^z$
- Note: $m \gg T$ $\quad (E_{\text{gap}} \sim 1eV \;,\; T \sim 10 \to 10^3 K)$



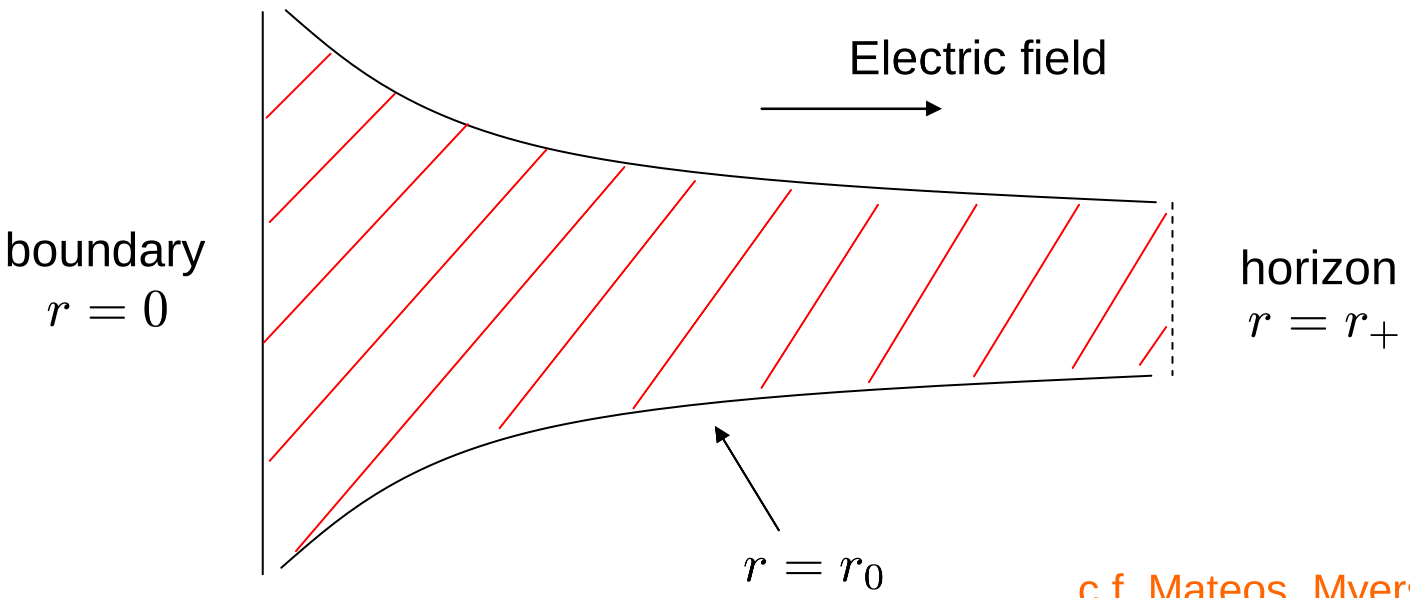
Charge Carriers

- Finite charge density: $A_0 \to \mu - J^t r^{2-z} + \ldots$

chemical potential

charge density

- Brane now stretches to horizon. θ(r) profile looks like tube.



c.f. Mateos, Myers et. al. '06

DC Conductivity

Karch, O'Bannon '07

$$A_0 \to \mu - J^t r^{2-z} + \dots$$

$$A_x \to Et + J^x r^z + \dots$$

electric field

current

- Compute full non-linear conductivity

$$J^x = \sigma(E, T)\, E$$

- Without doing any work….!

DC Conductivity

$$\sigma(E,T) = \sqrt{\text{const.} + r_\star^4 (J^t)^2}$$

due to pair creation

due to background charge

- where $f(r_\star) = E^2\, r_\star^{2z+2}$
- when $E_{\text{gap}},\ (J^t)^{z/2} \gg T$ ……….

$$\boxed{\rho = \frac{1}{\sigma} \sim \frac{T^{2/z}}{J^t}}$$

- Comments:
  - Finite DC conductivity only because we're ignoring backreaction
  - Linear for z=2
  - This follows from dimensional analysis *if* we assume linearity in charge density.
    - Unclear if this linearity is seen experimentally (charge density = doping)

Hall Conductivity

O'Bannon '07

- Similar technique:

$$\boxed{\frac{\sigma_{xx}}{\sigma_{xy}} \sim \rho}$$

- This is the Drude result: no sign of anomalous behaviour.
  - Caveat: The term from pair creation actually scales as

$$\frac{\sigma_{xx}}{\sigma_{xy}} \sim \frac{T^{4/z}}{J^t B}$$

  - Good for z=2!
  - But…not clear why this term would dominate
  - Experimental results show this ratio is independent of doping

AC Conductivity

$$A_x(\omega) = \frac{E_x(\omega)}{i\omega} + J^x(\omega) r^z + \ldots$$

- All ω –– Numerical Results:
- Large ω –– Analytic Results: $T \ll \omega \ll E_{\rm gap}$
  - Small density, with probe string computation
  - Finite density, with brane computation
  - Both give same ω dependence

(c.f. $\omega \sim 0.1 \to 1$ eV)

AC Conductivity

$$\sigma(\omega) \sim \begin{cases} (J^t)^{z/2}\,\omega^{-1} & z < 2 \\ J^t\,(\omega \log \omega)^{-1} & z = 2 \\ J^t\,\omega^{-2/z} & z > 2 \end{cases}$$

- Note: z=3 matches data!
- Linearity in J^t is now dynamical
- Agreement between small and large densities due to brane spike

Comment on the z=2 Crossover

- Dimensional Analysis: $[x] = -1 \qquad [t] = -z$
- Operators are relevant if $[\mathcal{O}] \leq d + z$

$$\int dt d^d x \quad \mathcal{O}$$

- Examples:
  - Charge density: $[J^t] = d \Longrightarrow (J^t)^2$ relevant for z>d
  - Probe Inertia: $\int dt \ \dot{x}^2 \Longrightarrow$ irrelevant for z>2
    - For z>2, all inertia due to stuff that particle drags around with it

Summary

- Charge carriers moving in strongly coupled Lifshitz backgrounds yields:
  - DC conductivity: $\rho \sim T^{2/z}$
  - Hall conductivity: $\sigma_{xx}/\sigma_{xy} \sim \rho$
  - AC conductivity $\sigma(\omega) \sim 1/\omega^{2/z}$ for z>2
- Starting point for model building or merely failure?!