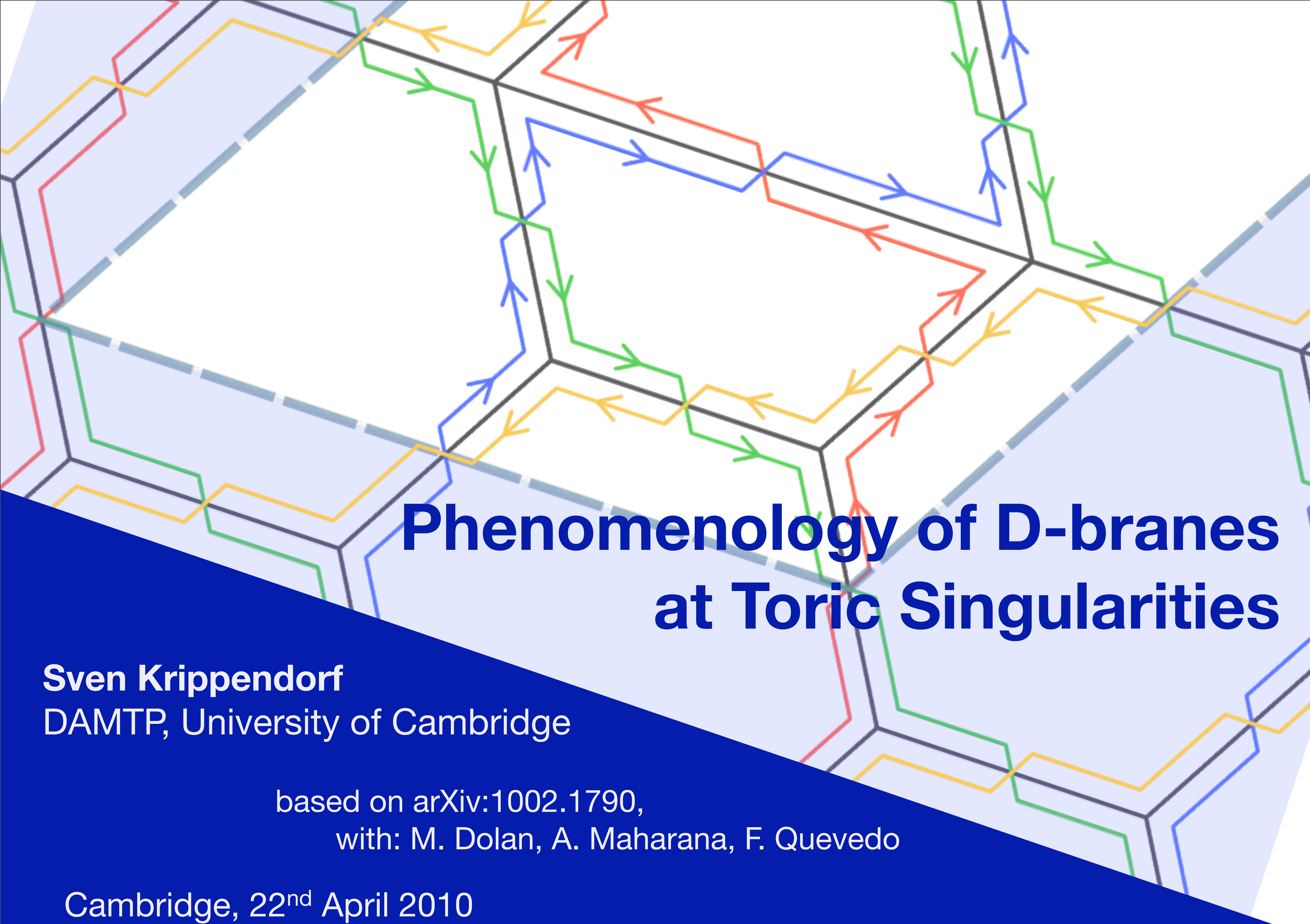

Phenomenology of D-branes at Toric Singularities

Sven Krippendorf
DAMTP, University of Cambridge

based on arXiv:1002.1790,
with: M. Dolan, A. Maharana, F. Quevedo

Cambridge, 22nd April 2010

Disclaimer: This talk is somewhat independent of the LHC.

... string theorists could have talked about this 20 years ago


WANTED
Quarks
u c t
d s b
Forces
Z γ
W g
e μ τ
Leptons
STANDARD MODEL
FROM STRING THEORY
ImageChef.com

for more than 25 years...

for more than 25 years...

Heterotic String Theory

for more than 25 years...

Heterotic String Theory

Type II brane models

for more than 25 years...

Heterotic String Theory

Type II brane models

e.g. intersecting branes in IIA,
magnetised branes in IIB

F-theory

for more than 25 years...

0502005, 0702094

Heterotic String Theory

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0502005, 0702094

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moduli stabilisation

Heterotic String Theory

Type II brane models

0502005, 0702094

e.g. intersecting branes in IIA,
magnetised branes in IIB

moduli stabilisation

under control in Type IIB
(KKLT, LVS)

0502005, 0702094

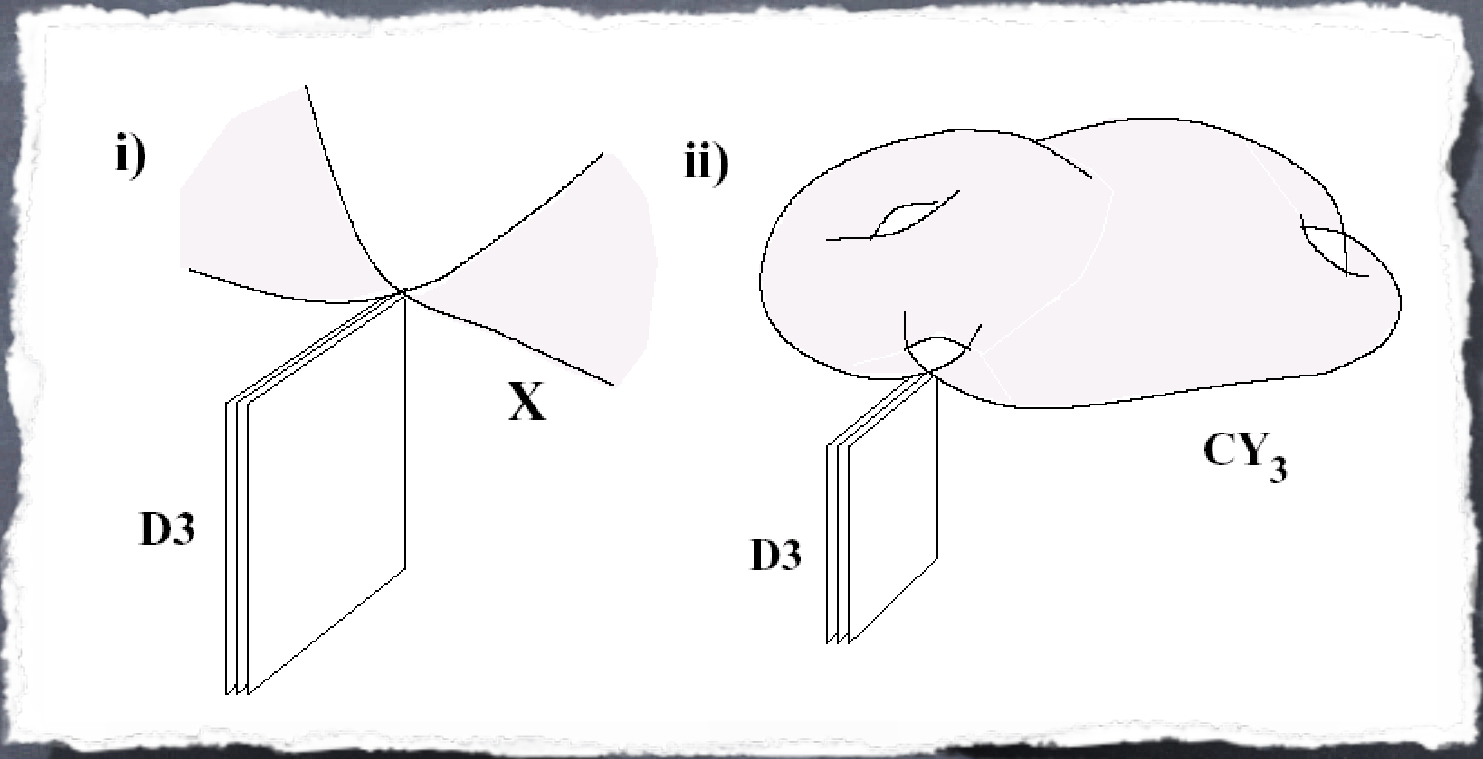
Heterotic String Theory

Type II brane models

e.g. intersecting branes in IIA,
magnetised branes in IIB

Perspective: Local Brane Models (within IIB)

... bottom-up model building



Modular String Model Building (in LARGE volume)

e.g.: aldazabal, ibanez, quevedo, uranga (10 years ago),
verlinde, wijnholt (5 years ago)

and what can we get?

Standard (like) Models with fractional (D3/D7) branes at singularities

e.g.: aldazabal, ibanez, quevedo, uranga (10 years ago),
verlinde, wijnholt (5 years ago)

Brane models: how good are they to date?

- chiral matter, adjoint and (bi-)fundamental matter
- U(n), SO(n), Sp(n) gauge groups (exceptional gauge groups in F-theory)
- models with the correct matter content

e.g. madrid model, 0105155

- **BUT structure of yukawa couplings, e.g.:**
  hierarchy of masses, ckm matrix

... this applies also to F-theory models

Content

- Why branes at singularities, what gauge theories do we get?

  ...background and history

- Studying gauge theories with dimers
- What general properties do gauge theories of the infinite class of toric singularities obey?
- How good are the models regarding flavour physics, i.e. can we get the CKM-matrix?

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Upper bound of #families (≤3)

- e theories with dimers

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Mass structure (0, m, M)

- What general properties do gauge theories of the infinite class of toric singularities obey?

- How good are the models regarding flavour physics, i.e. can we get the CKM-matrix?

yes, we can...
there is enough structure around
to build conrete models with the right ckm.

Motivation for branes at singularities

- Local models -> a lot of information without addressing moduli stabilisation
- Effective field theory (although distances below string scale) well under control
- Gauge coupling unification (in principle)
- Powerful (dimer) techniques for toric singularities
- Gauge theories highly restricted (unlike intersecting branes in IIA)

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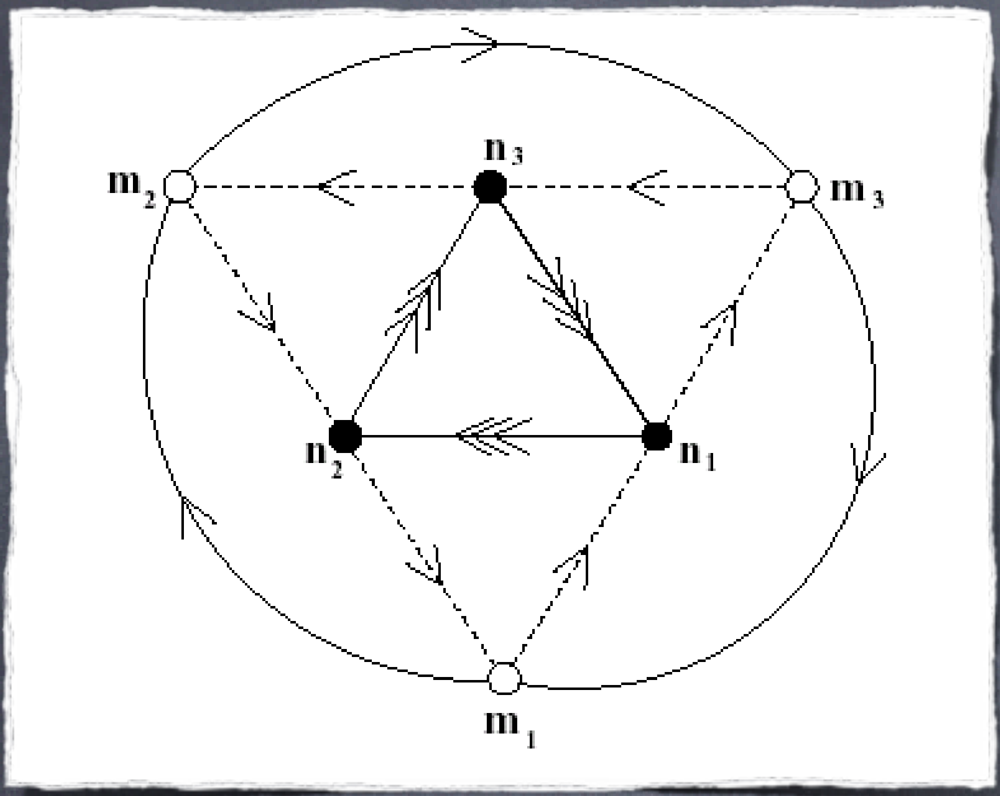
Hanany et al.

Classic Example: $\mathbb{C}^3/\mathbb{Z}_3$



- n_i D3-branes: $U(n_1)\times U(n_2)\times U(n_3)$
- m_i D7-branes: $U(m_1)\times U(m_2)\times U(m_3)$
- Arrows: bi-fundamental matter
- Anomaly cancellation

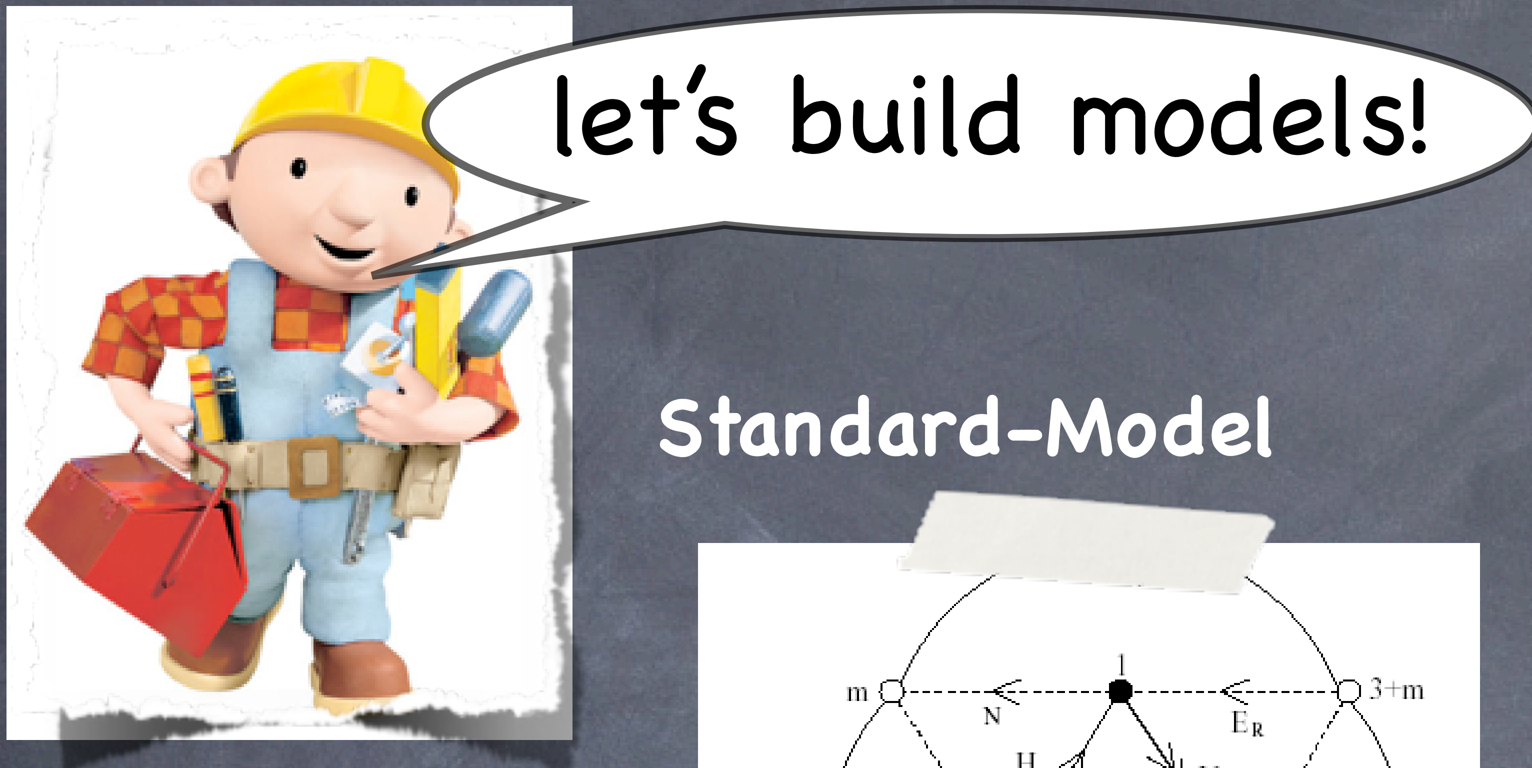
$$m_2 = 3(n_3 - n_1) + m_1,$$
$$m_3 = 3(n_3 - n_2) + m_1$$

D3 matter content:

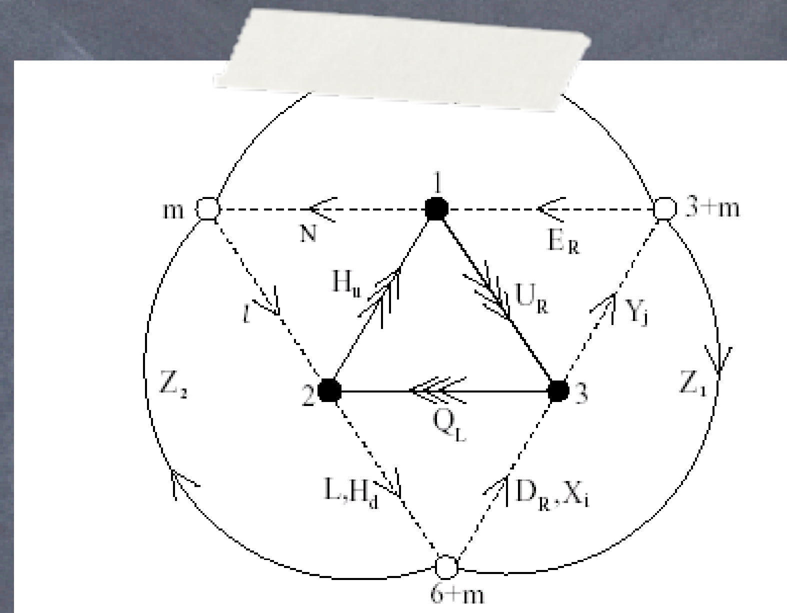
$$3[(n_1, \bar{n}_2, 1), (1, n_2, \bar{n}_3), (\bar{n}_1, 1, n_3)]$$

- Hypercharge:

$$Q_{\text{anomaly}-\text{free}} = \sum_i \frac{Q_i}{n_i}$$



Standard-Model



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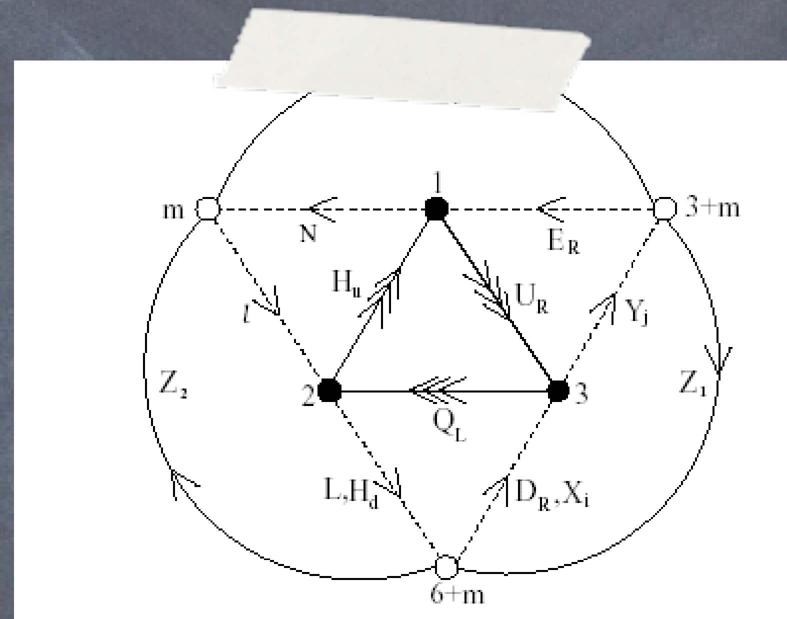
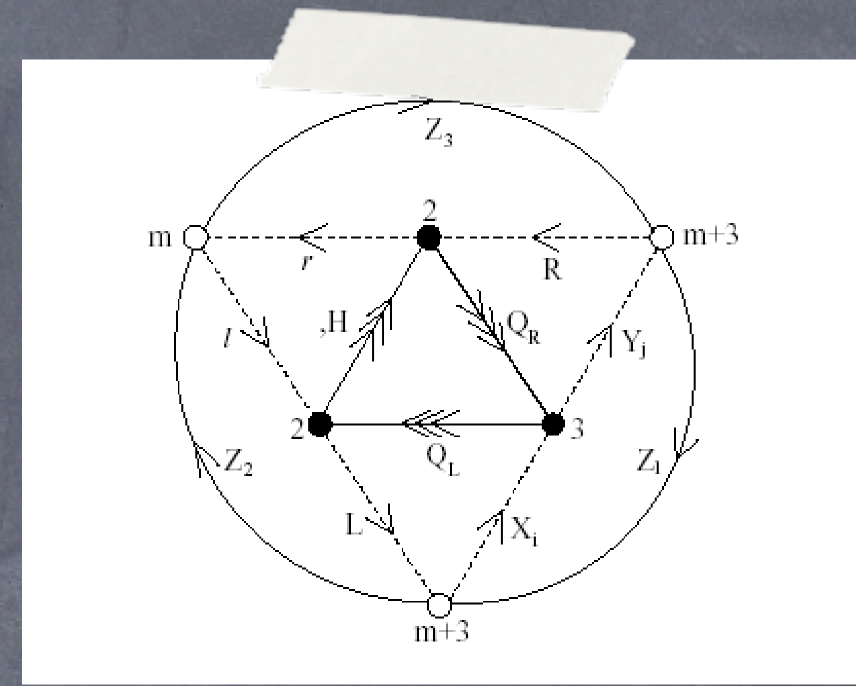


Left-Right



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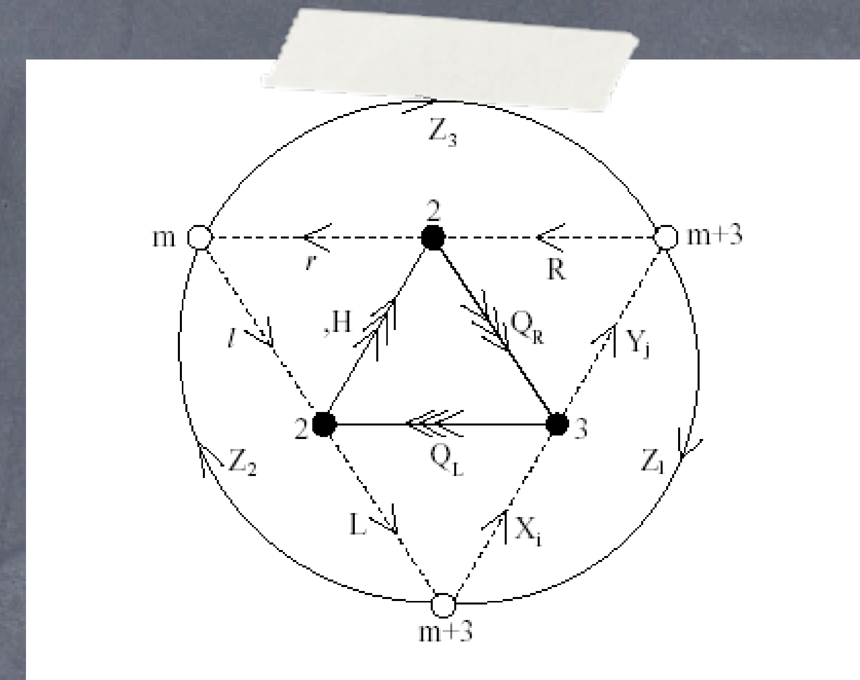

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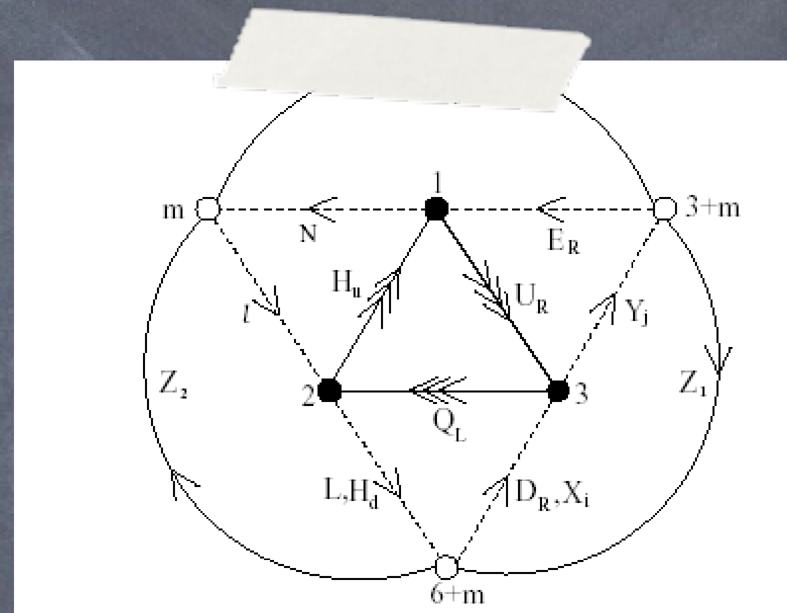
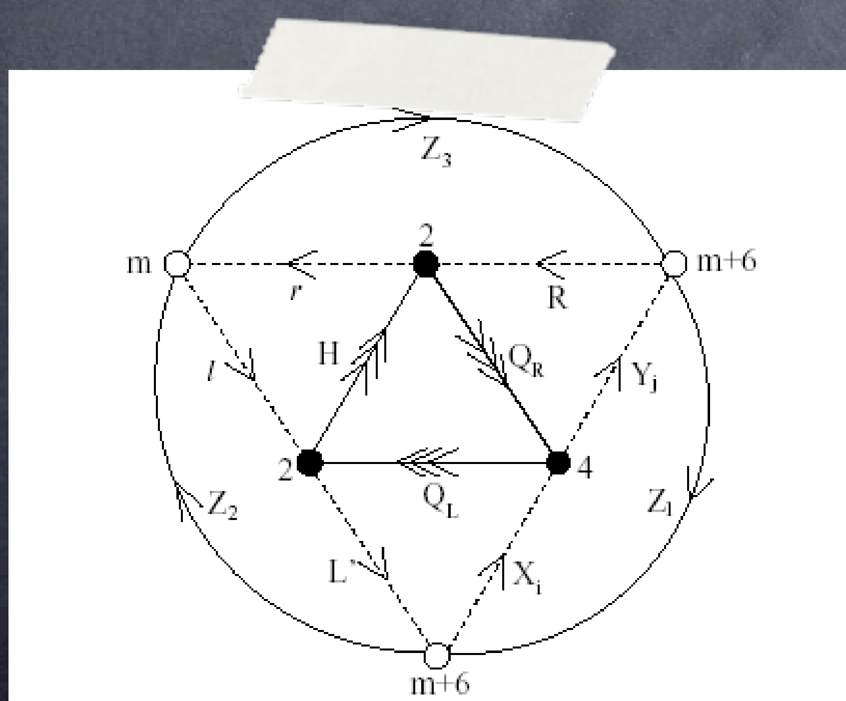


Pati-Salam



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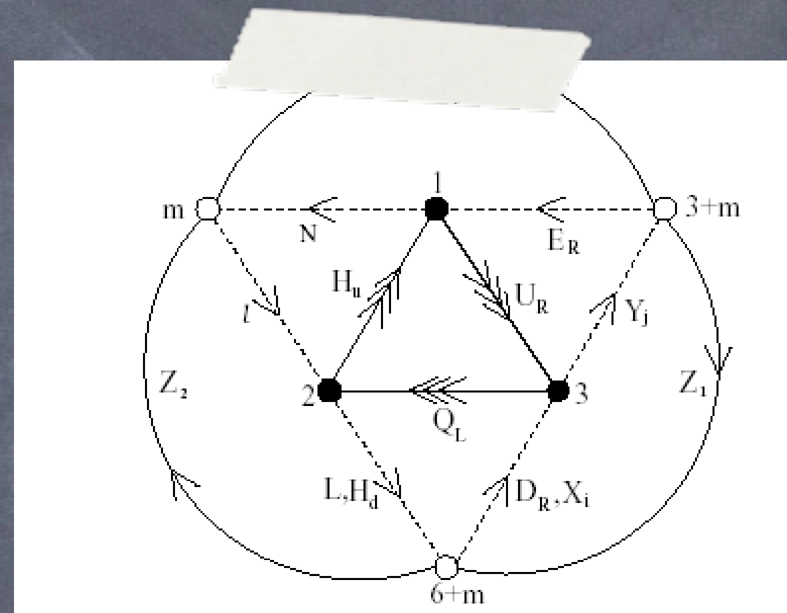

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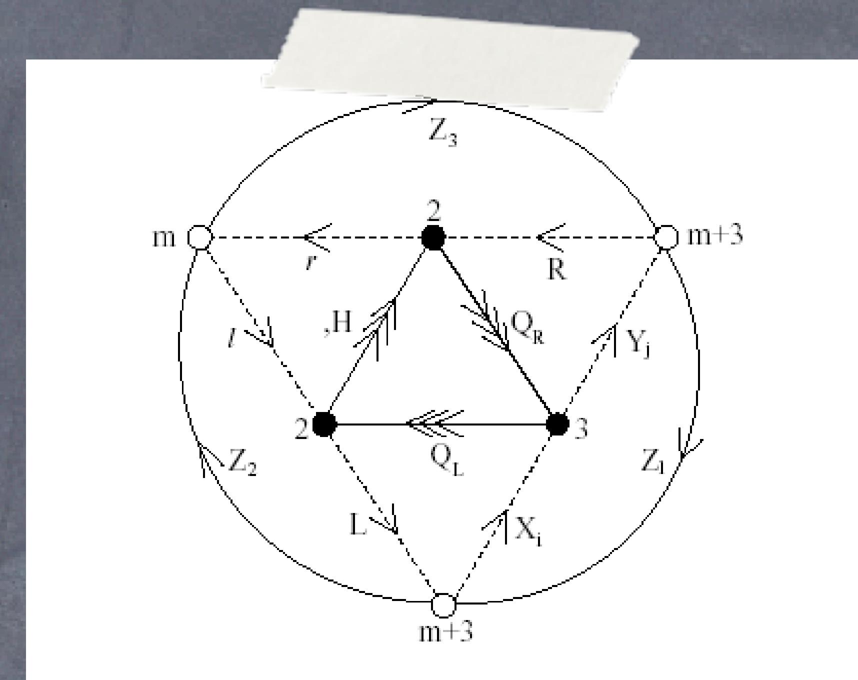

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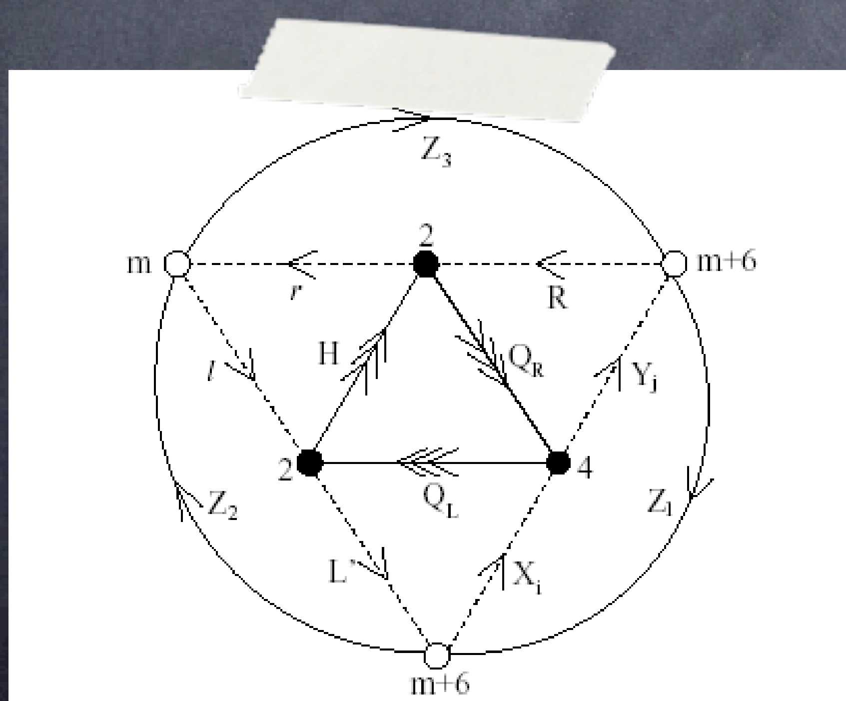
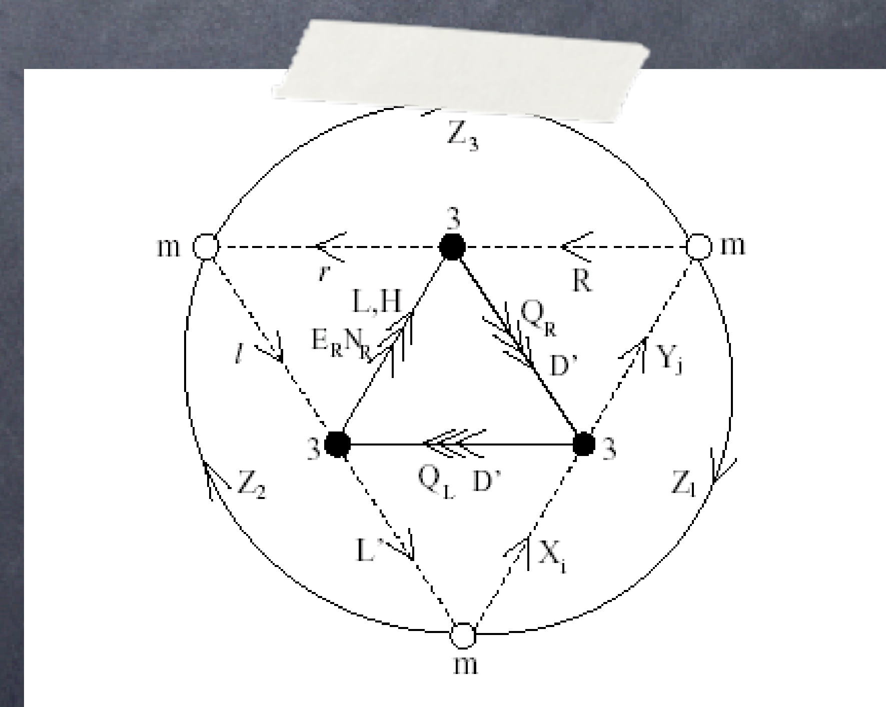
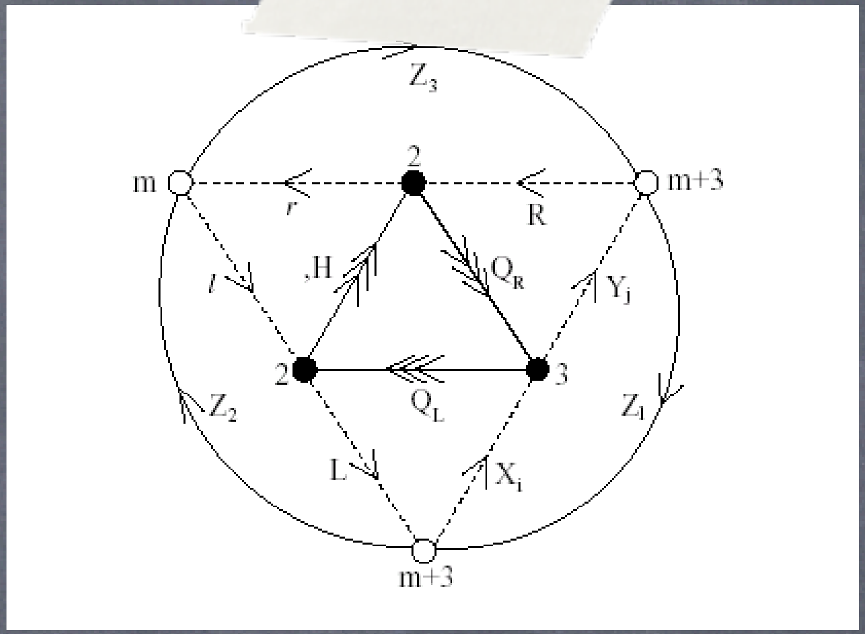


Trinification



e.g. left-right model



- 3 families $SU(3)xSU(2)_LxSU(2)_RxU(1)_{B\text{-}L}$
- B-L normalisation 32/3: $\sin^2\theta = \frac{3}{14} = 0.214$

  + 3 Higgses -> unification at 10^{12} GeV
- Proton stability from global $U(1)_{B\text{-}L}$
- embedded in compact CY

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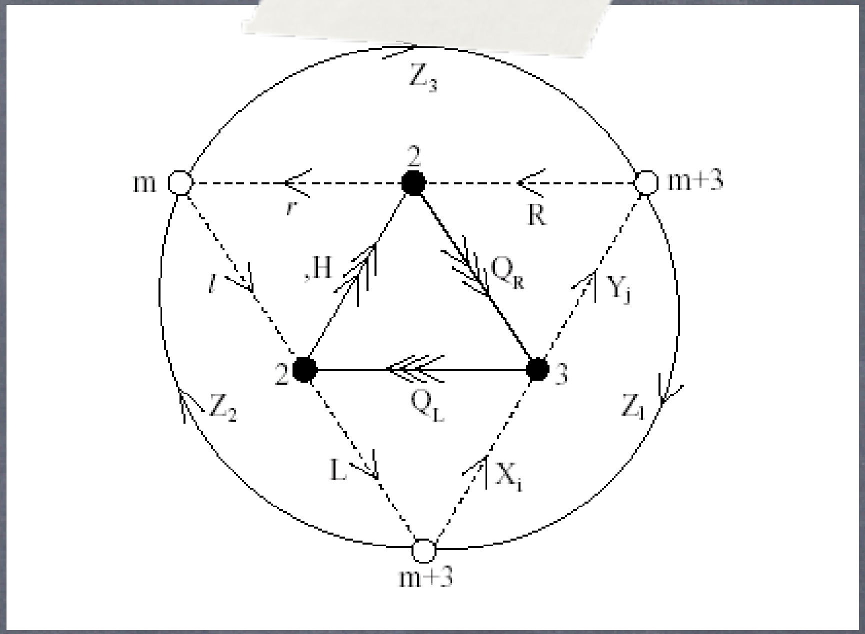

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details in Aldazabal, Ibanez, Quevedo, Uranga hep-th/0005067

What's bad about the model?

Problem: Yukawa couplings

$$W = \epsilon_{ijk}\, Q^i_L H^j_u u^k_R = \begin{pmatrix} Q^1_L \\ Q^2_L \\ Q^3_L \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{12} \\ -Z_{12} & 0 & X_{12} \\ Y_{12} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} u^1_R \\ u^2_R \\ u^3_R \end{pmatrix}.$$

Masses: (0, M, M)

$$M^2 = |X_{12}|^2 + |Y_{12}|^2 + |Z_{12}|^2$$

Resolution: Embed into singularities with more structure

(in this case del Pezzo 1)

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cmq: 0810.5660

$.\mathbb{C}^3/\mathbb{Z}_3 = dP0$

What types of singularities are there?

- Orbifold Singularities
- del Pezzo singularities (P^2 blown-up), Conifold

  -> TORIC SINGULARITIES
- non-toric singularities

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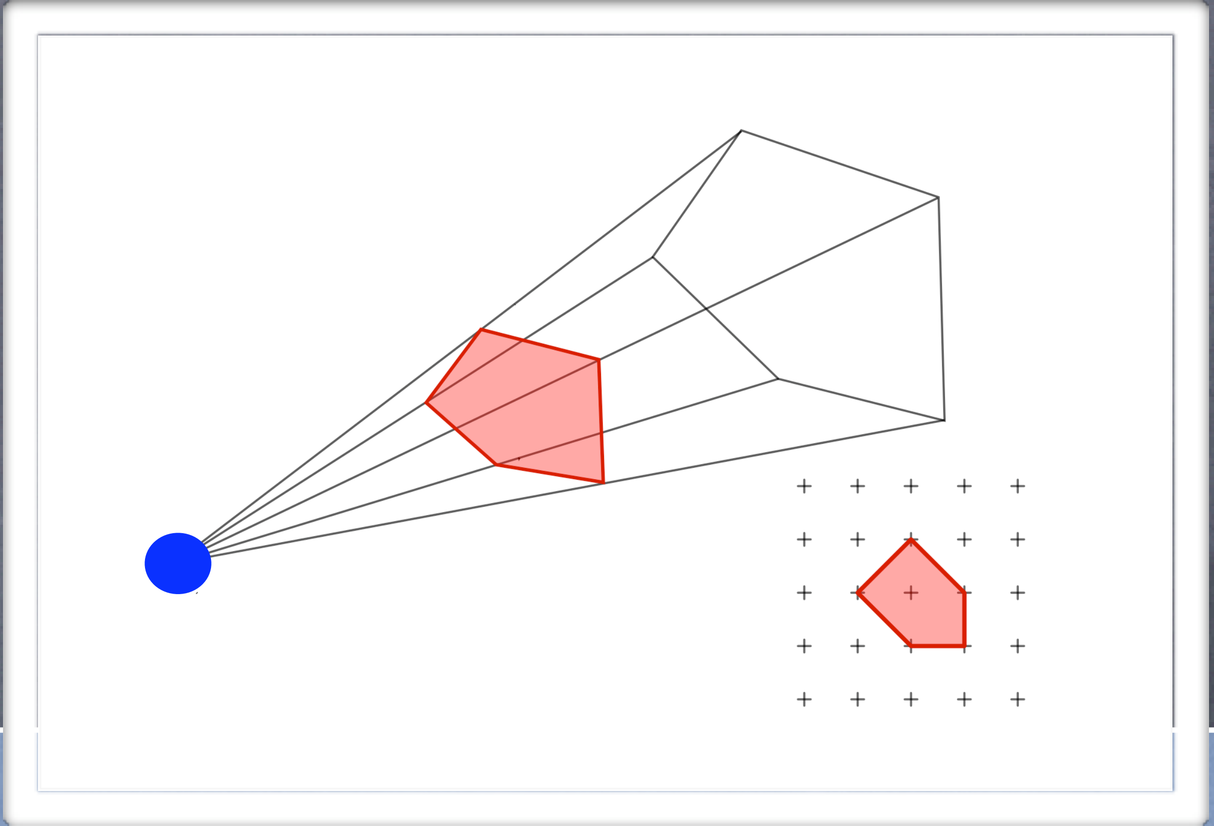
  -> **TORIC SINGULARITIES** ← infinite class, techniques
- non-toric singularities ← limited techniques

Gauge theories probing toric singularities

Toric CY-Cone:

- represented as T^3 fibration over rational polyhedral cone
- D3 branes (at the tip of the cone)
- D7 branes (wrapping 4-cycles passing through singularity)



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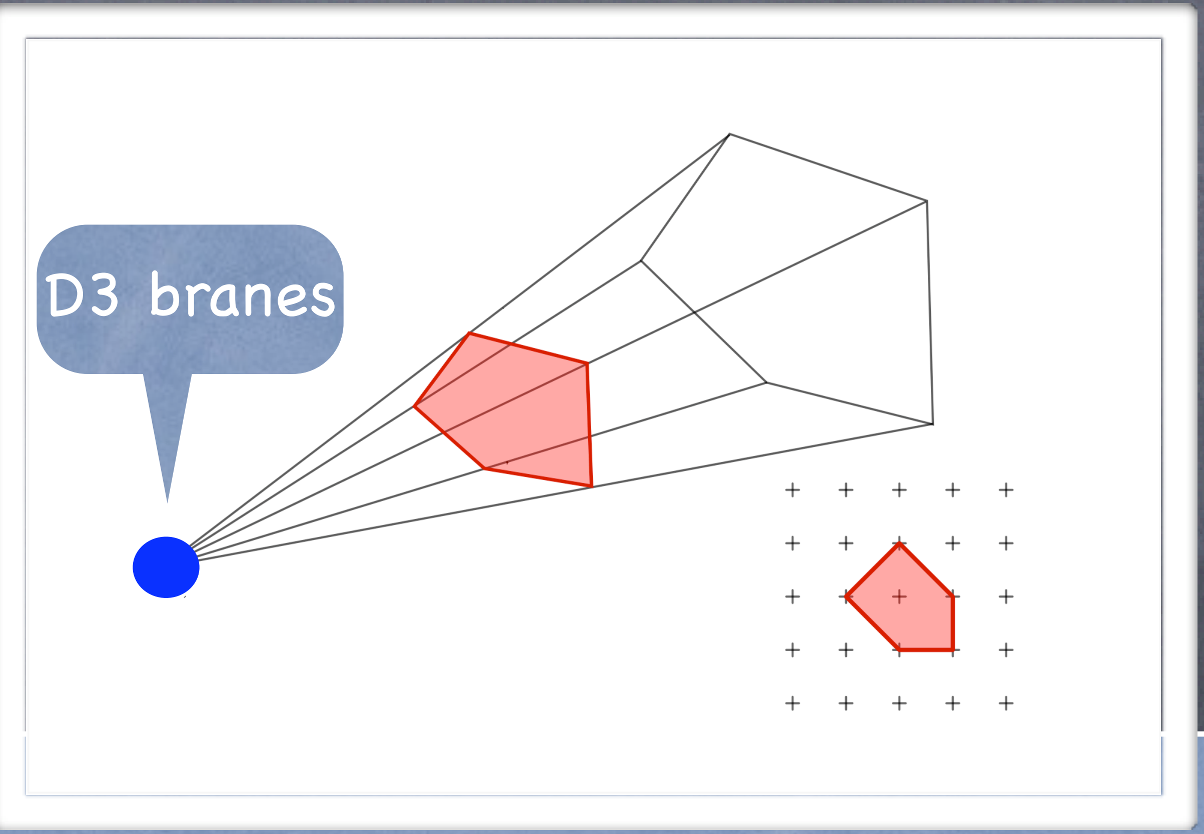

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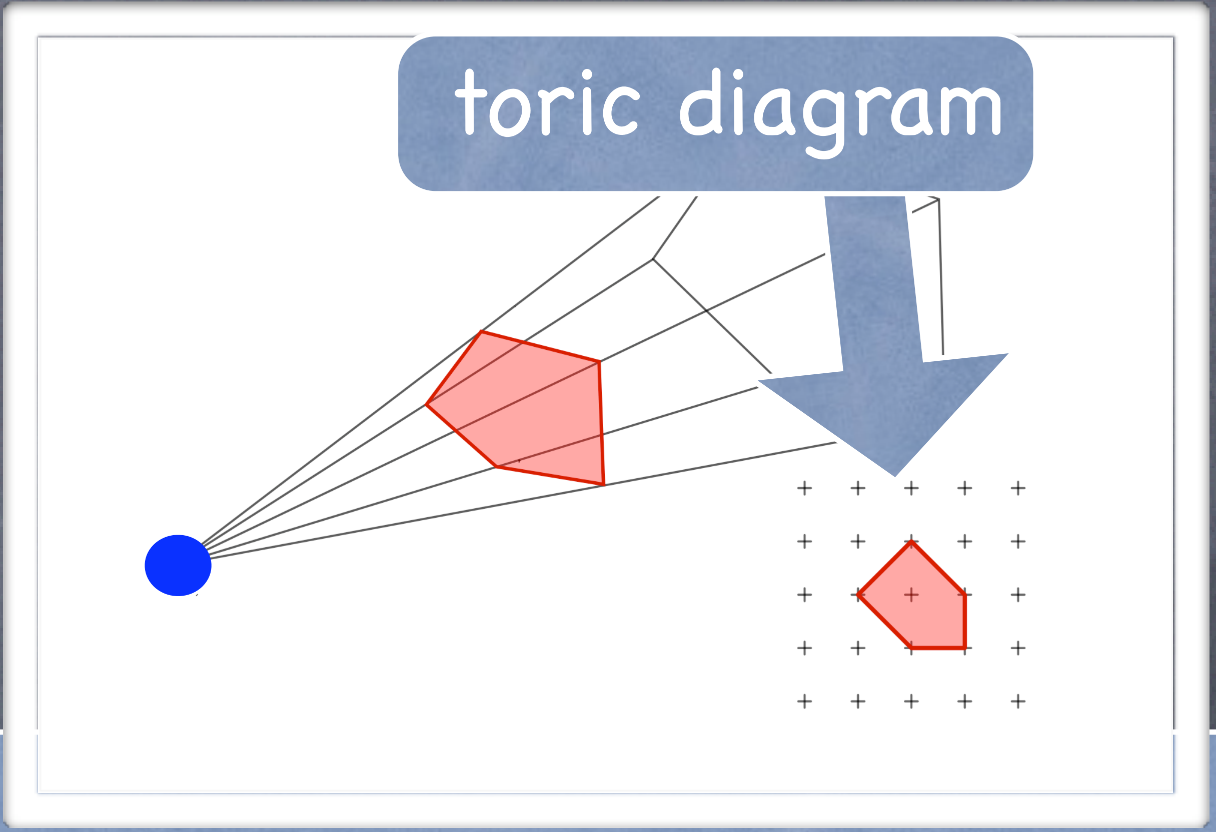

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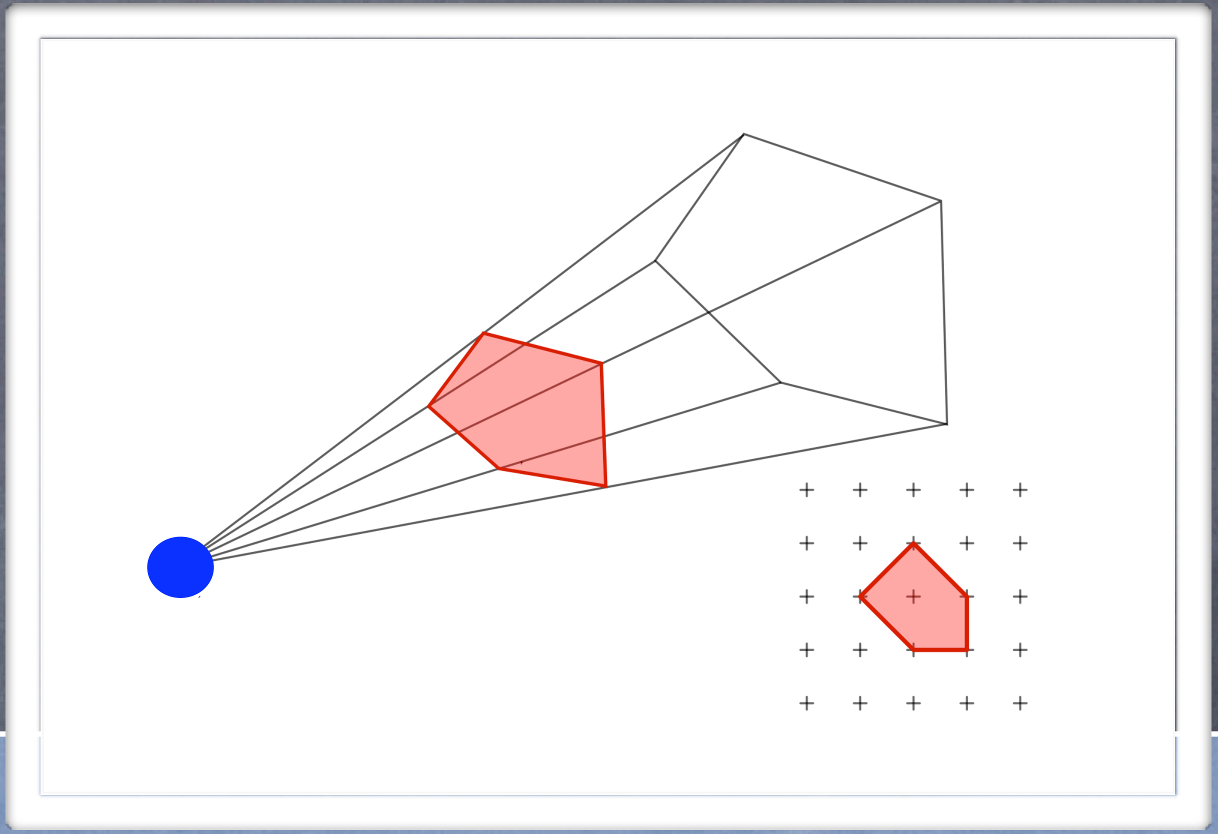

Gauge theories of toric singularities are always quiver gauge theories!

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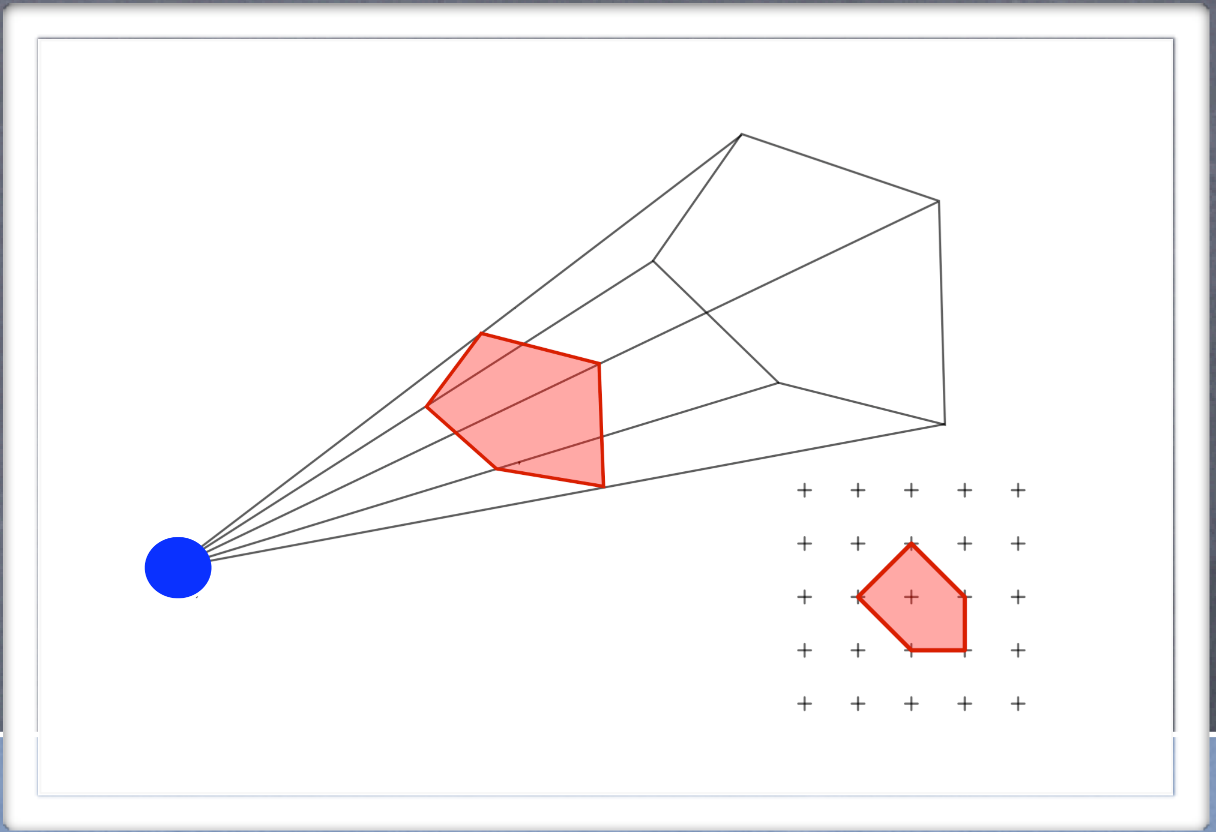

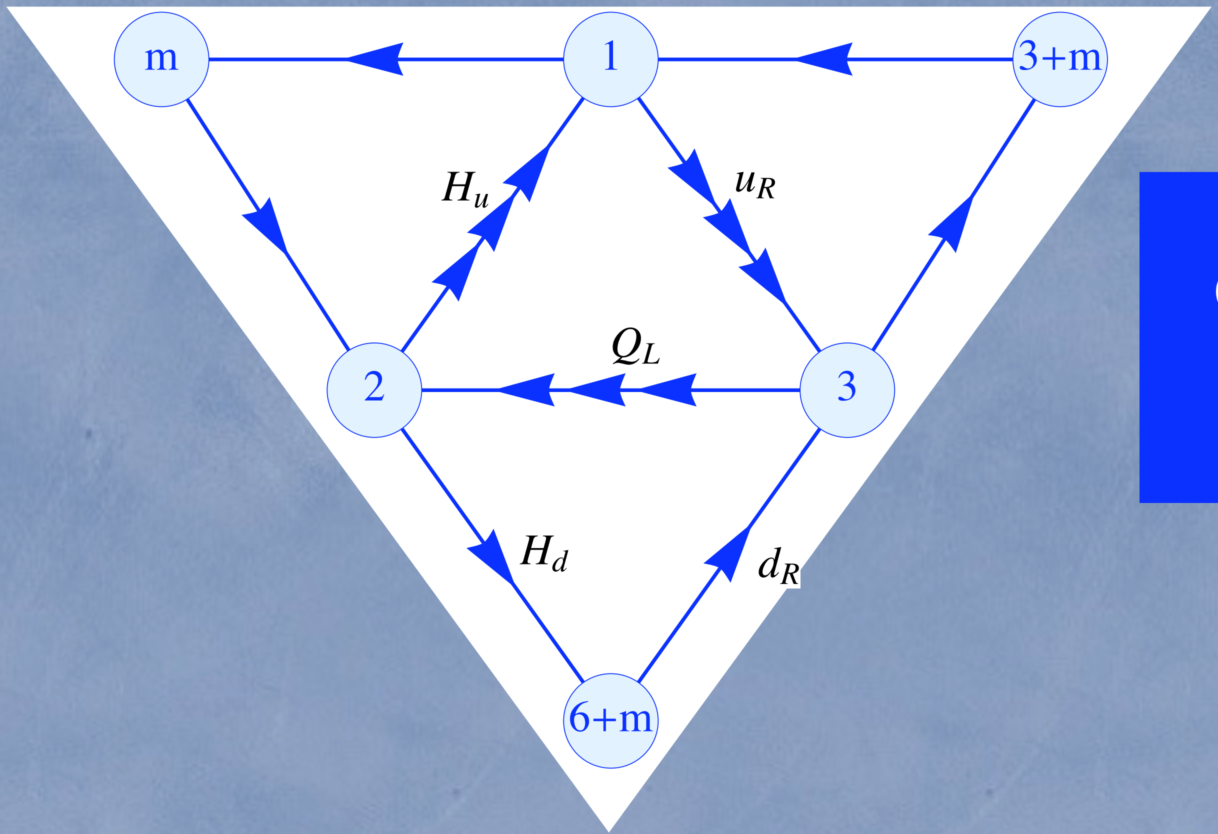

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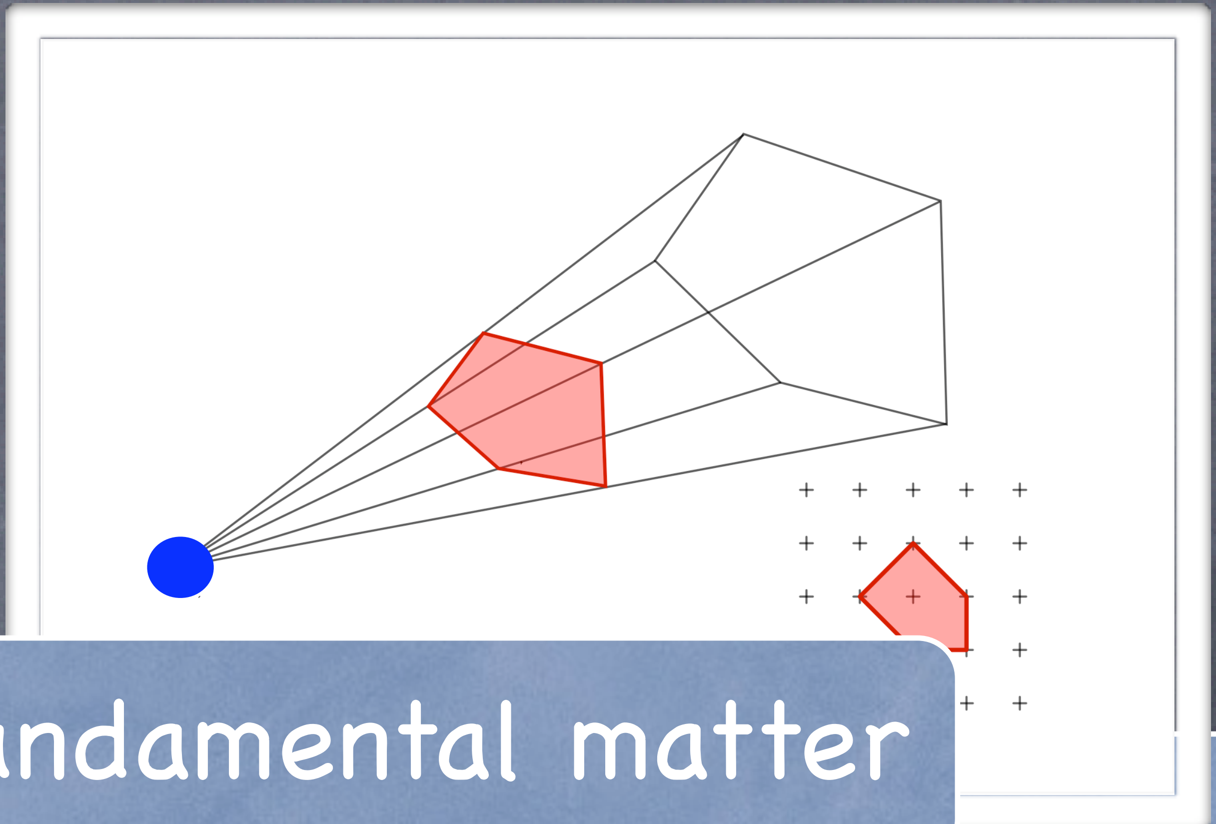
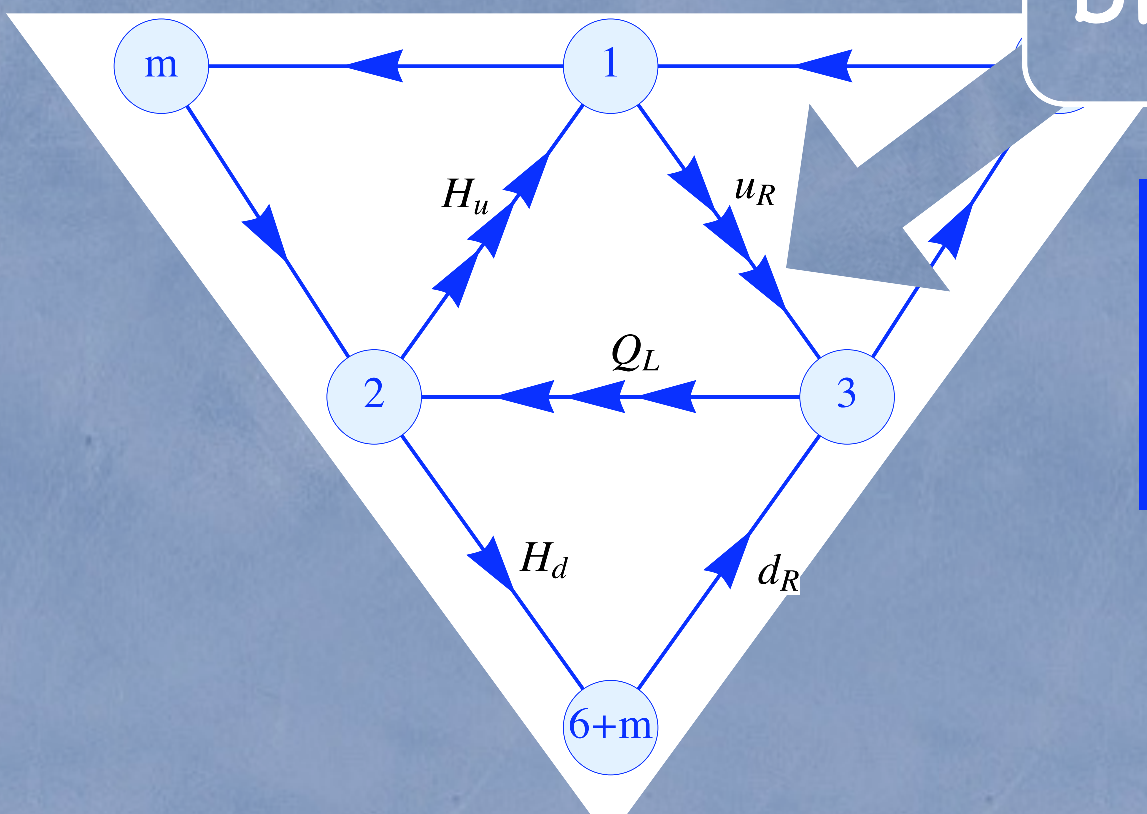


bi-fundamental matter



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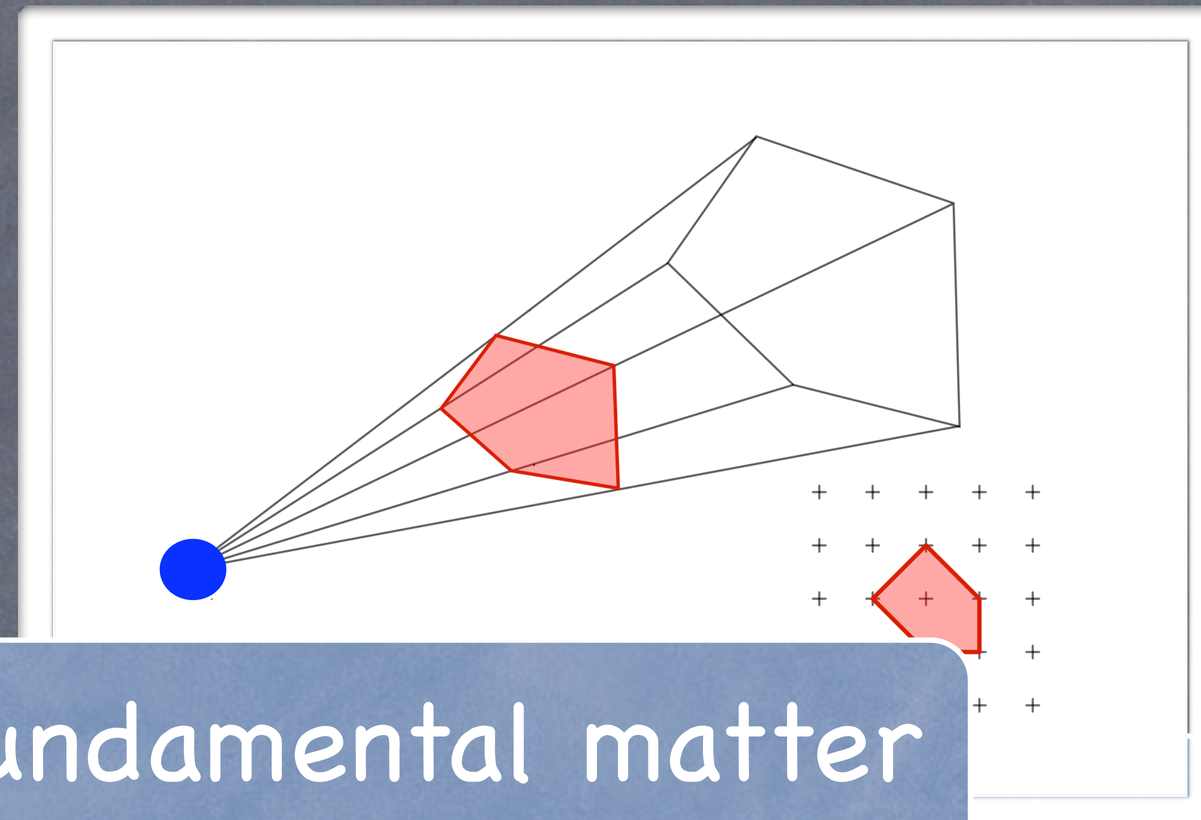

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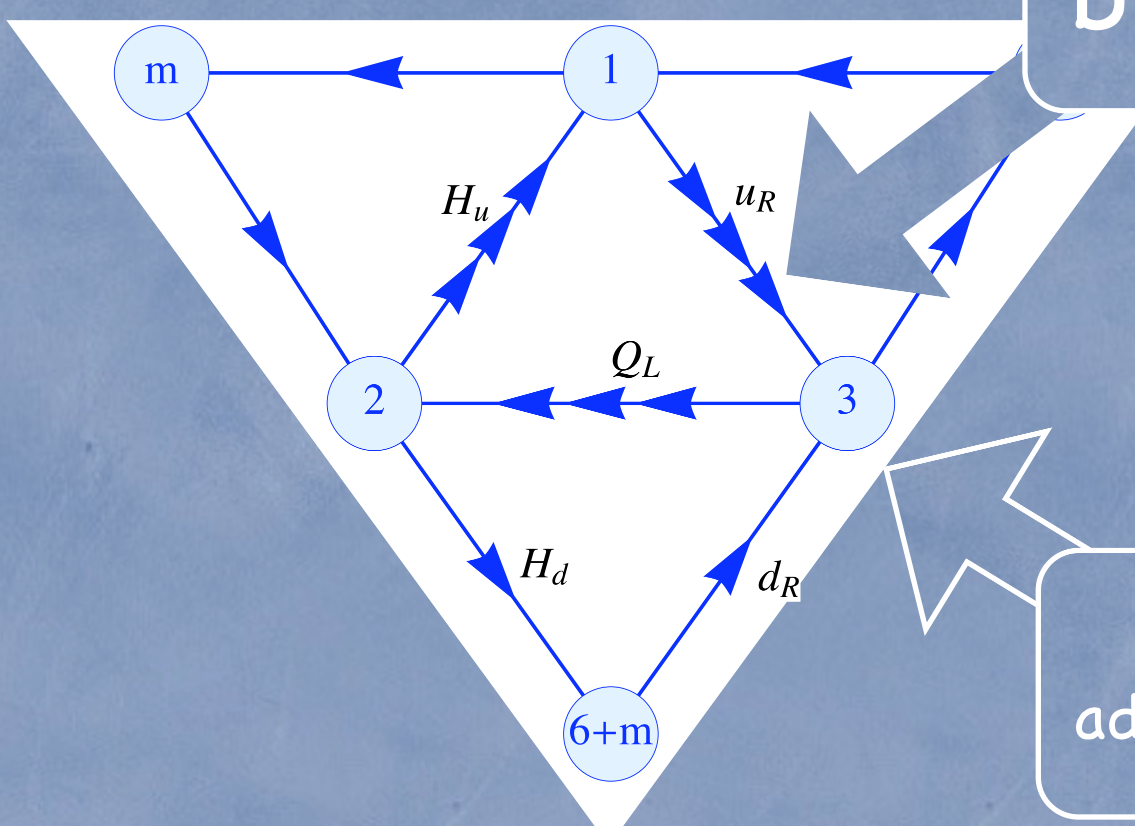

Gauge theories of toric singularities are always quiver gauge theories!

change rank by
addition of fractional branes

Gauge theories probing toric singularities

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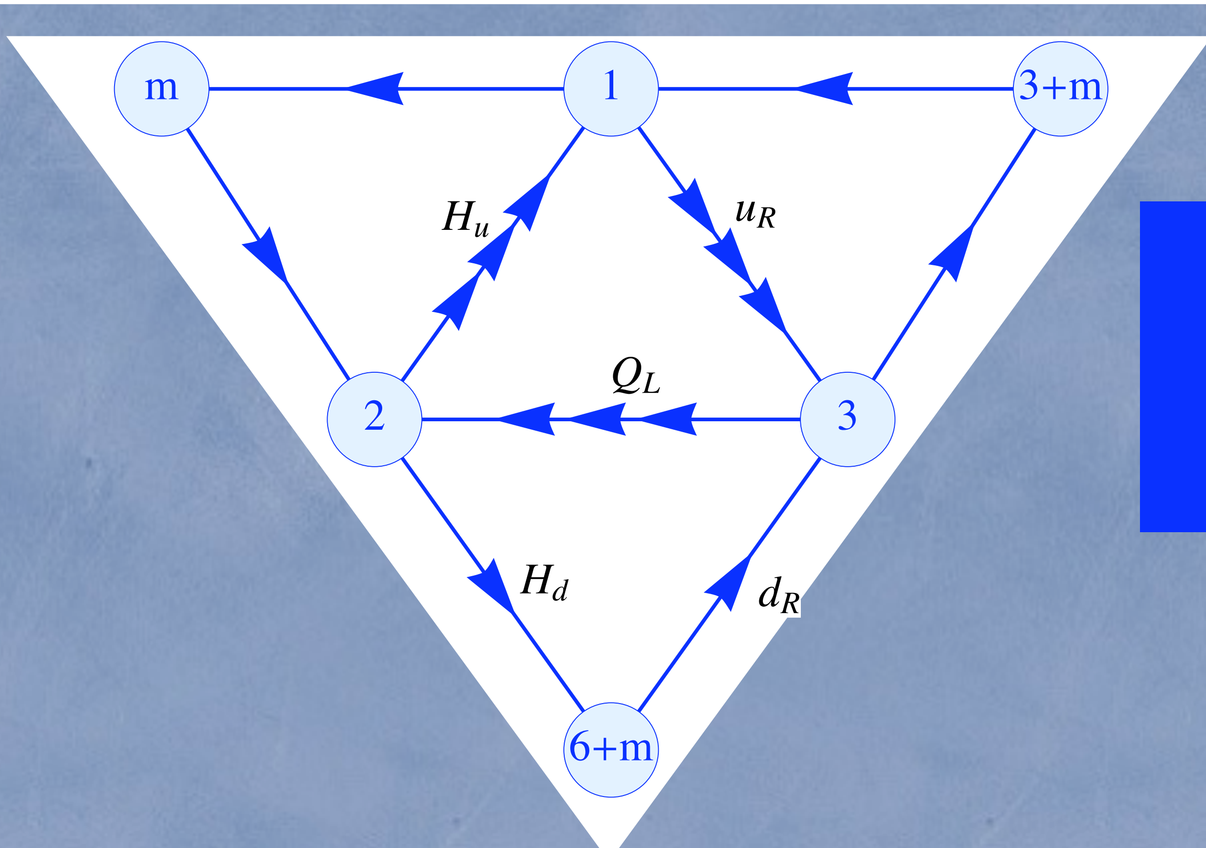


Gauge theories of toric singularities are always **quiver gauge theories!**

Question:

- How to determine the gauge theory associated to a given singularity (inverse problem)?
  -> most efficient solution via dimers

- Aside: Dimer techniques useful in understanding gauge theories of M2 branes at singularities (no inverse algorithm)

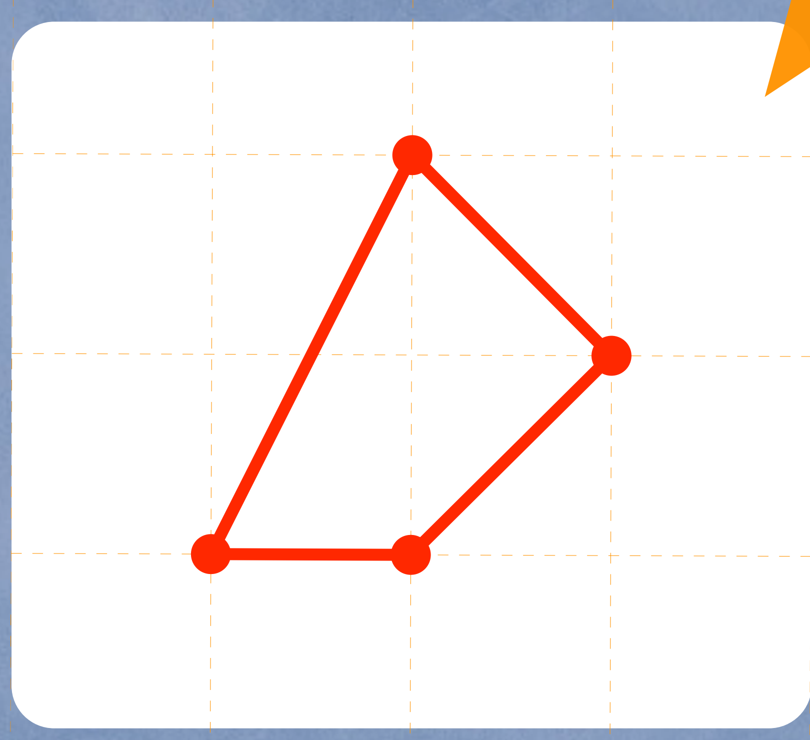
Hanany, Kennaway, Franco, Vegh, Wecht, Martelli, Sparks, Feng, He, Vafa, Yamazaki
0503149, 0504110, 0505211, 0511063, 0511287, 0604136, 0706.1660, 0803.4474

The Dimer Language

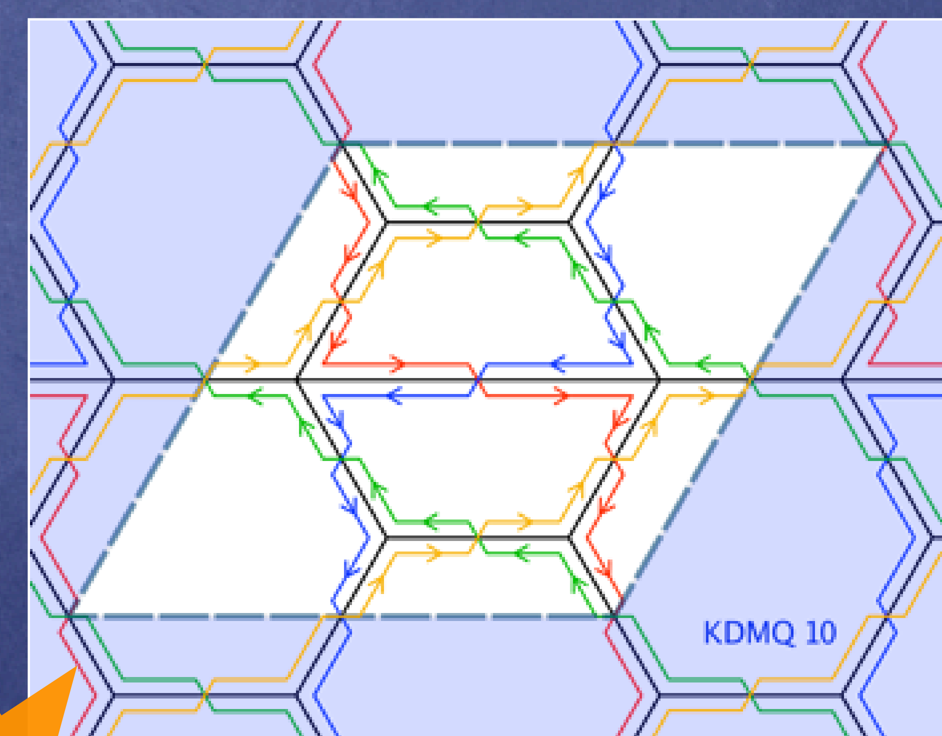
Dimers visualise the gauge theory of toric singularities.

Geometry: Toric Diagram

inverse slopes in toric diagram



Gauge Theory: Dimer



winding numbers of zigzag paths

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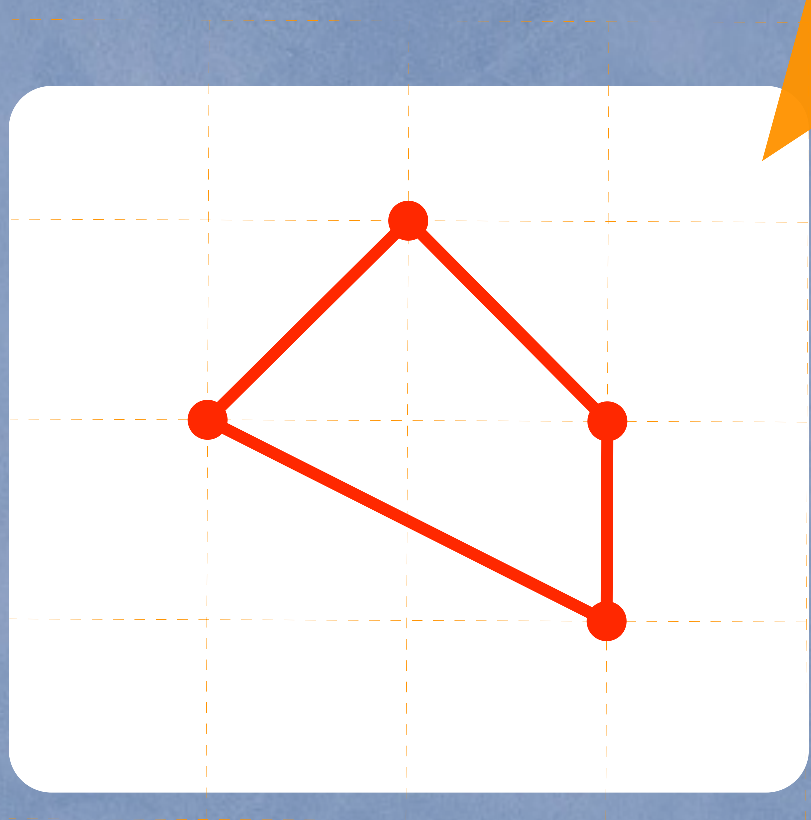

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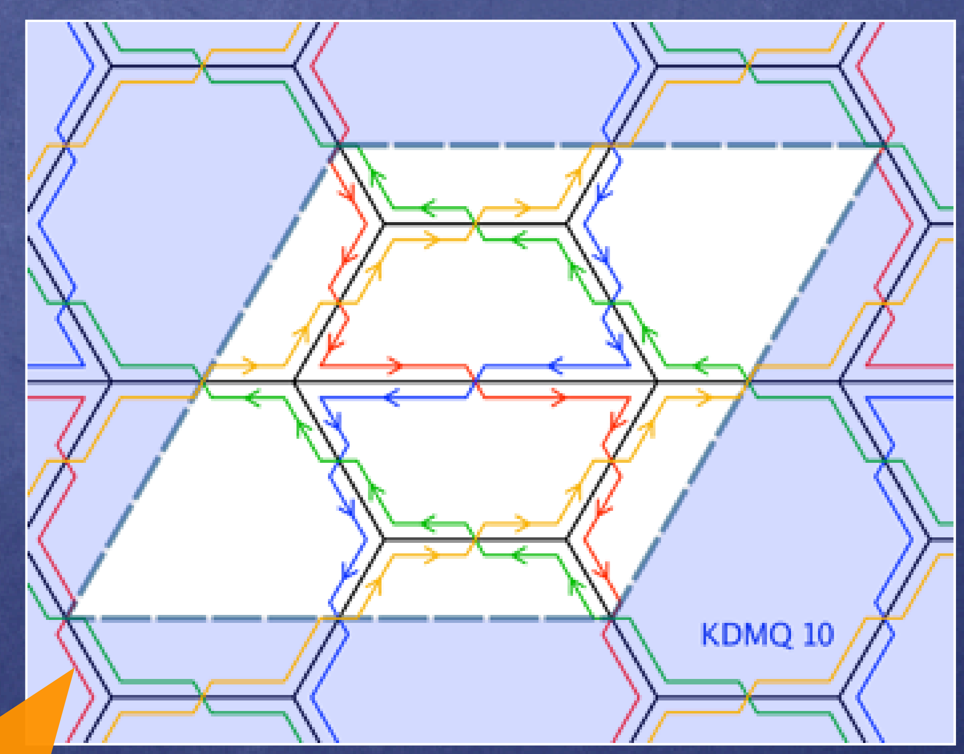

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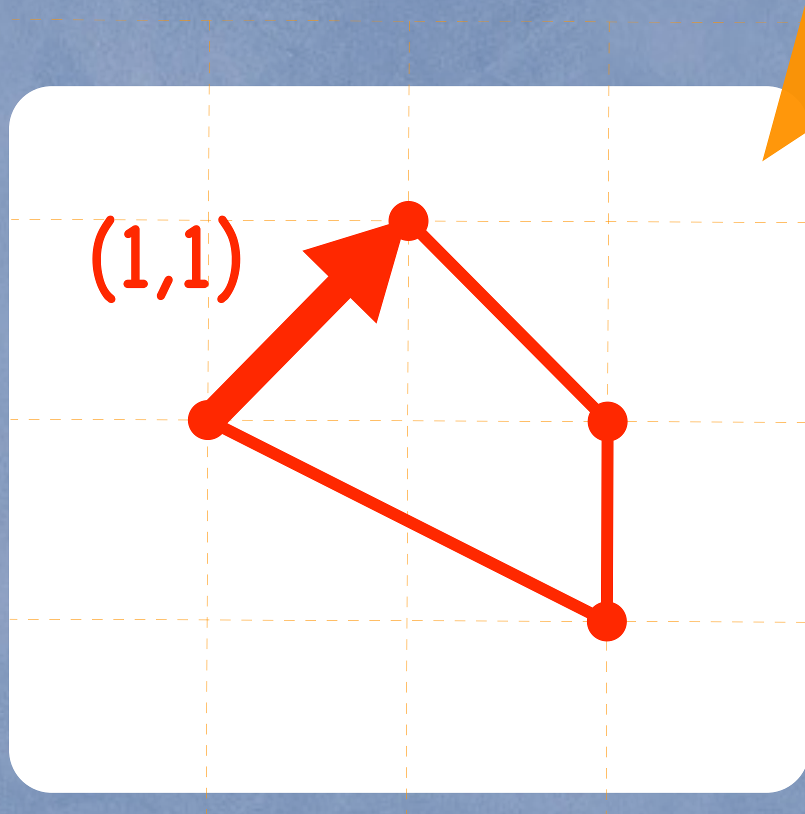

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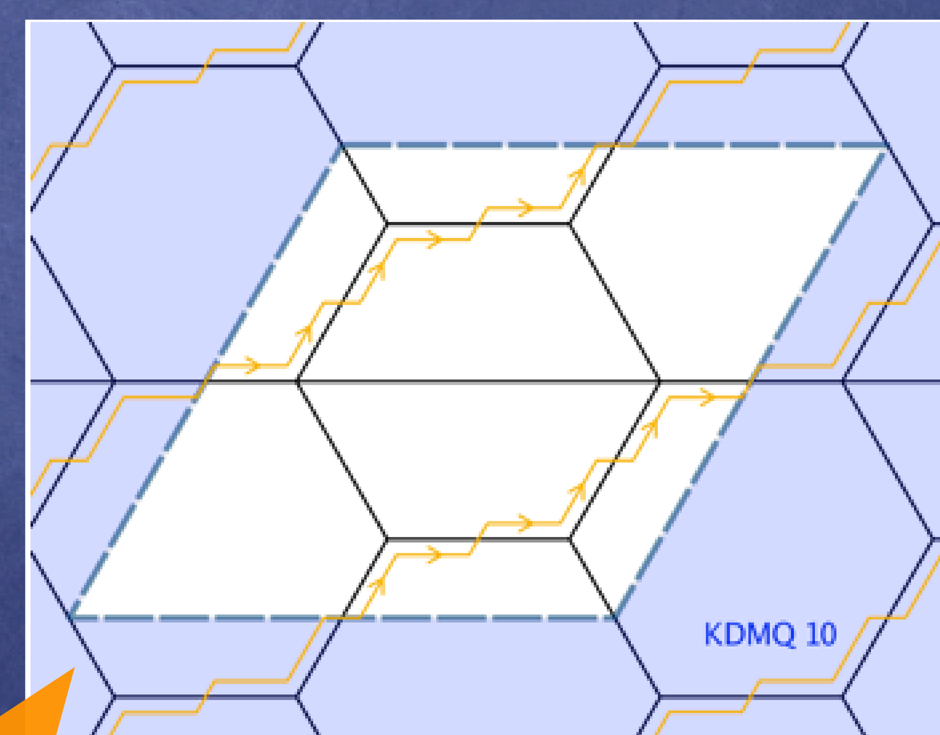

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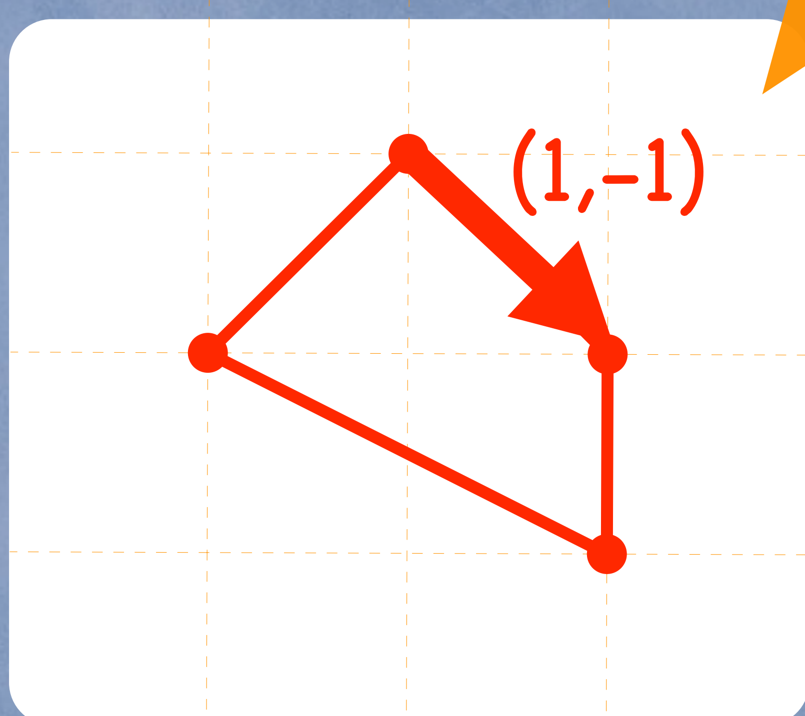

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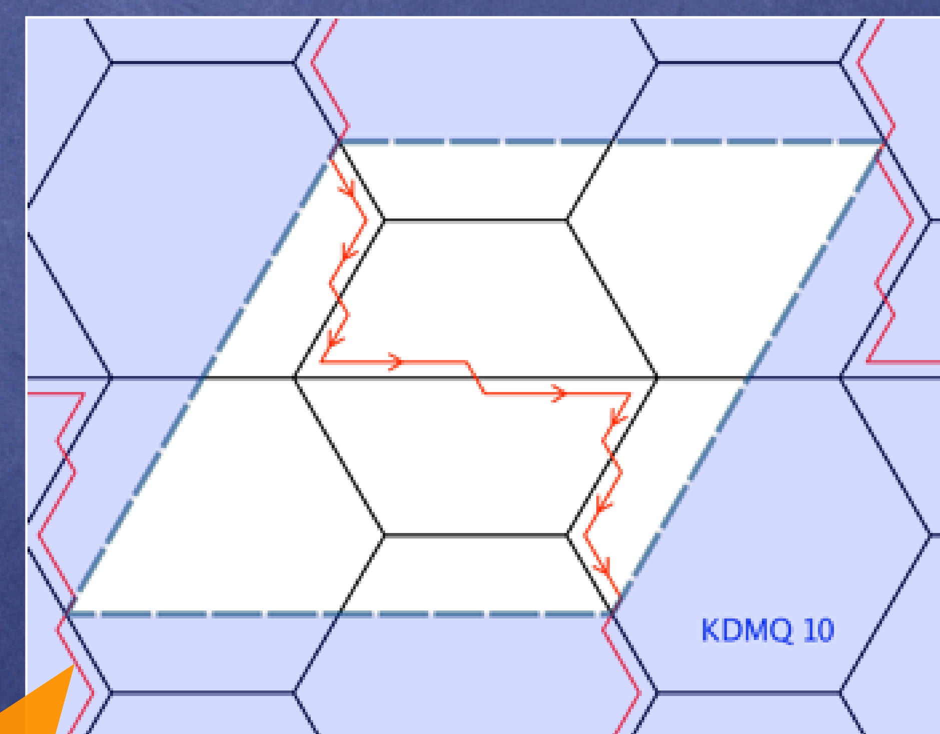

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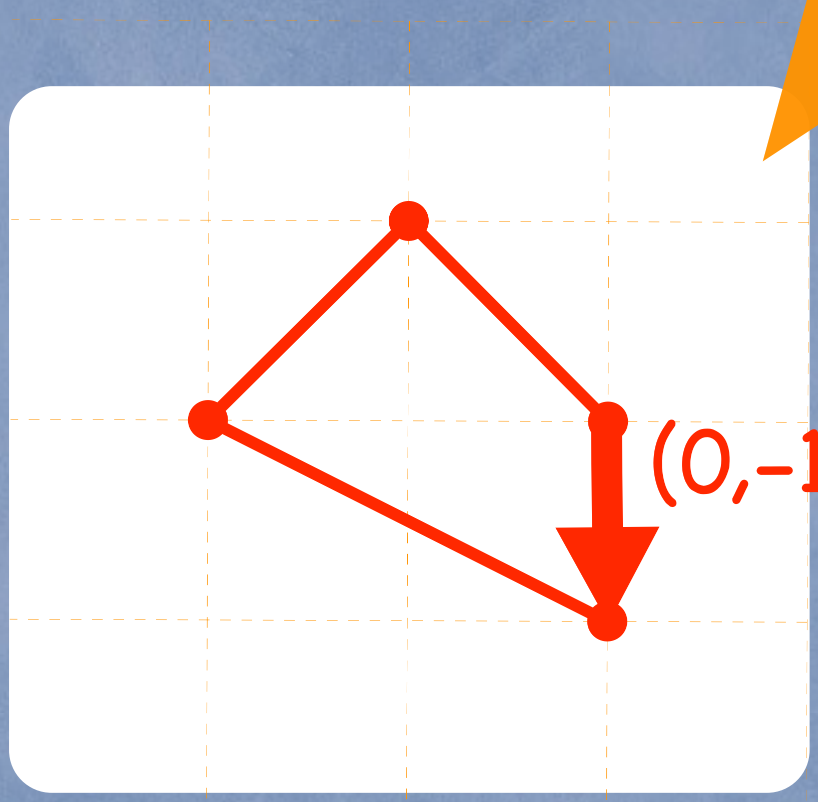

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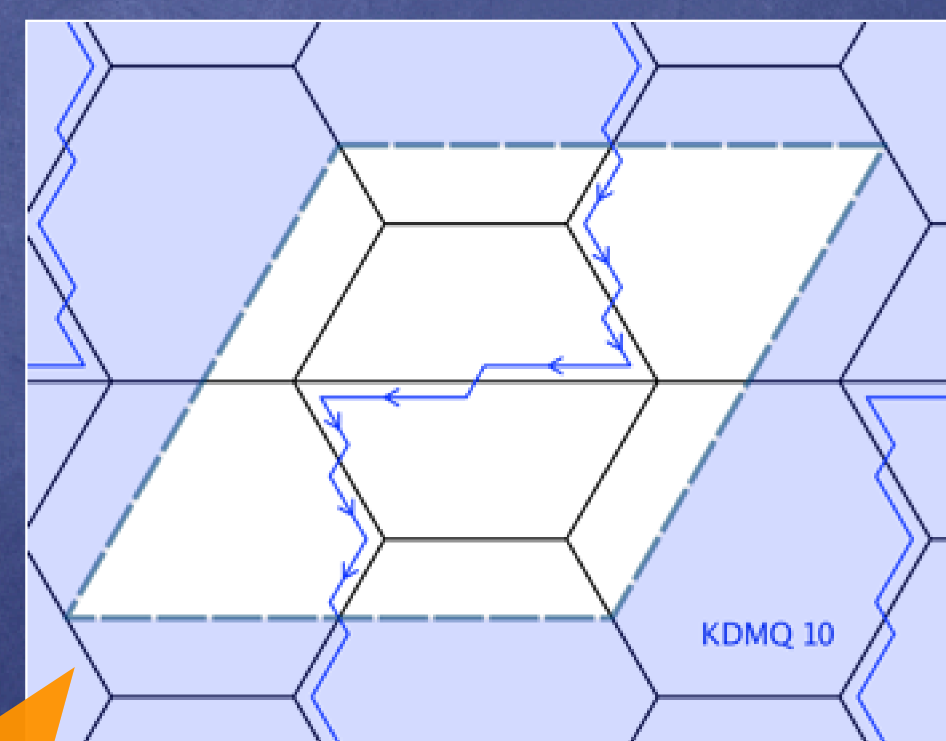

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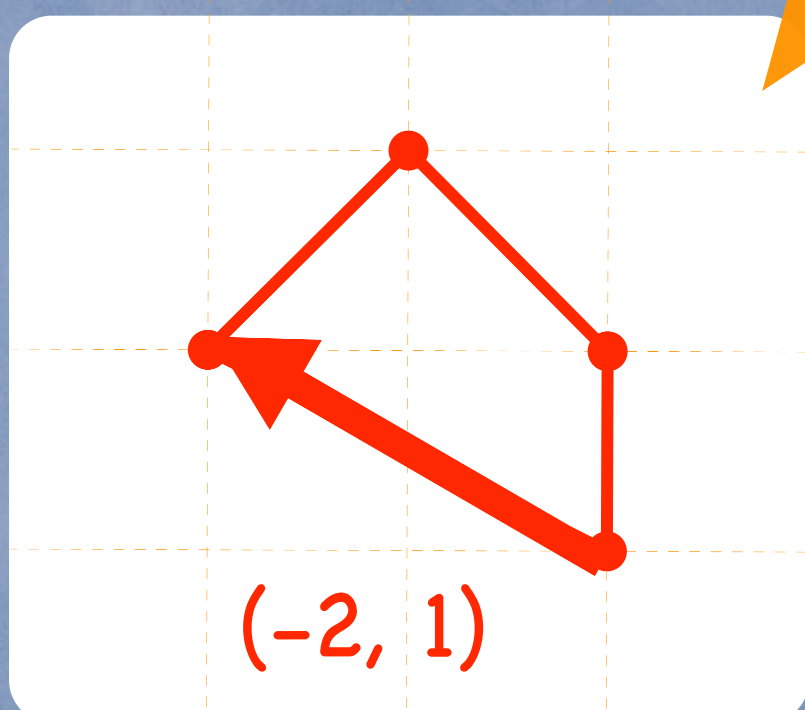

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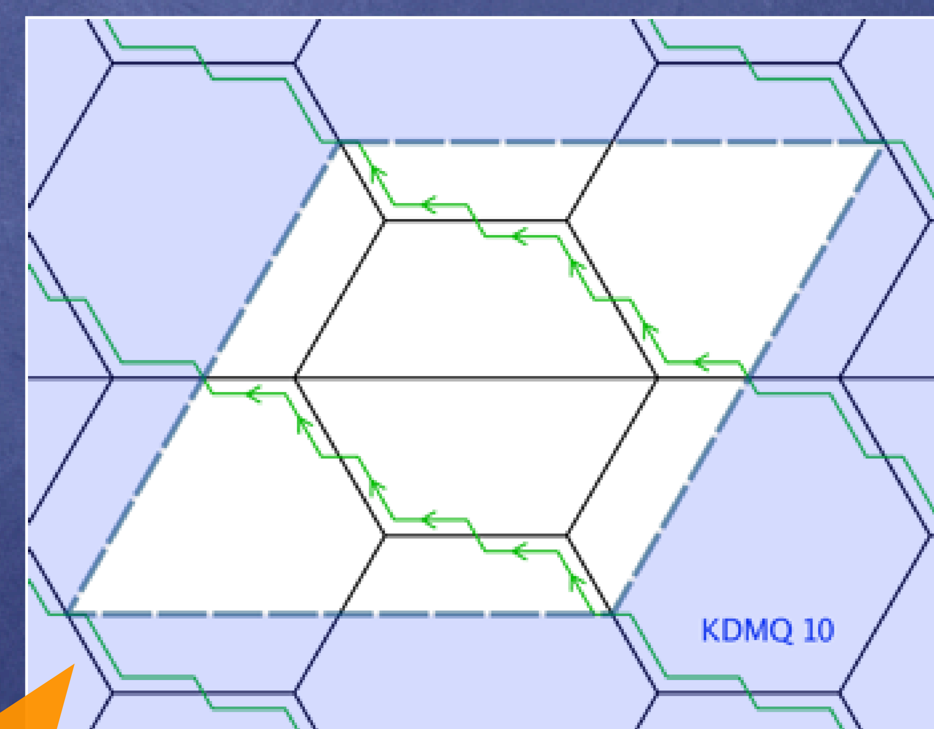

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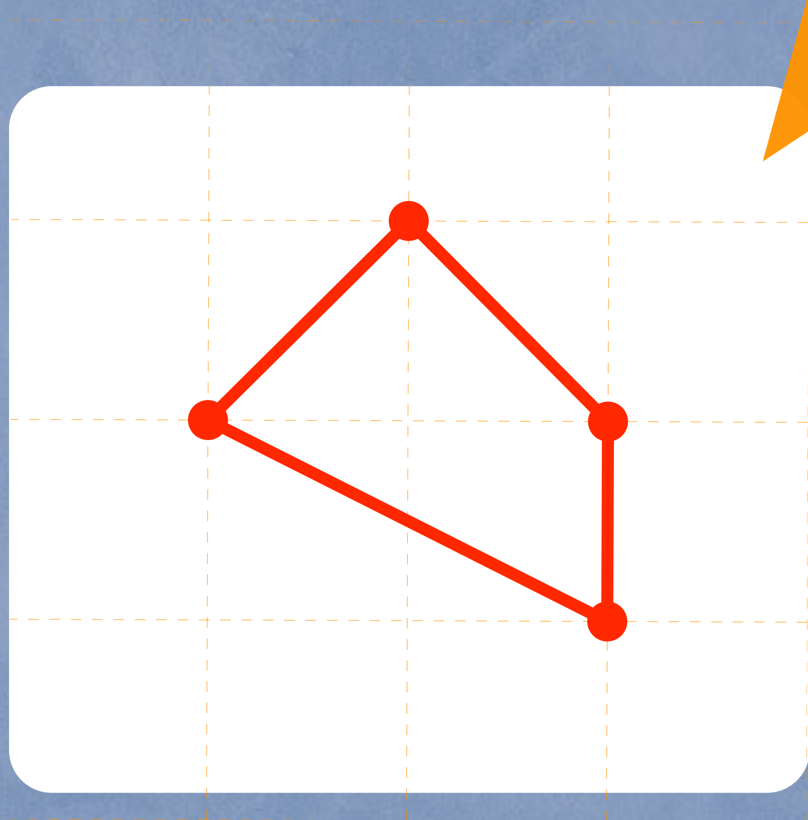


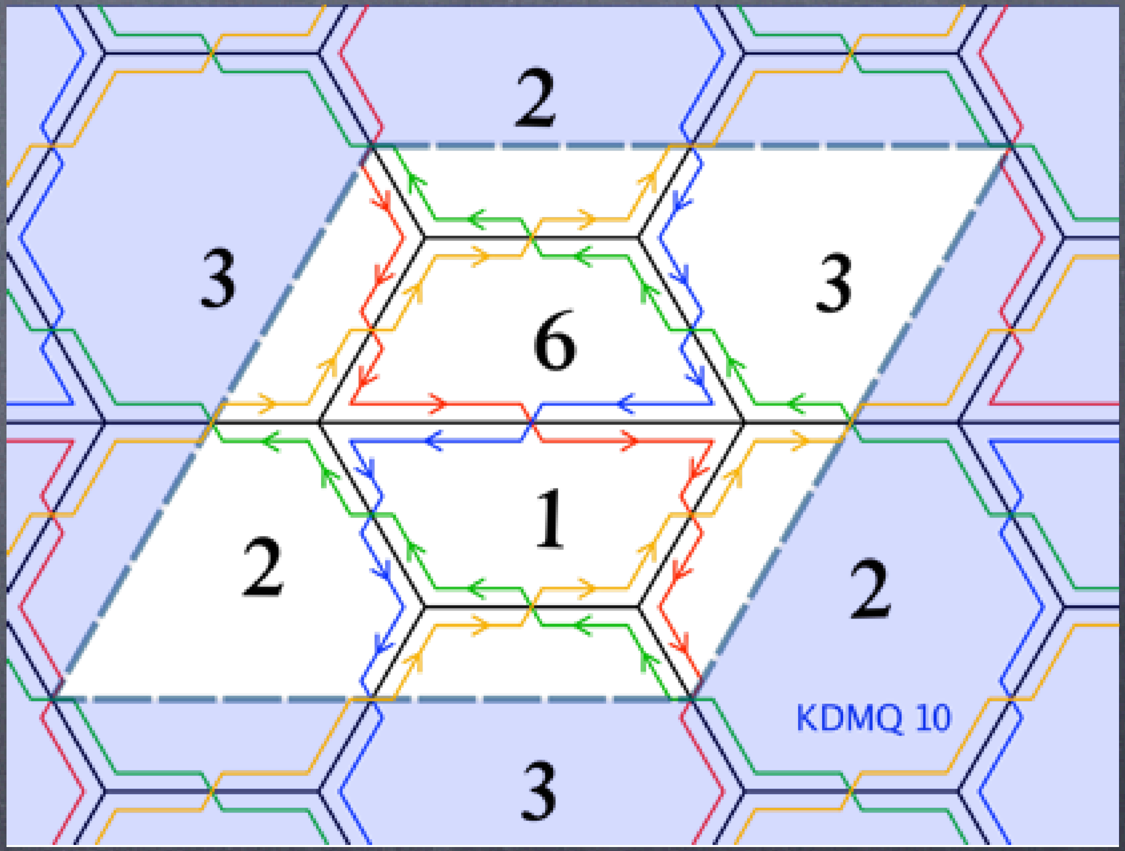
Gauge Theory: Dimer

winding numbers of zigzag paths

Dimer Language II

Reading off the gauge theory

- Faces
  = gauge groups
- Intersection of zigzag paths
  = bi-fundamental matter
- Vertices (faces orbited by zigzag paths)
  = superpotential terms



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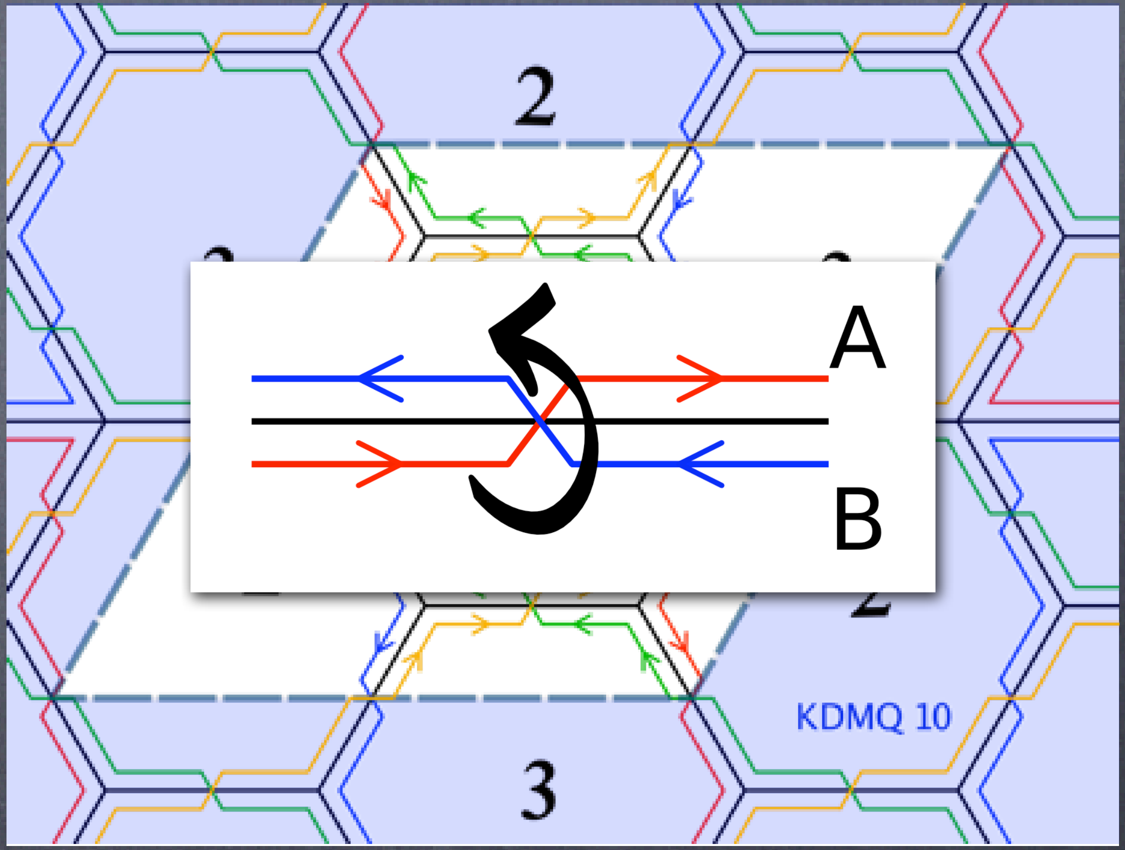

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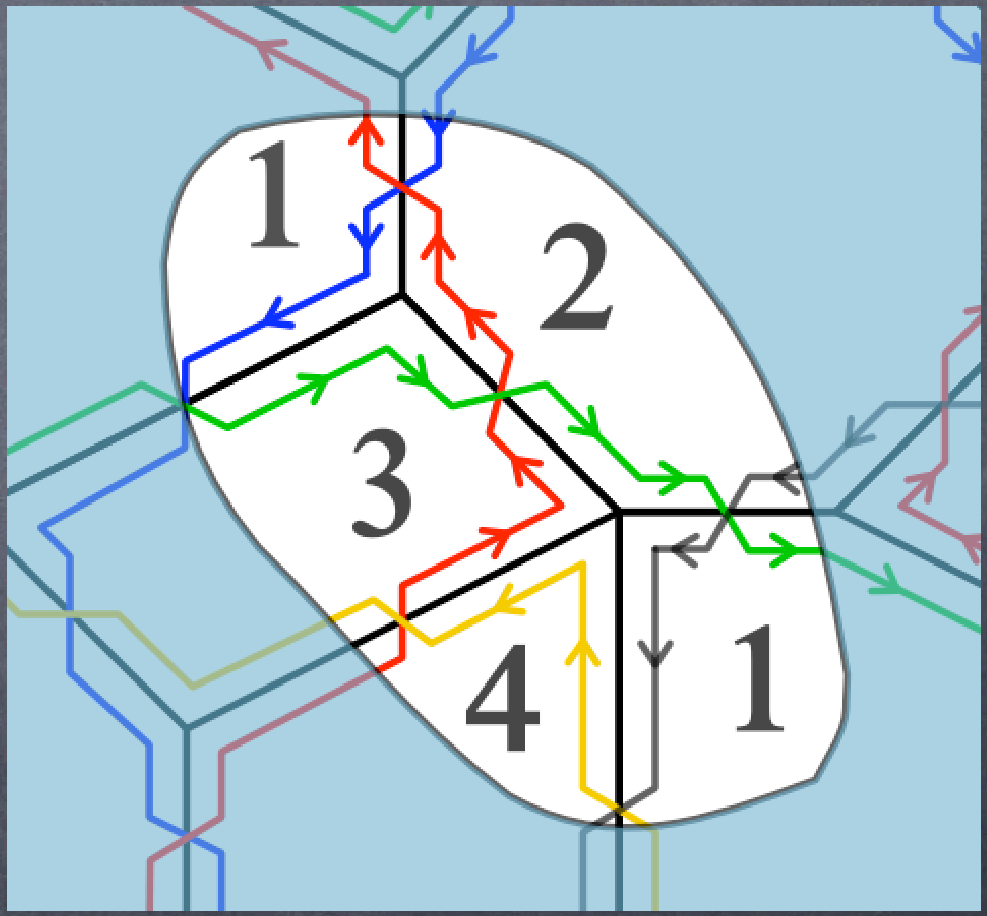
  = gauge groups
- Intersection of zigzag paths
  = bi-fundamental matter
- Vertices (faces orbited by zigzag paths)
  = superpotential terms



$$W = X_{13}X_{32}X_{21} - X_{14}X_{43}X_{32}X_{21}$$

Gulotta 0807.3012

Dimer Language III: How do I get a dimer?

- embed toric singularity in orbifold of conifold whose dimer is known (chess-board).
- collaps cycles in singularity (= cutting toric diagram)
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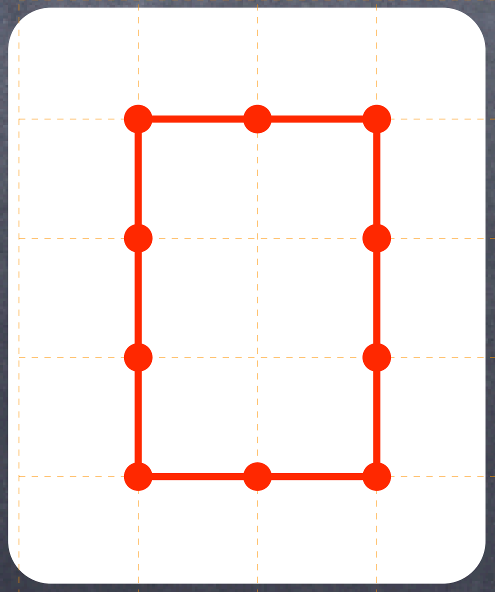

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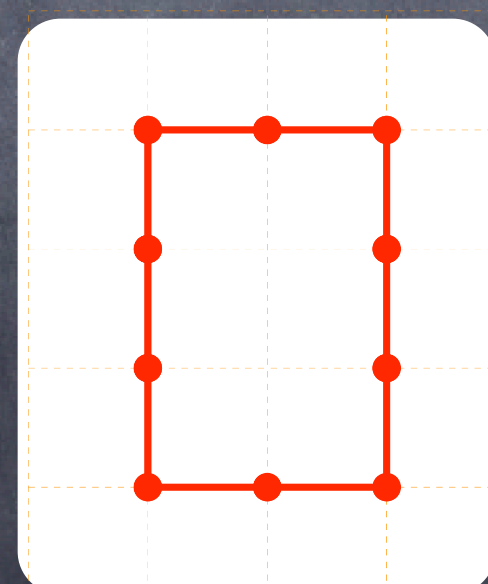

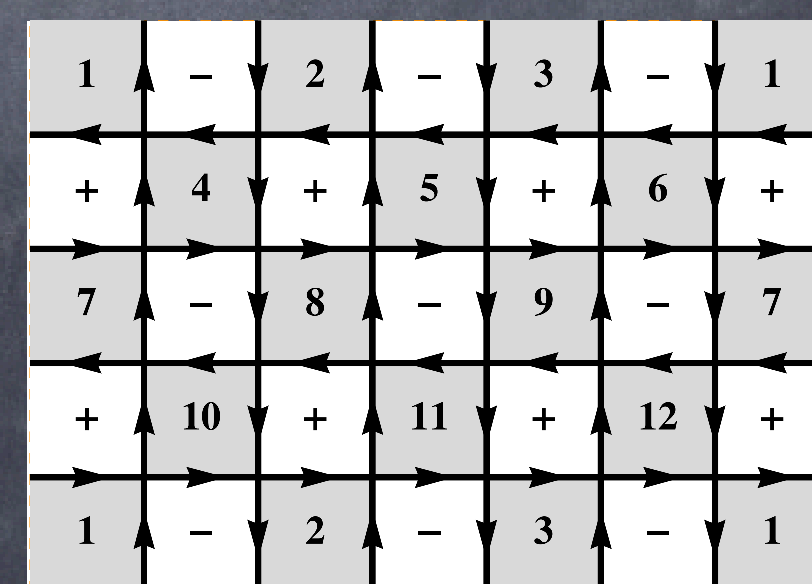

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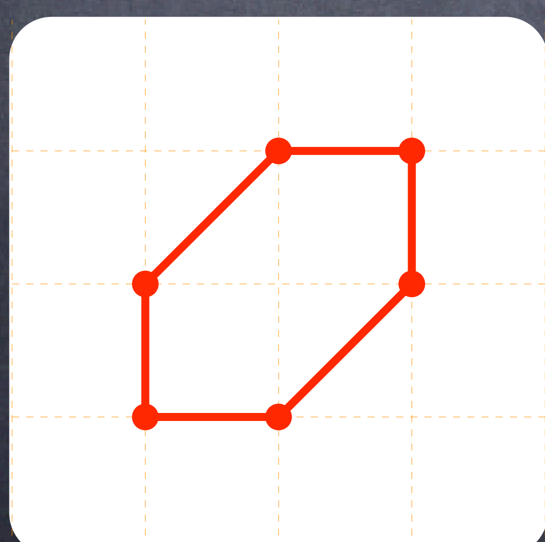

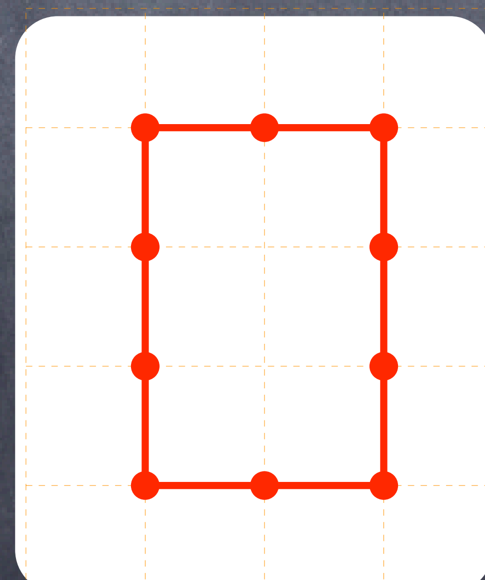

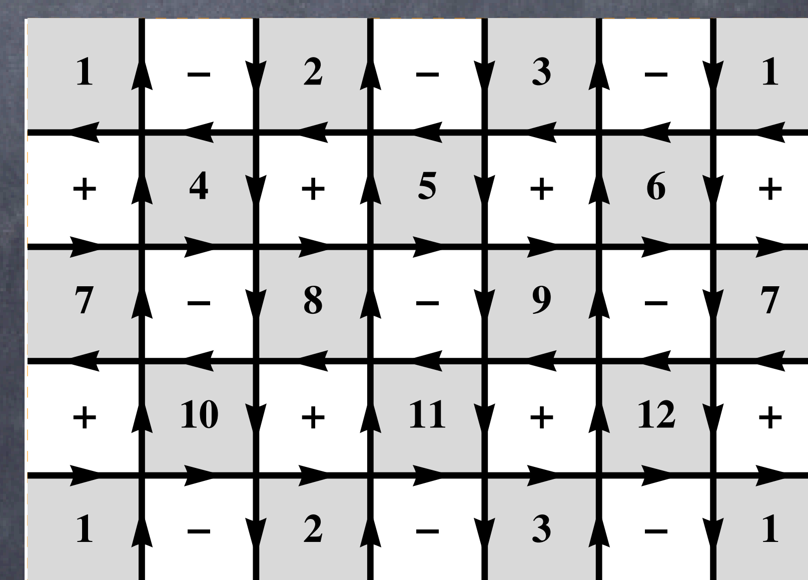

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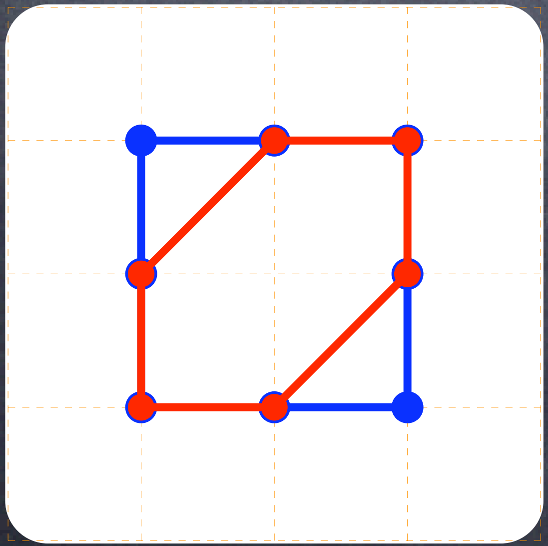

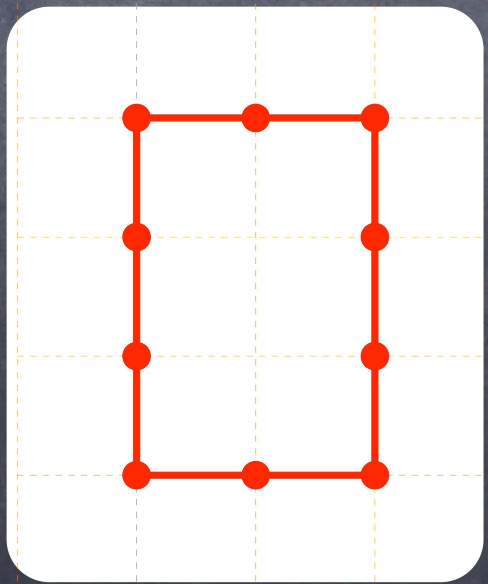

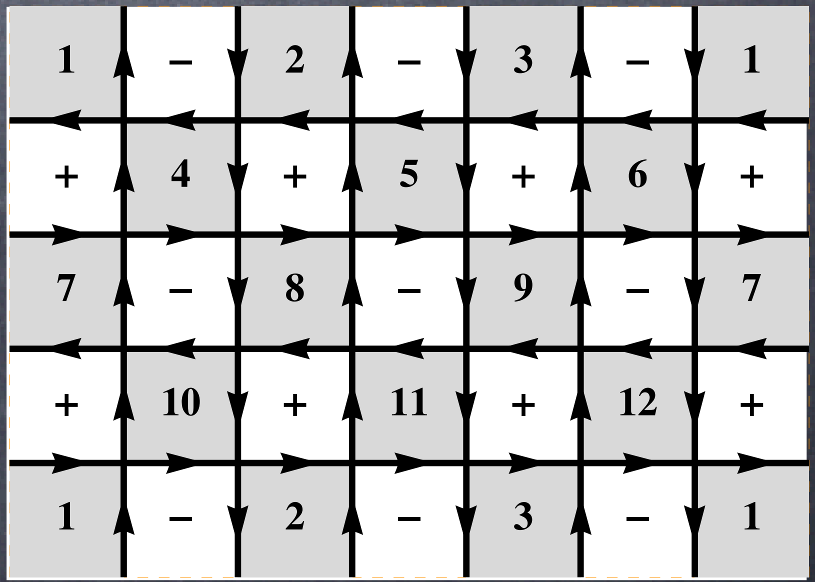

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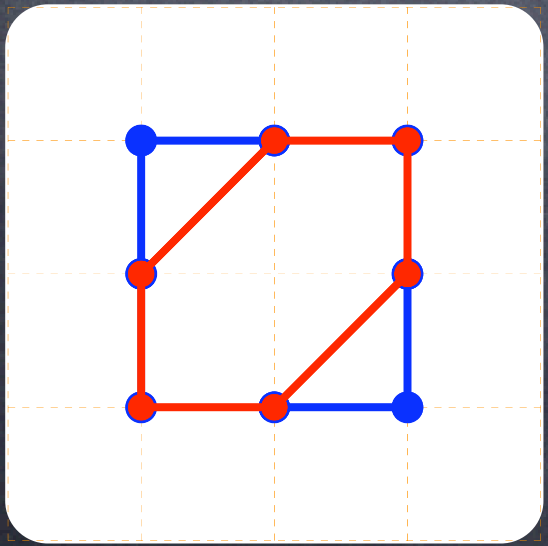

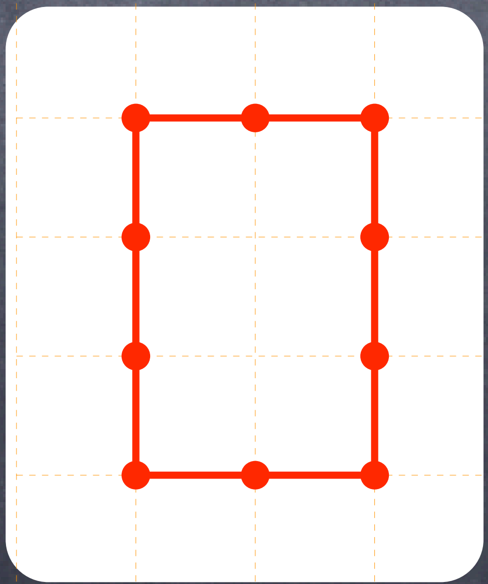

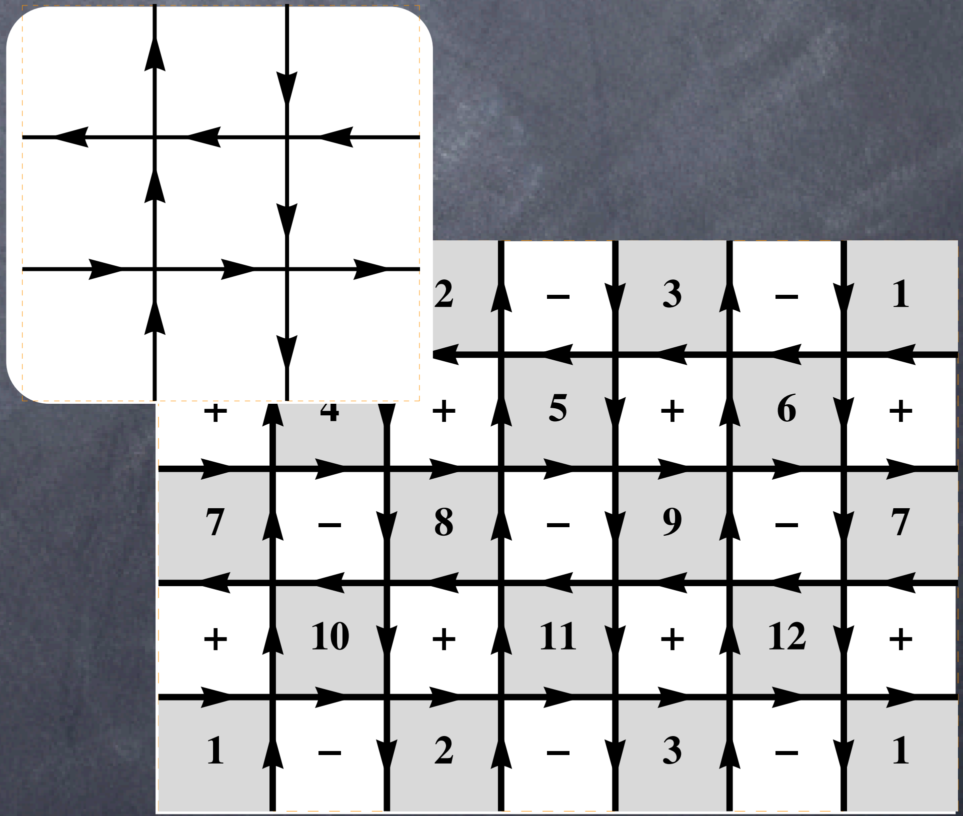

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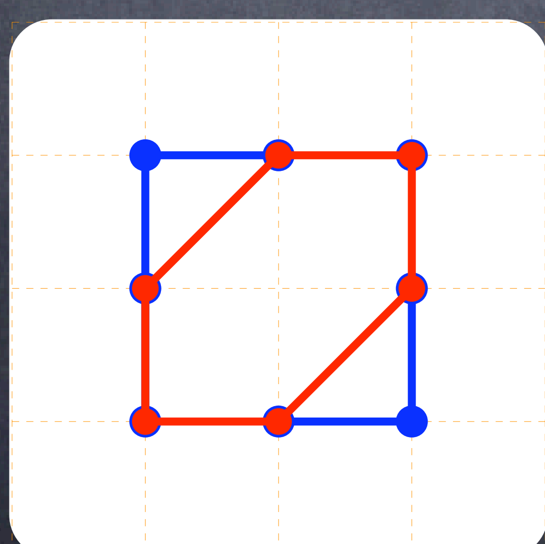

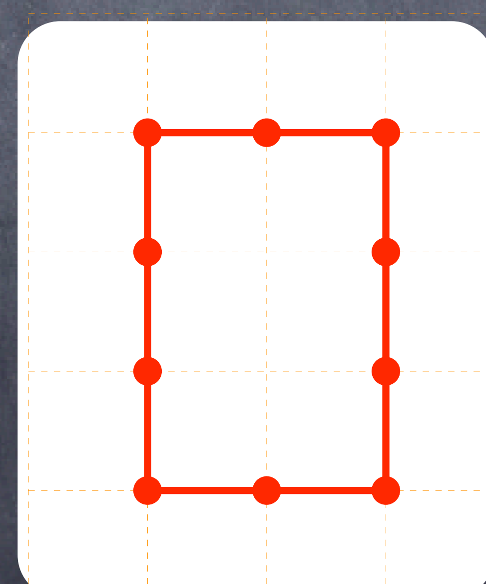

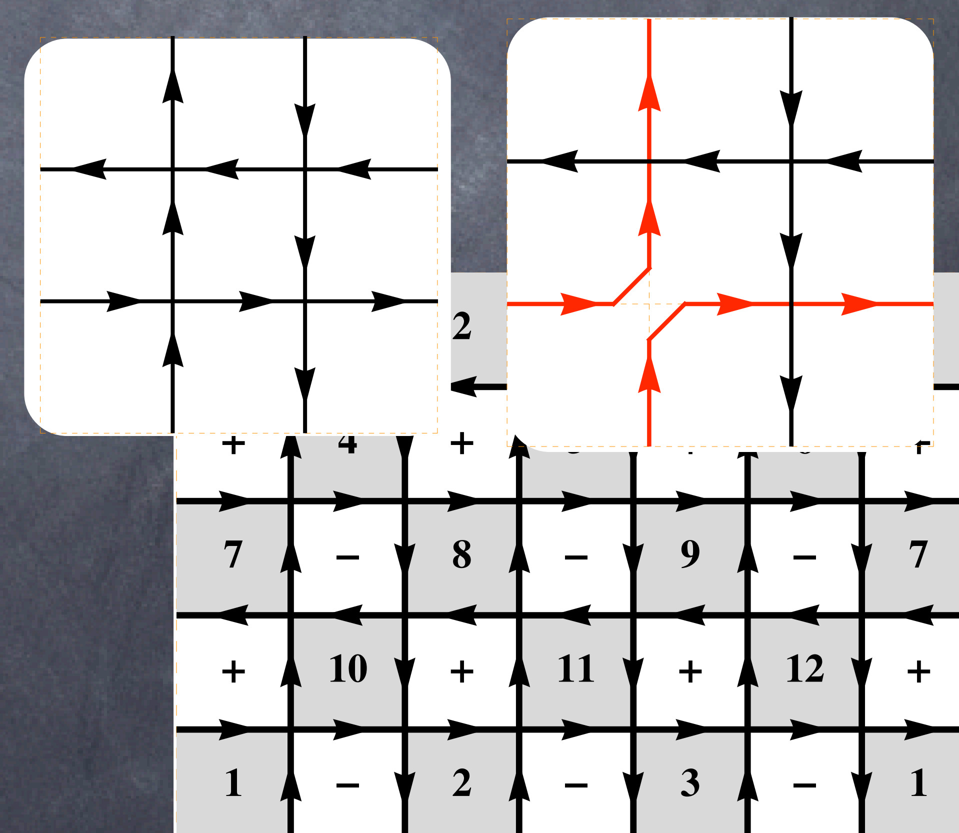

Operations on the dimer

Operation 1:

(1,0) + (0,1) -> (1,1)

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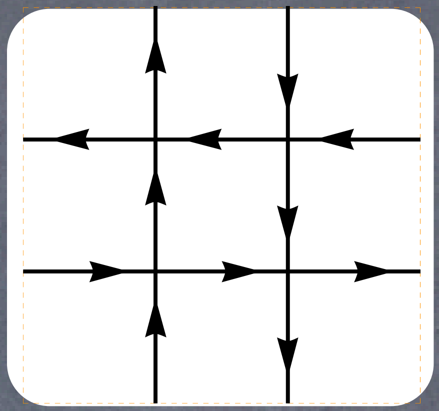

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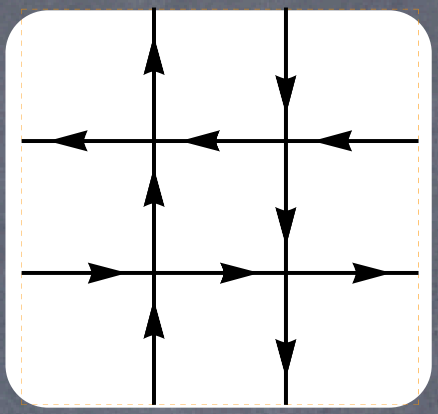

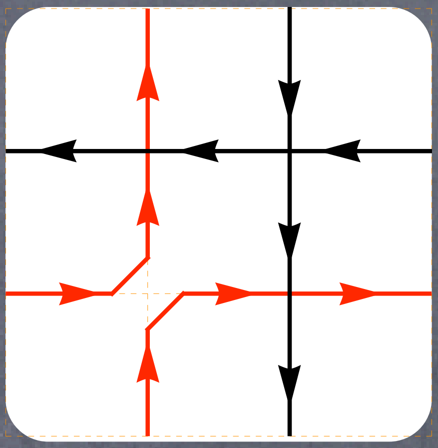

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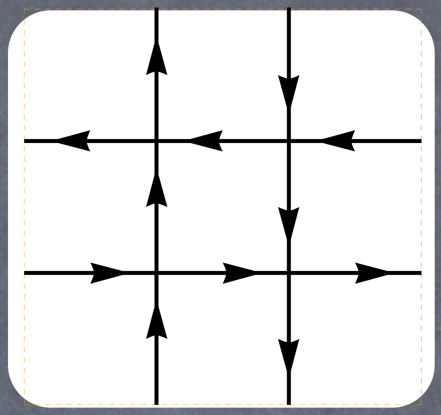

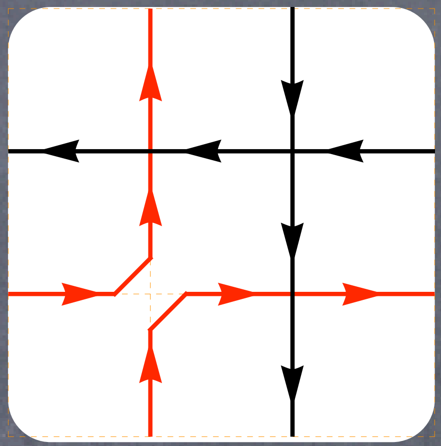

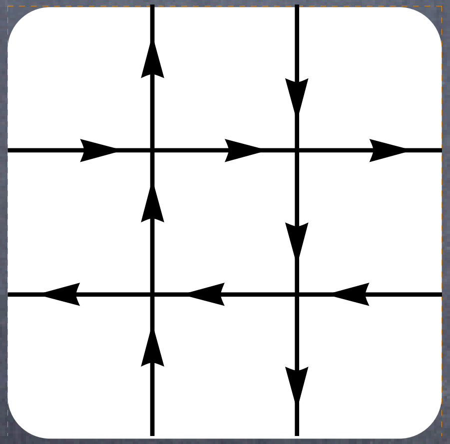

Operation 2:

(1,0) + (0,1) &
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$$I_{(1,1),(-1,-1)} = 0$$

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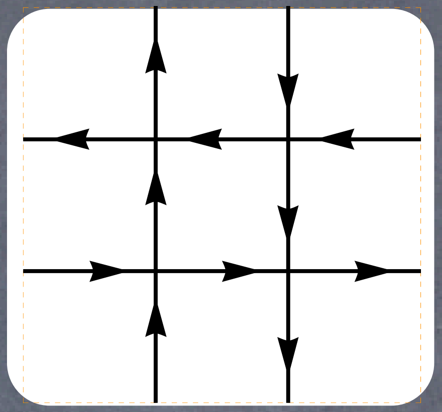

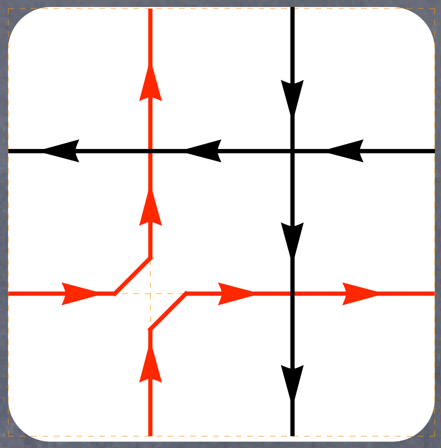

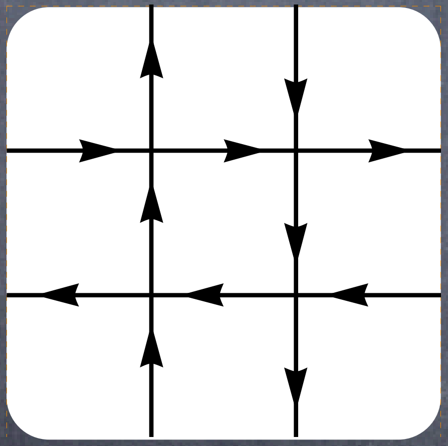

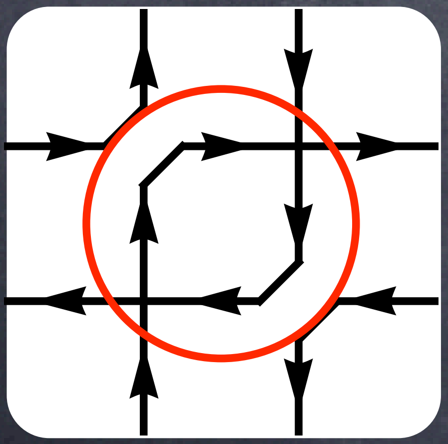

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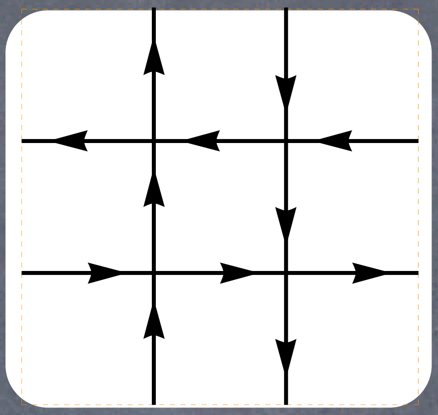

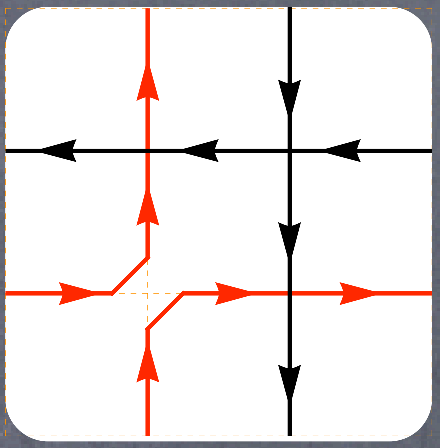

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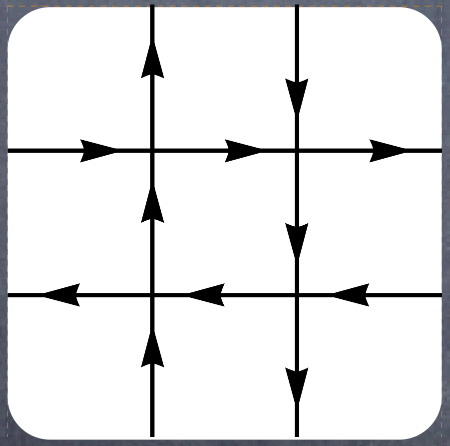

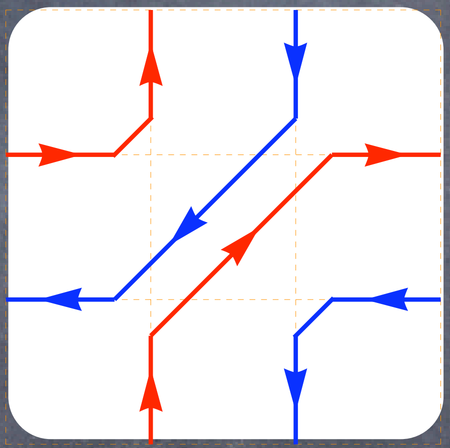

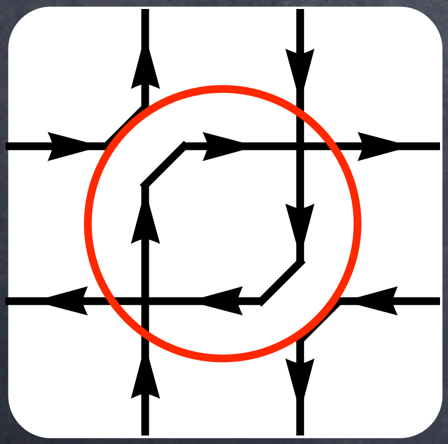

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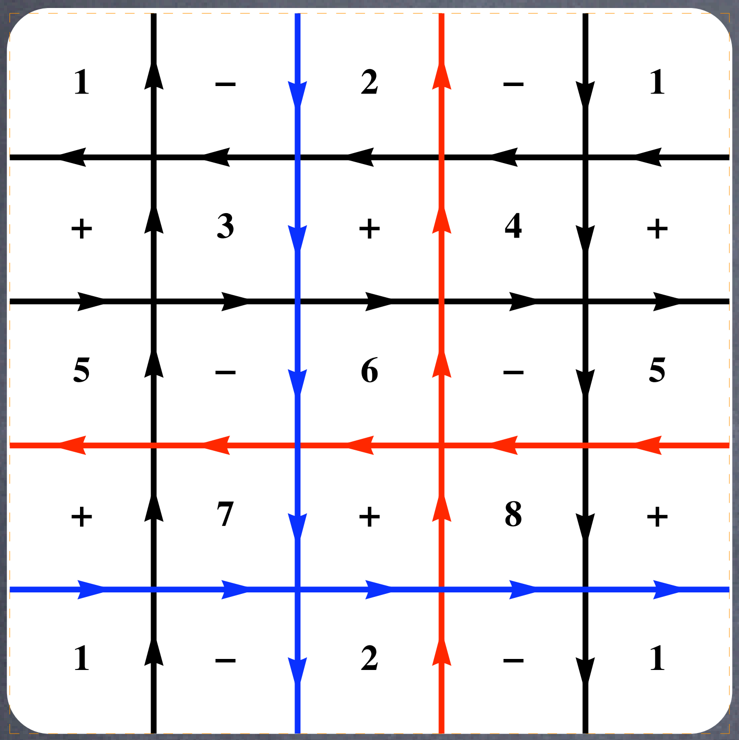
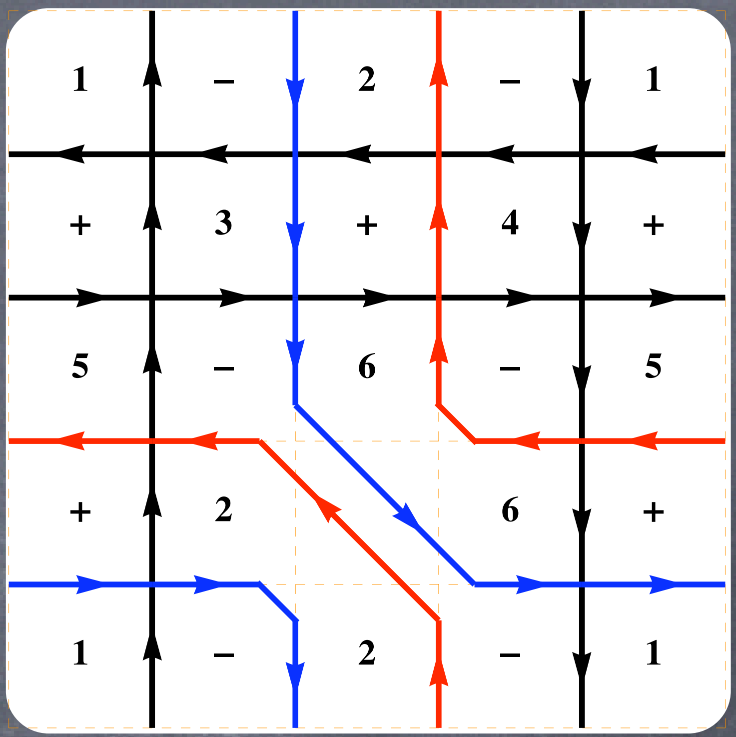
e.g. del Pezzo 3





$$
\begin{aligned}
W_{dP_3} \quad = \quad & -X_{12}Y_{31}Z_{23} - X_{45}Y_{64}Z_{56} + X_{45}Y_{31}Z_{14}\rho_{53} + X_{12}Y_{25}Z_{56}\Phi_{61} \\
& + X_{36}Y_{64}Z_{23}\Psi_{42} - X_{36}Y_{25}Z_{14}\rho_{53}\Phi_{61}\Psi_{42}
\end{aligned}
$$

$$
= \begin{pmatrix} X_{45} \\ Y_{23} \\ Z_{25} \end{pmatrix} \begin{pmatrix} 0 & Z_{14}\rho_{53} & -Y_{64} \\ -Z_{14}\rho_{53}\Phi_{61}\Psi_{42} & 0 & X_{12}\Phi_{61} \\ Y_{64}\Psi_{42} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{56} \end{pmatrix} .
$$

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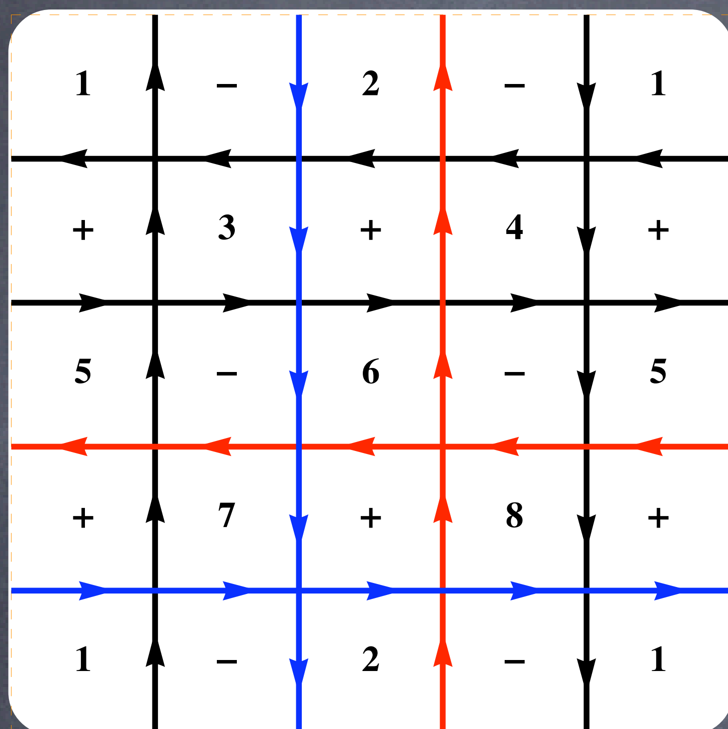

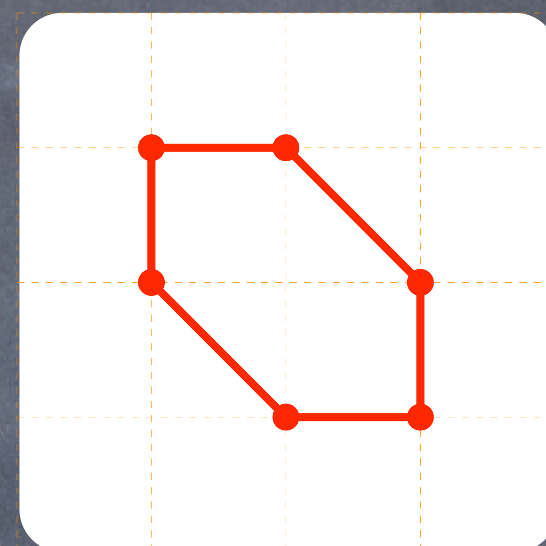

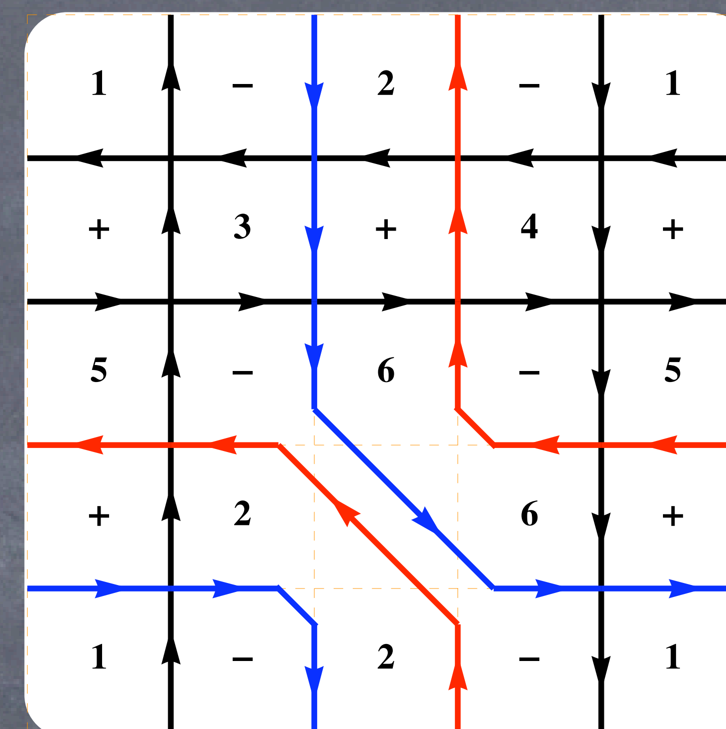

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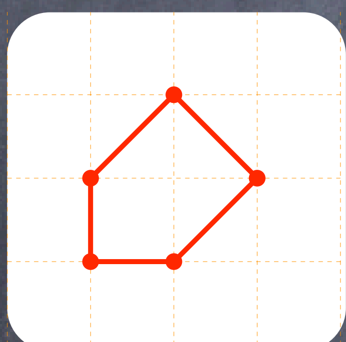
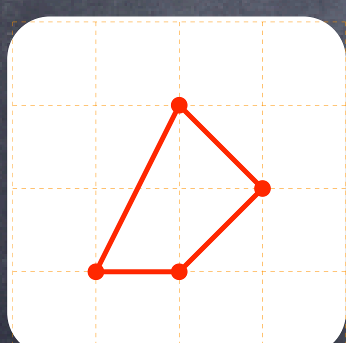
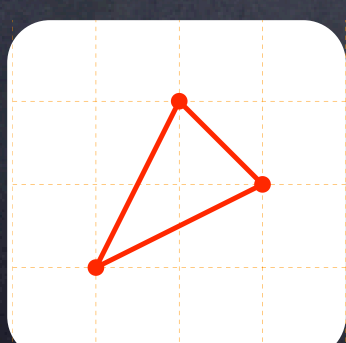
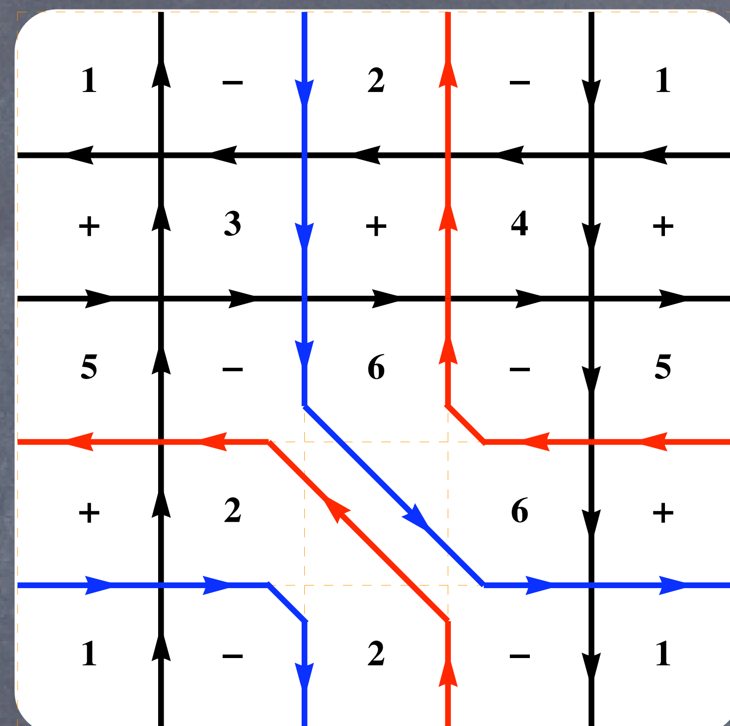
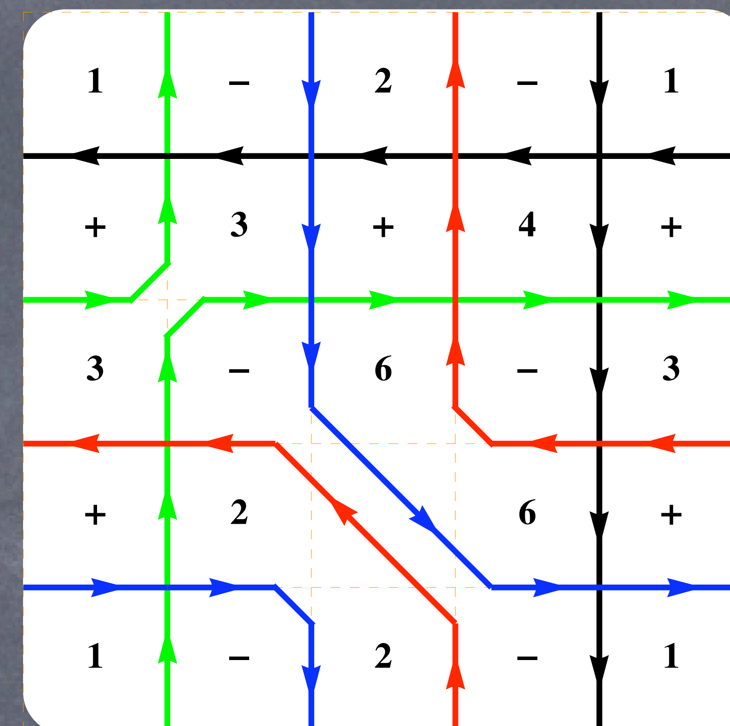
Application of Gulotta's algorithm to toric del-Pezzo surfaces

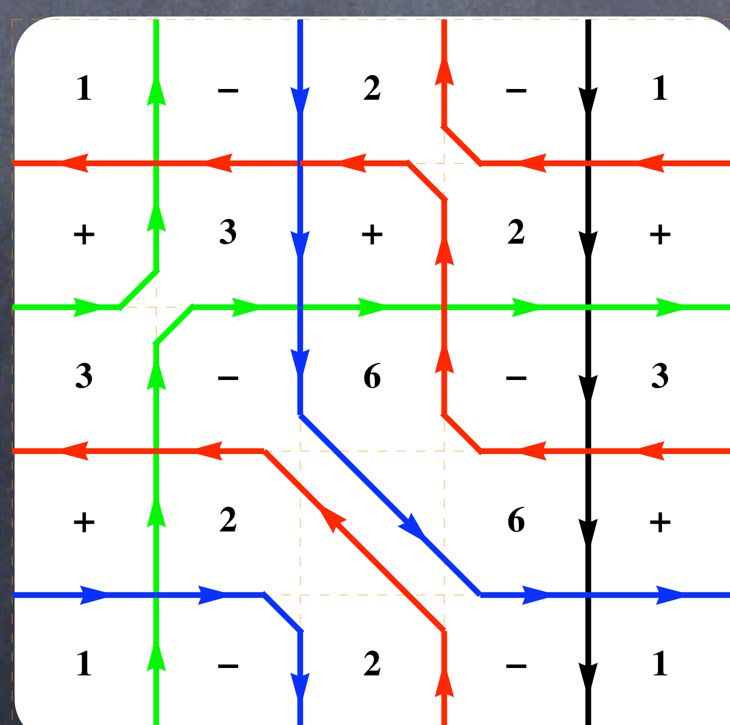


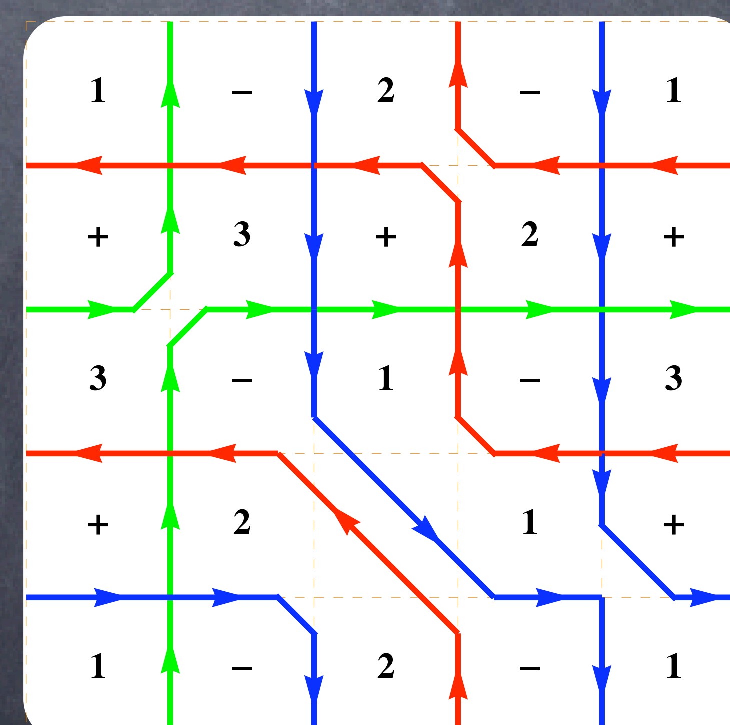




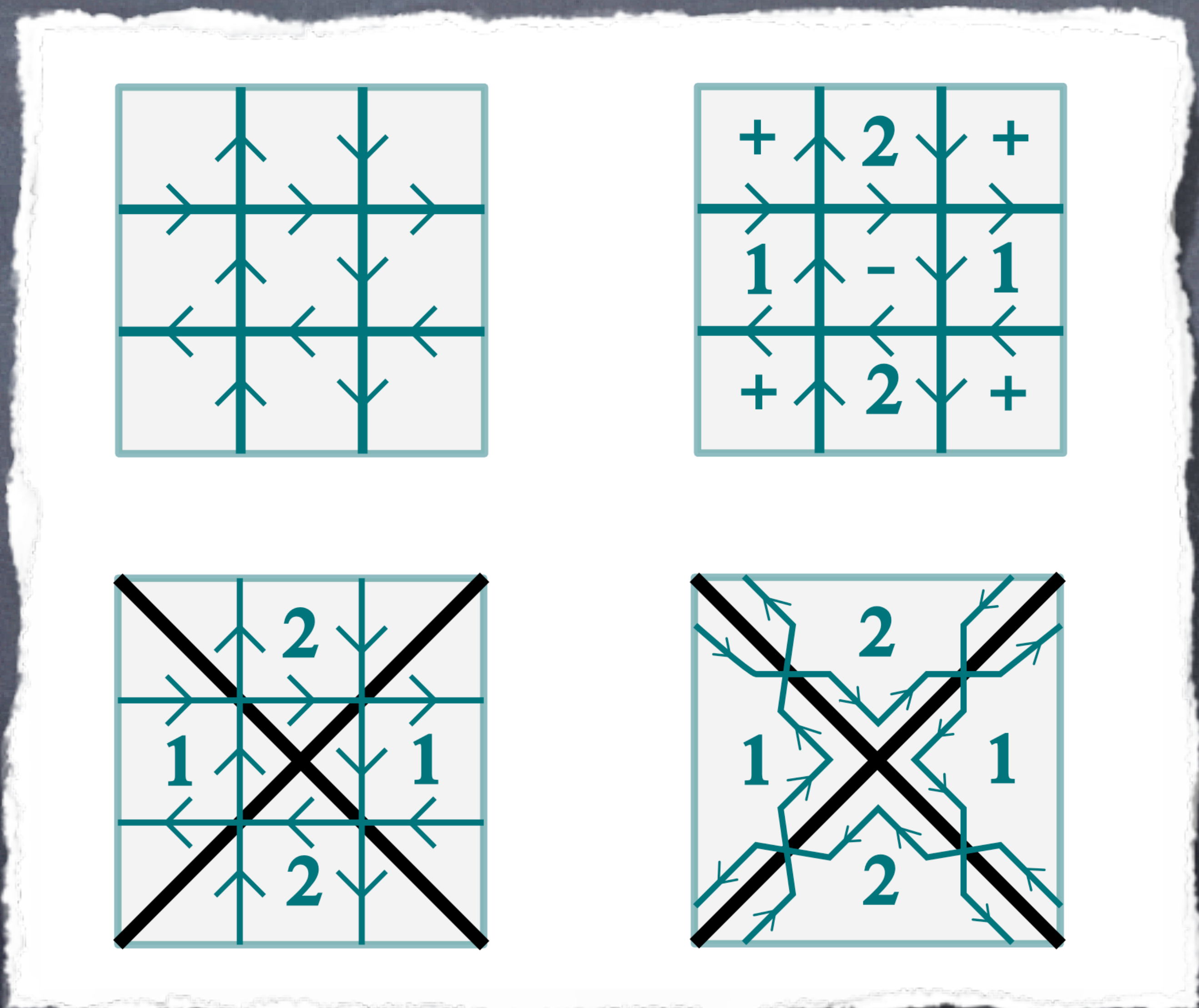








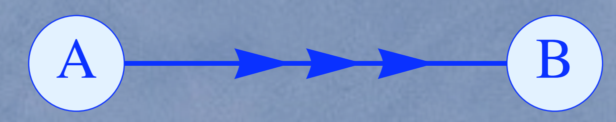
Gulotta's dimers = Traditional dimers



Philosophy: Use this algorithmic view of gauge theories to find general features for gauge theories probing toric singularities!

Are there properties revealed that are not apparent from looking at the superpotential and are they useful for model building?

Restricting the # of families



- The dimers obtained with the operations of Gulotta are highly restricted (otherwise: inconsistent dimers)
- example: additional crossing (-> mass term)
- without add. crossings 3 (left), with add. crossings 4 but unique (right)

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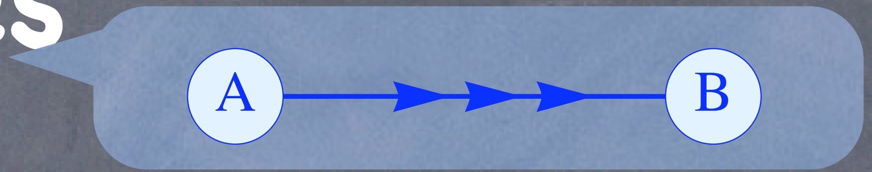

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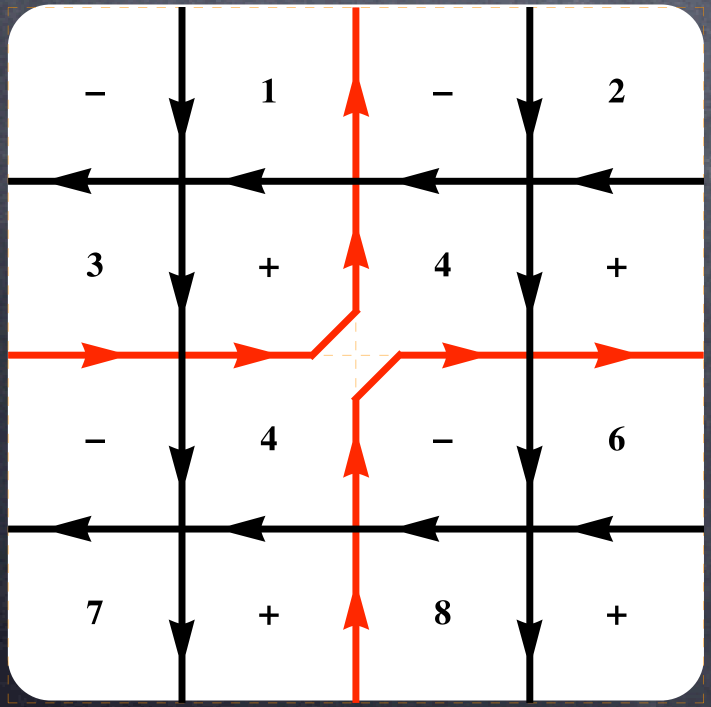

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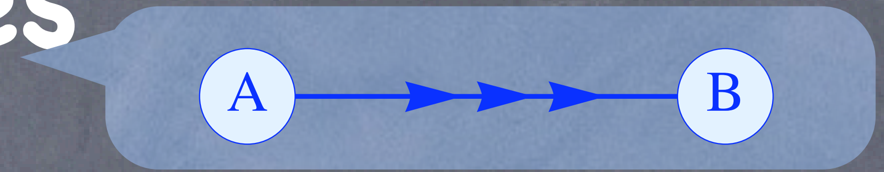

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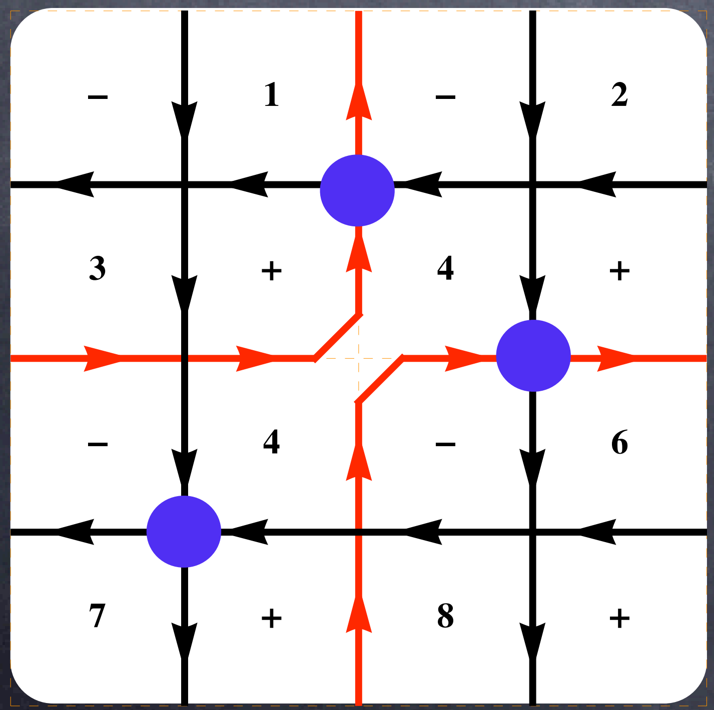

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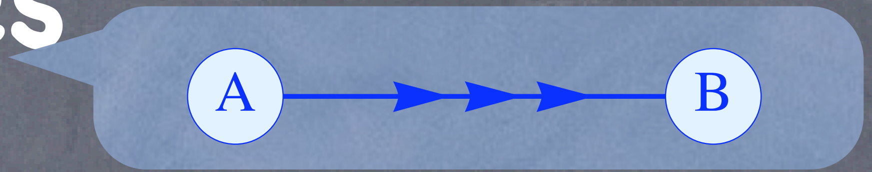

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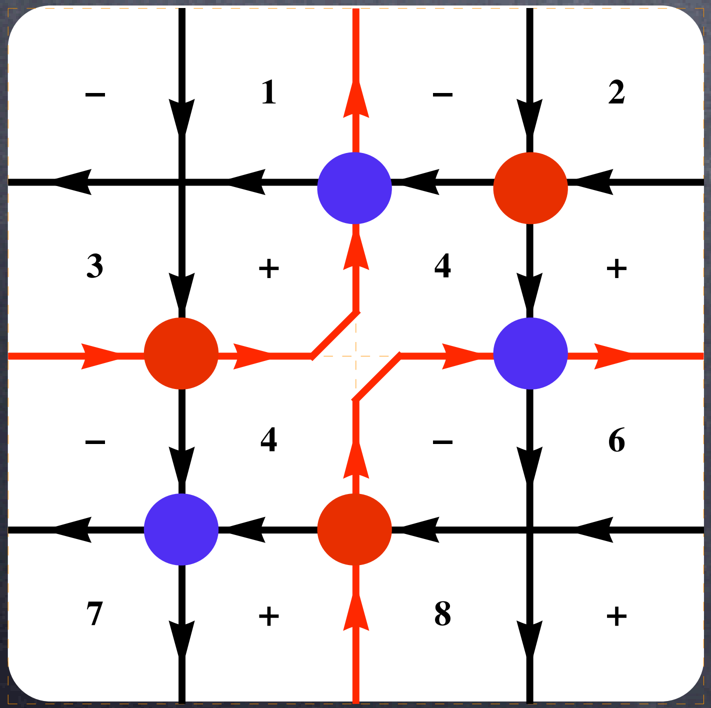

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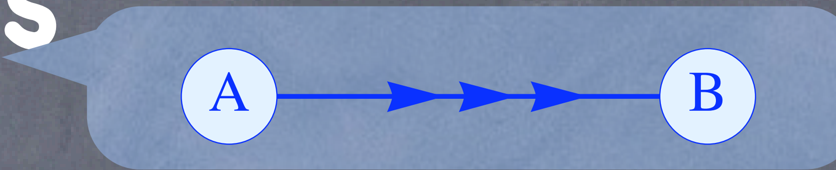

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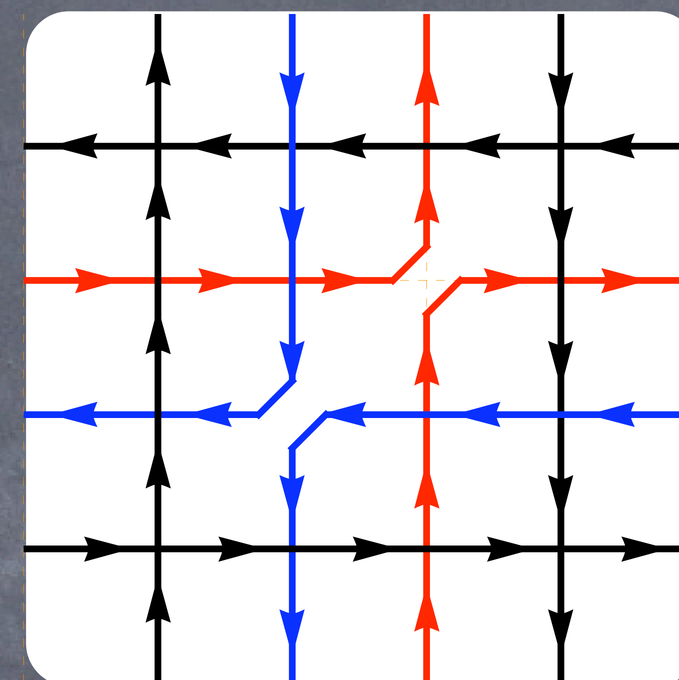

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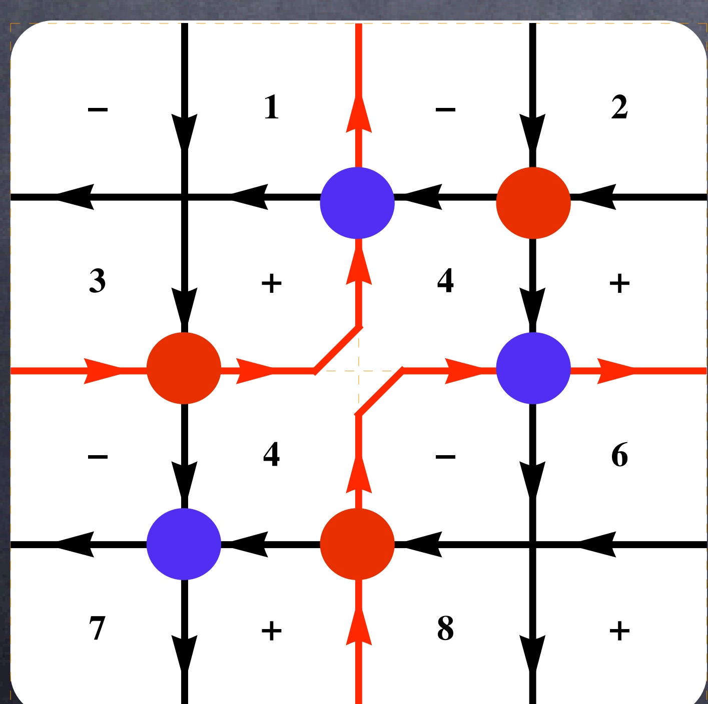

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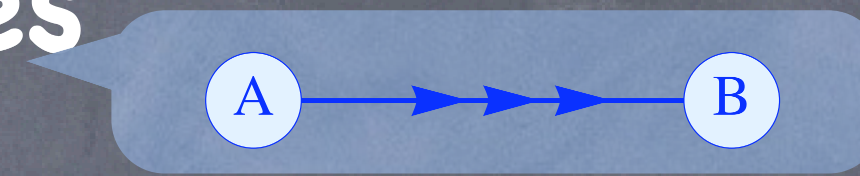

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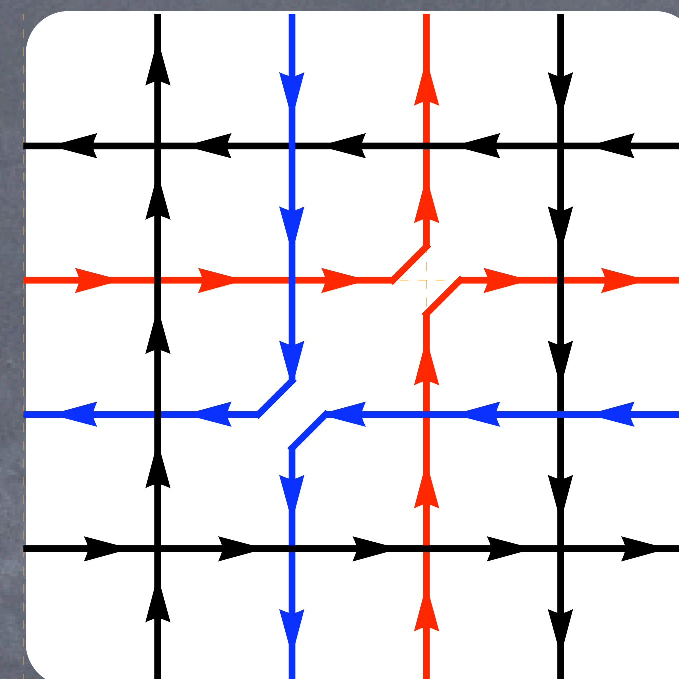

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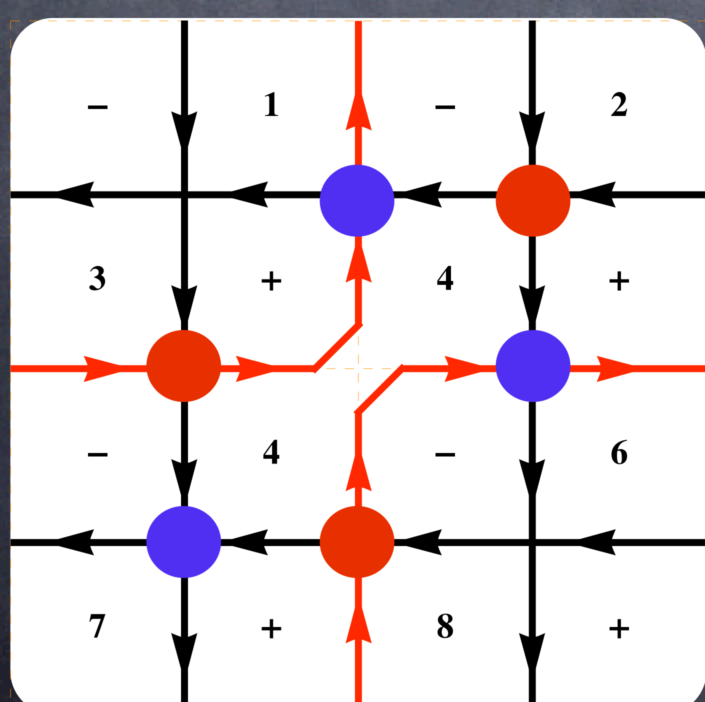
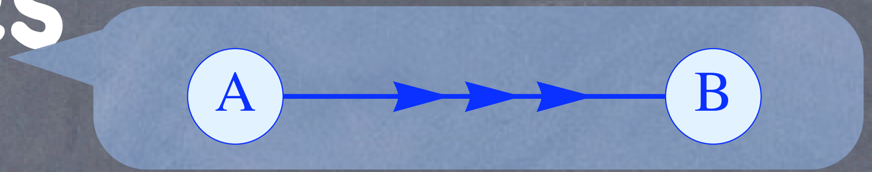


maximum of
3 fields in/out for
any gauge group

-> 3 families

Restricting the # of families



- The dimers obtained with the operations of Gulotta are highly restricted (otherwise: inconsistent dimers)
- example: additional crossing (-> mass term)
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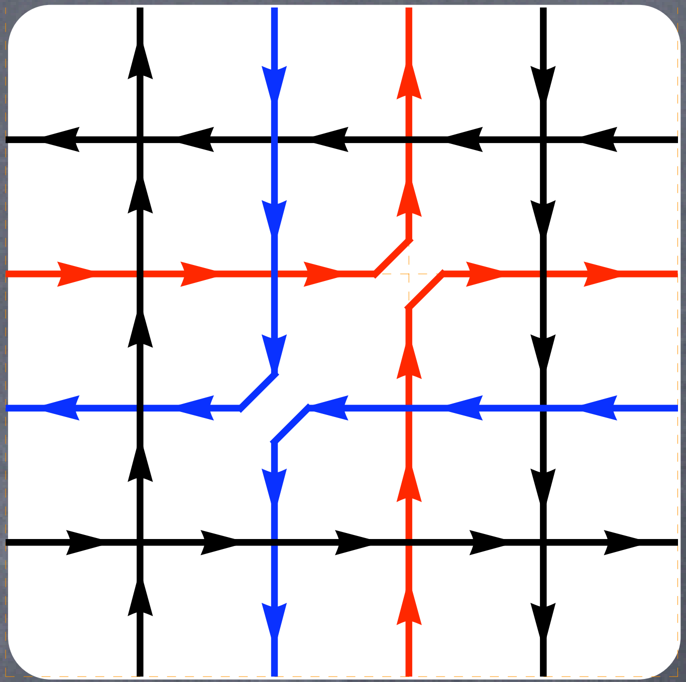
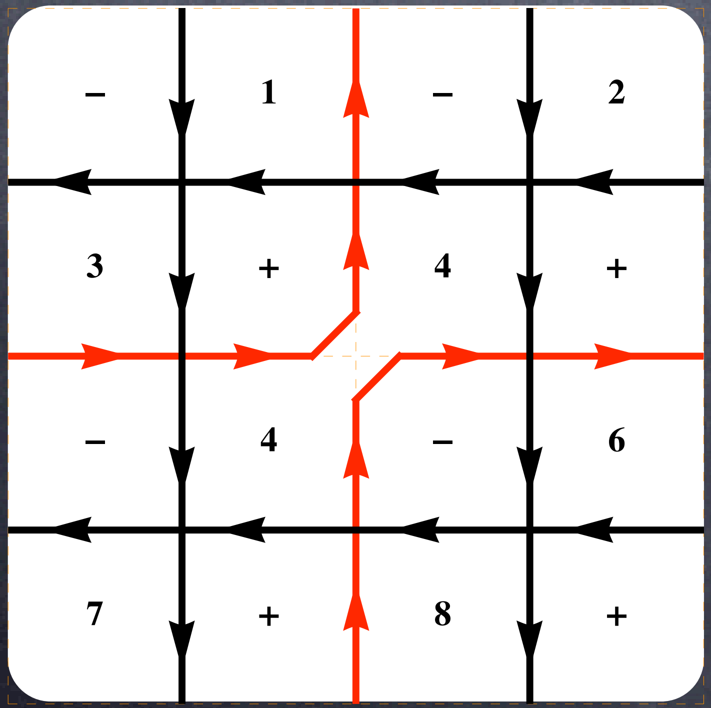
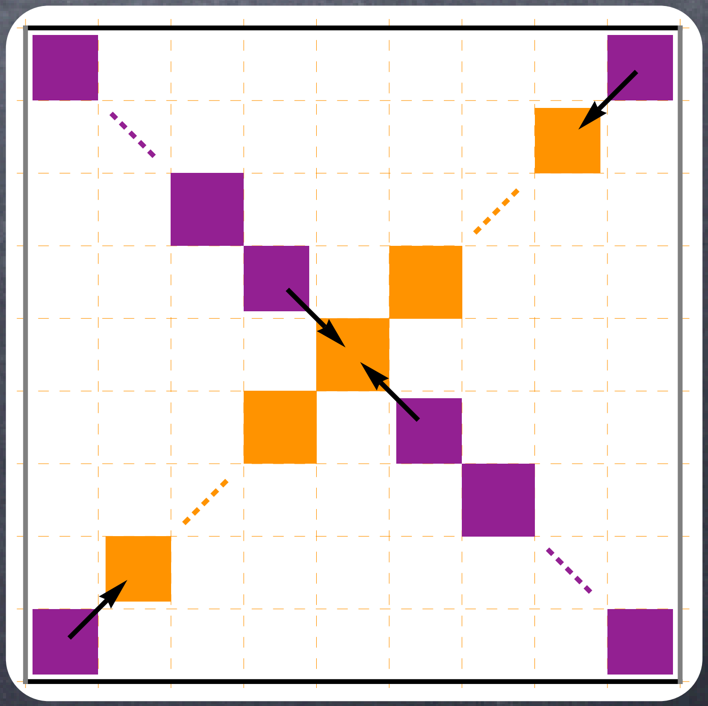


e.g.



maximum of 4 fields (no add. branches) between 2 gauge groups

-> 4 families



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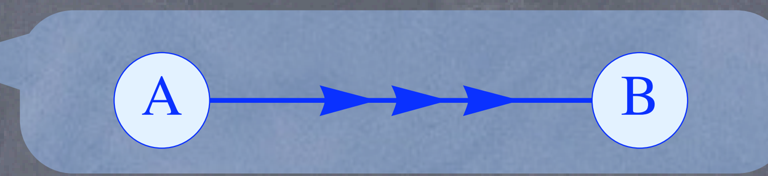

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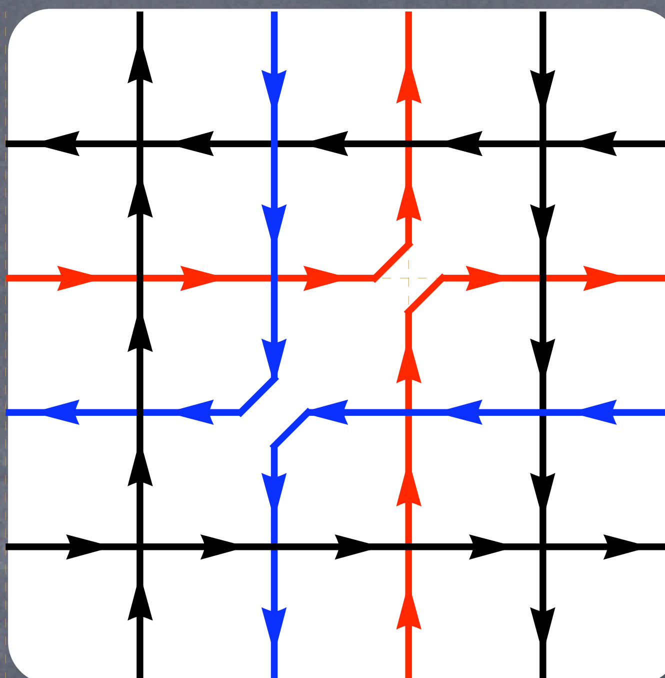

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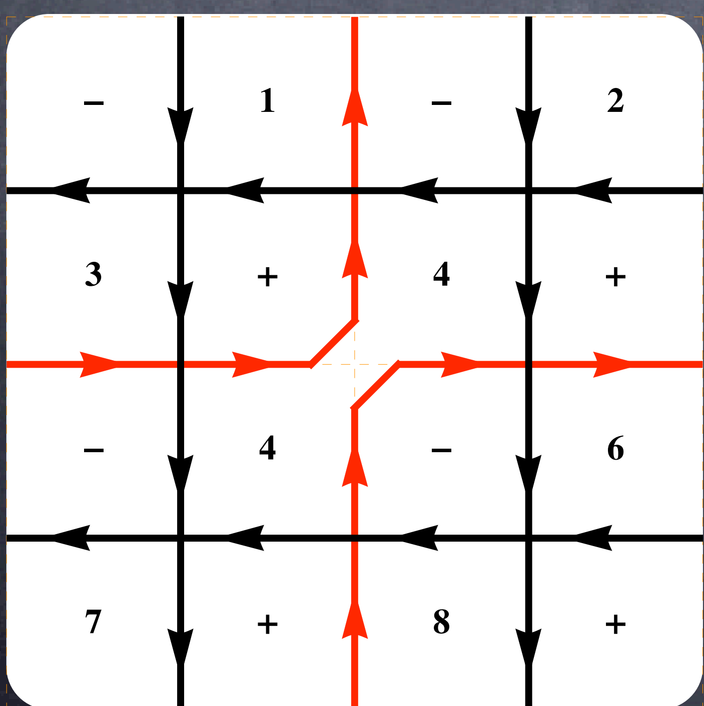

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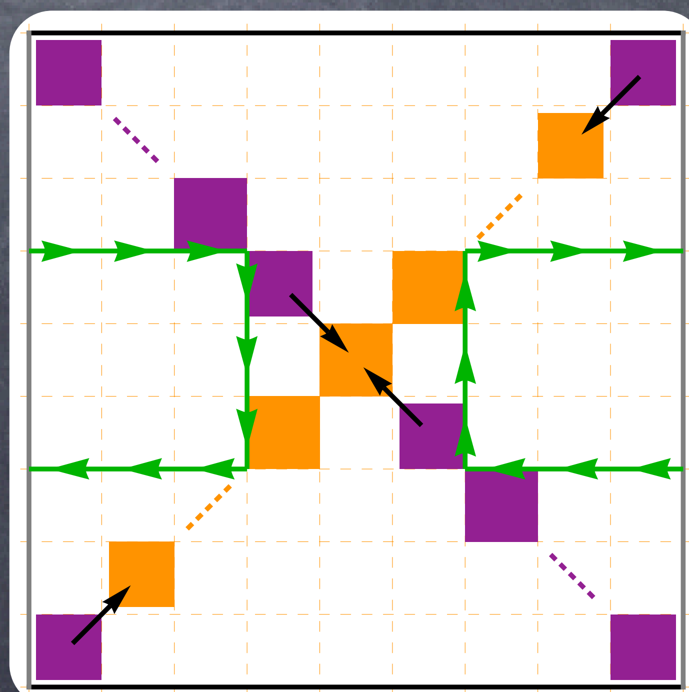

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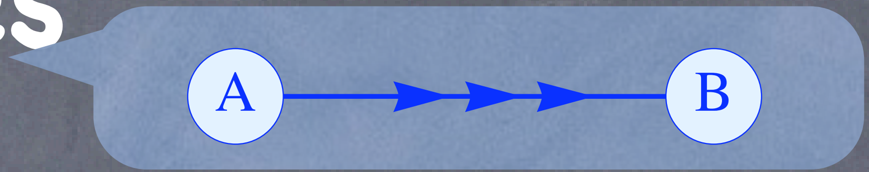

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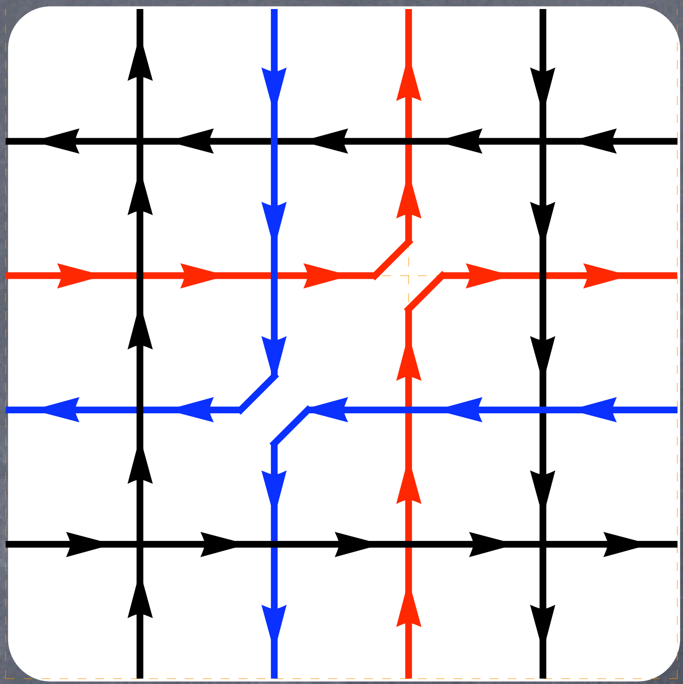

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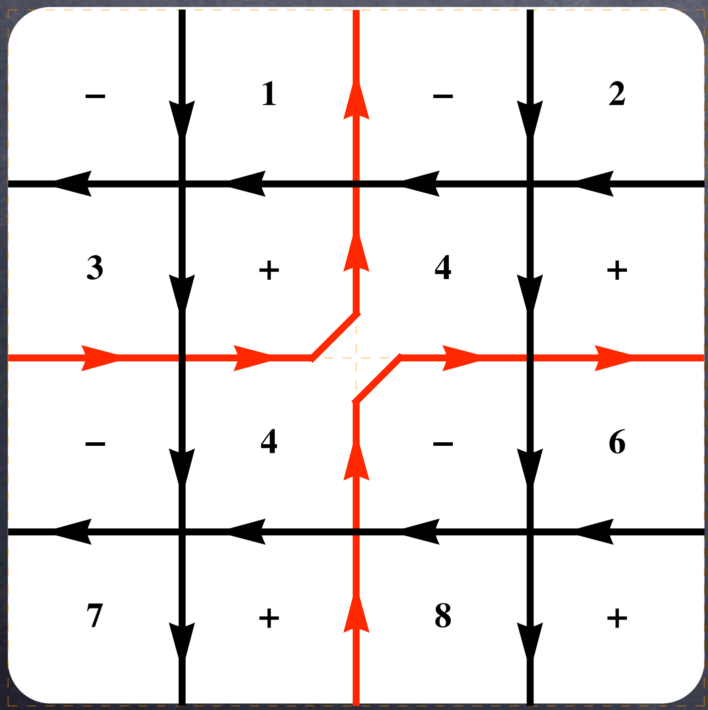
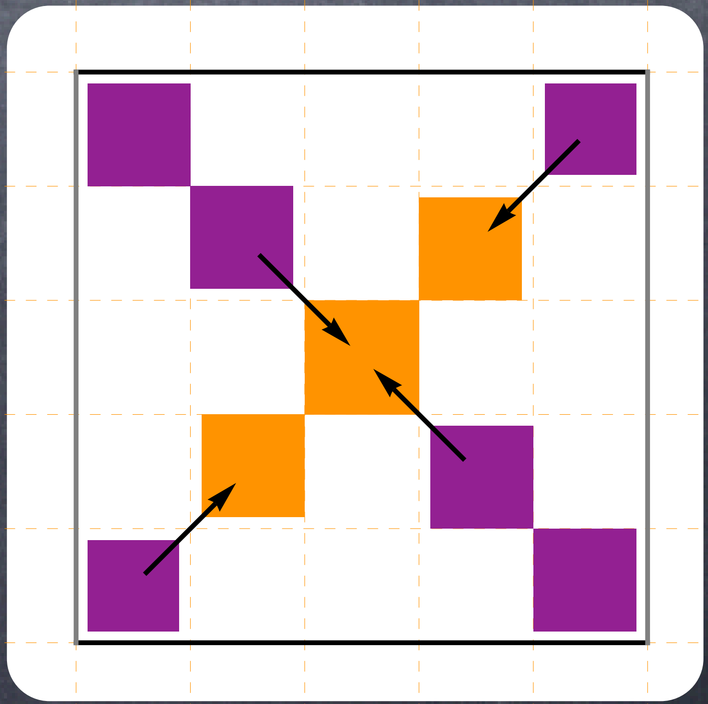


maximum of 4 fields (no add. branches) between 2 gauge groups

-> 4 families (unique F0)



Upper bound on the number of families given by the physically observed number with one exception (4 families in F0).

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Can you break this bound?

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Can you break this bound?

requires to go to non-toric phases/singularities.
there are examples with more families.

Mass Hierarchies

CMQ: 0810.5660

- Known result: dP0 (0, M, M); dP1 (0, m, M)
- the zero eigenvalue is present in all toric dPs
- appearing due to vanishing determinant of Yukawa matrix
- Is the structure (0, m, M) generic in models at toric singularities?

$$M = |Y_{12}|^2 + |X_{62}|^2 + |Z_{62}|^2$$

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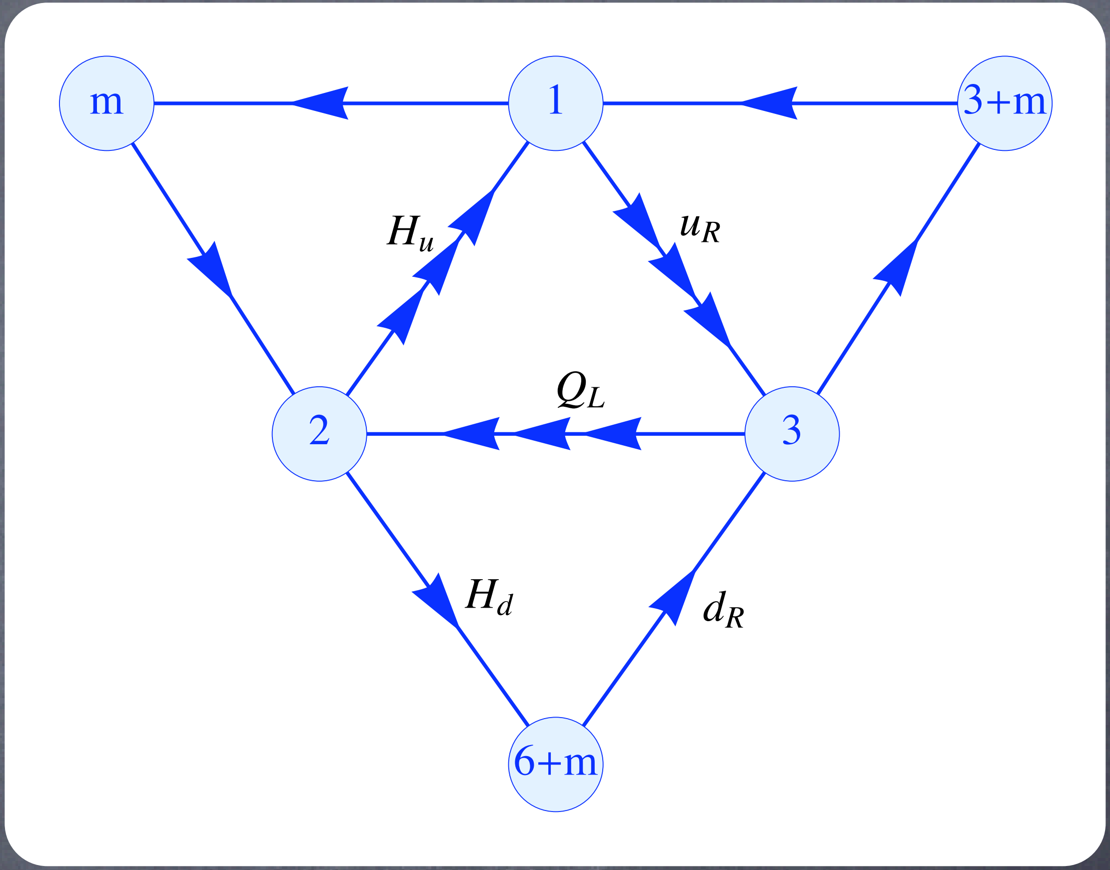

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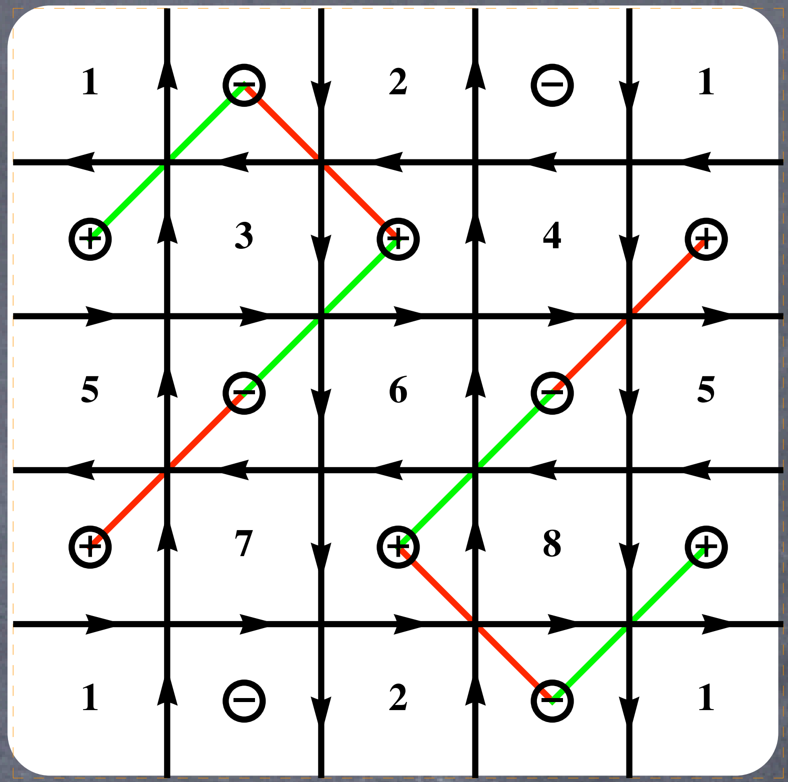
Mass hierarchies II



- How do we choose "quarks"?
  one left & right handed quark in every coupling with quarks.
  -> every superpotential term has two quarks
  -> quarks aligned in closed lines

- Connected or disconnected lines?
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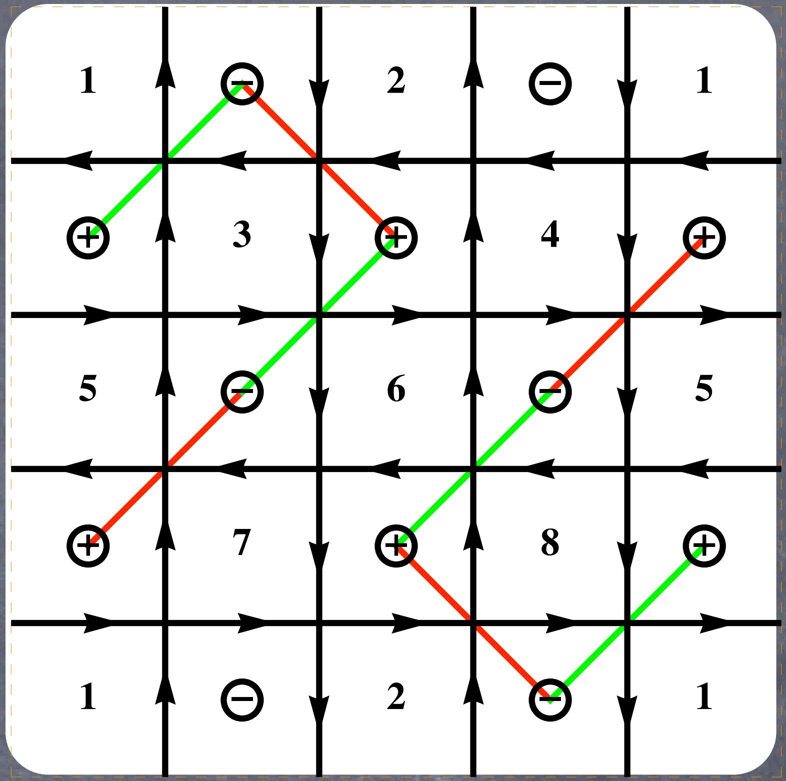

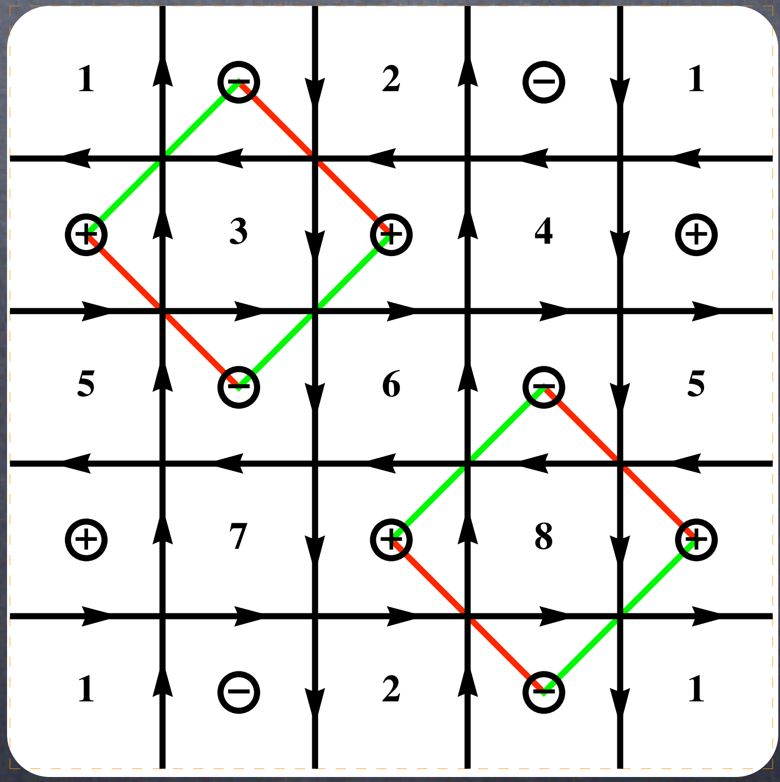

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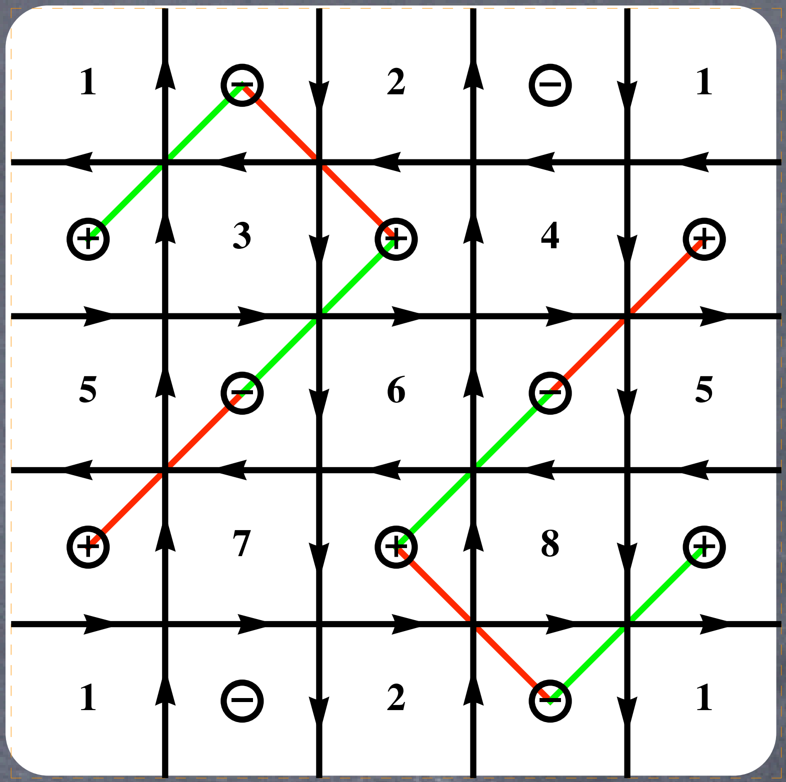

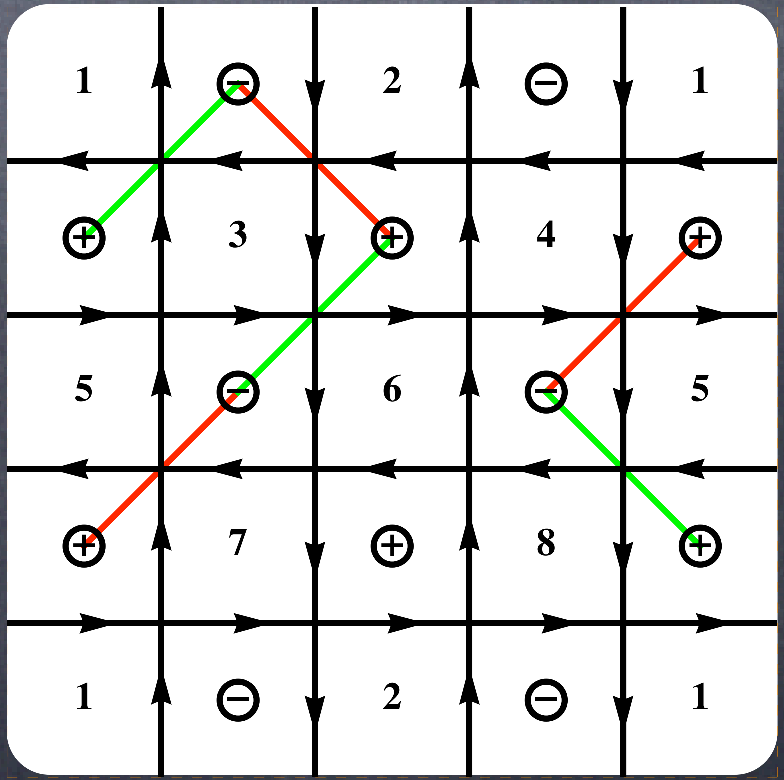

general structure (0, m, M)

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non-vanishing mass?

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non-vanishing mass?

corrections to Kähler potential, deformation of singularity (non-toric)

... back to model building

Flavour mixing: CKM

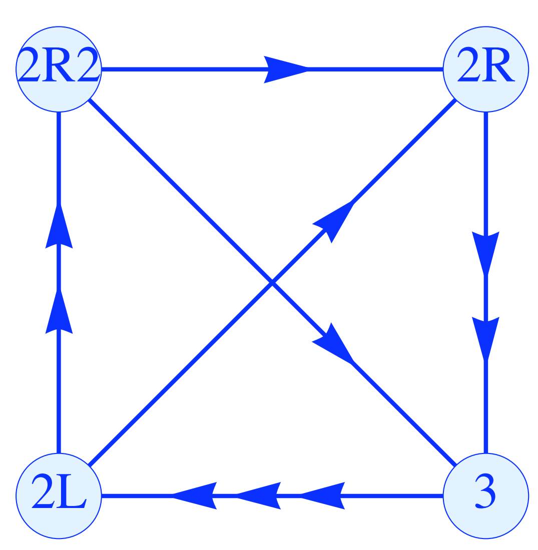
... we have a non-trivial superpotential (Yukawa structure) in these singularity models. What does this imply for model building?

Aim: construct models with the correct flavour mixing among quarks

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}.$$

2 types of models:

a) up & down from D3D3 states



b) up from D3D3 states
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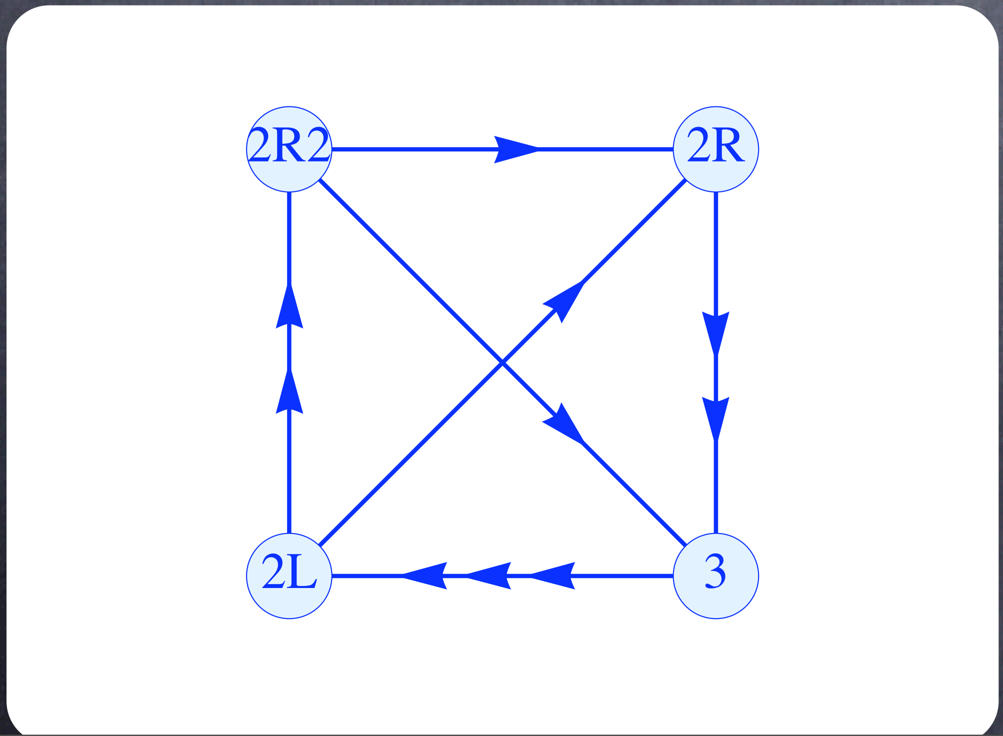

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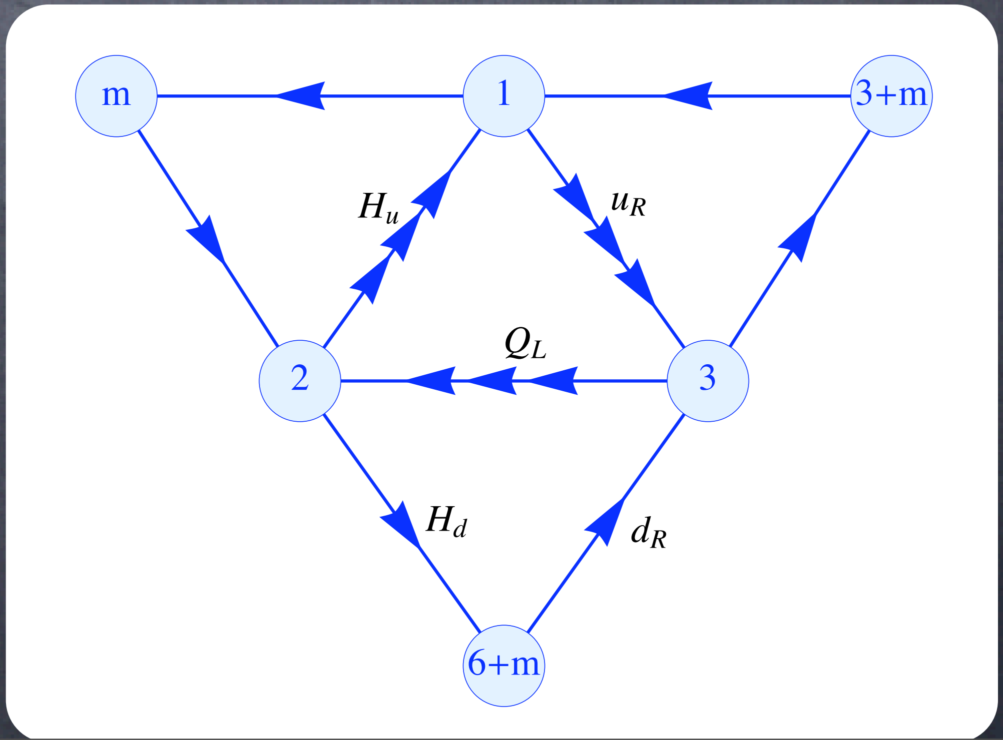
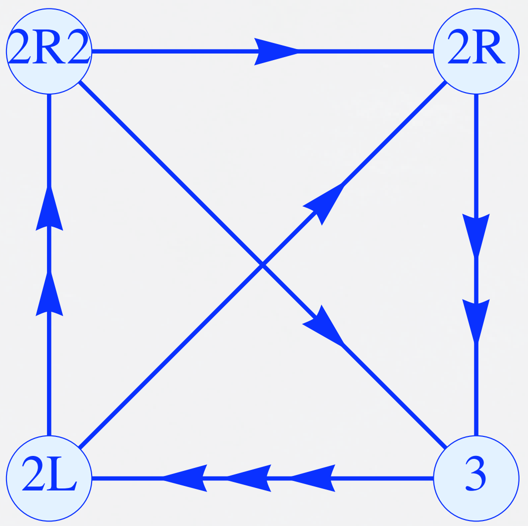


A left-right model with the right CKM

$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix} .$$



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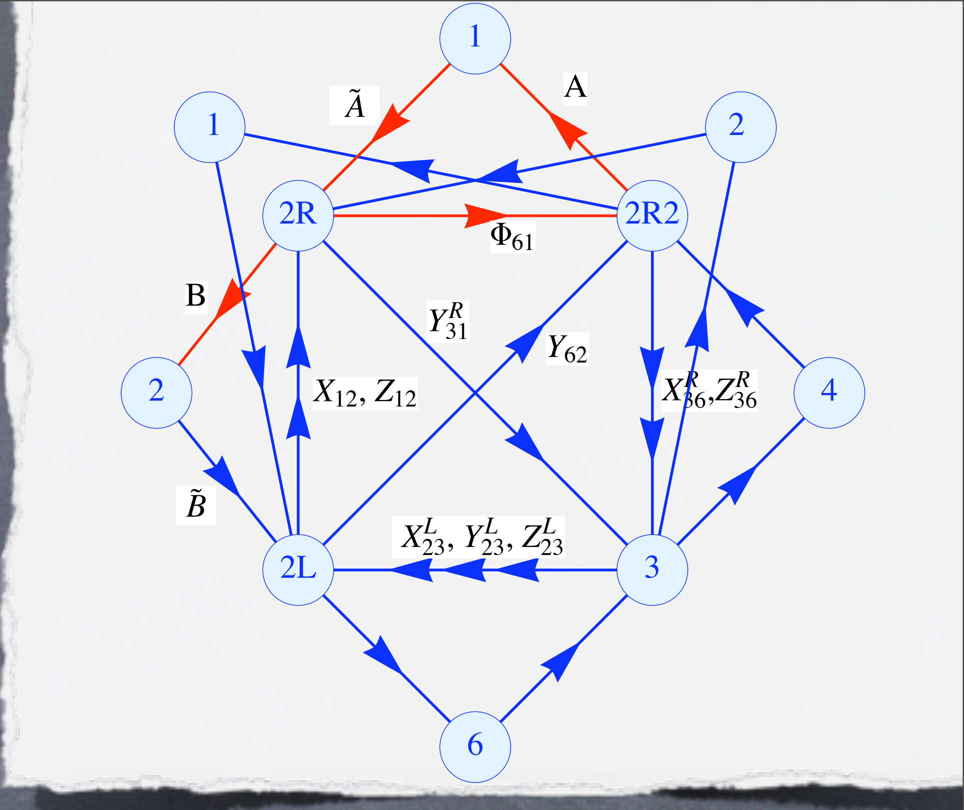

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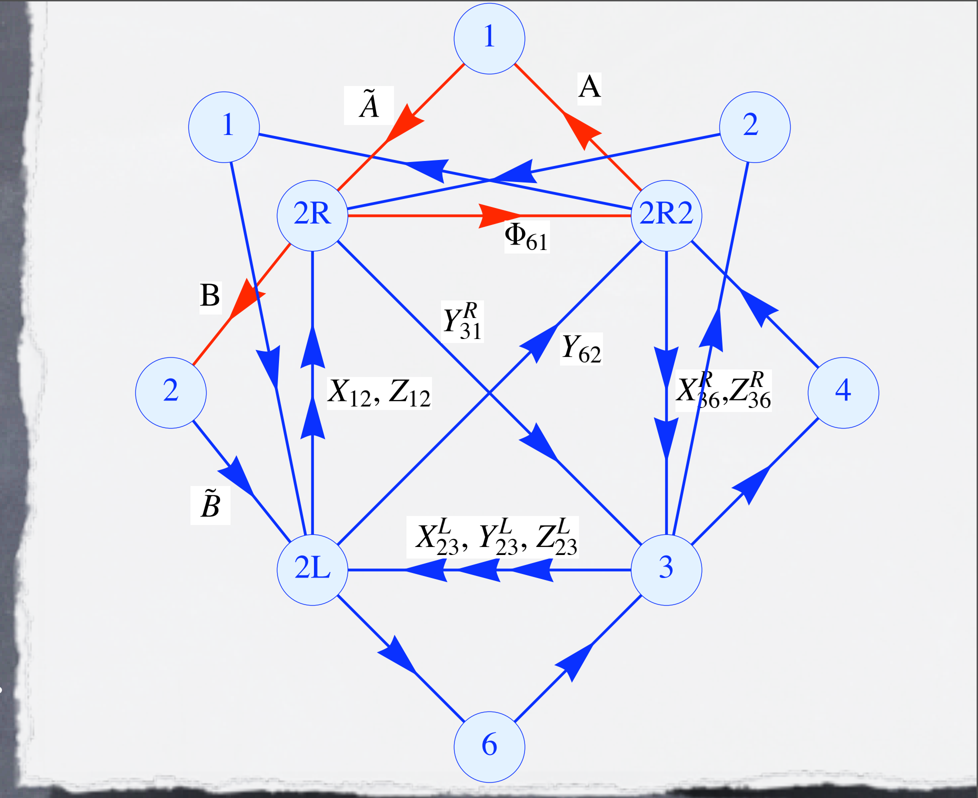

After breaking of $U(2)_R$

$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12}^u & -Y_{62}^u \\ -Z_{12}^u\frac{\varphi}{\Lambda} & 0 & X_{12}^u\frac{\varphi}{\Lambda} \\ Y_{62}^u & -X_{12}^u & 0 \end{pmatrix} \begin{pmatrix} X_{36}^u \\ Y_{31}^u \\ Z_{36}^u \end{pmatrix} + \begin{pmatrix} X_{23}^d \\ Y_{23}^d \\ Z_{23}^d \end{pmatrix} \begin{pmatrix} 0 & Z_{12}^d & -Y_{62}^d \\ -Z_{12}^d\frac{v_d}{\Lambda} & 0 & 0 \\ Y_{62}^d & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{36}^d \\ Y_{31}^d \\ Z_{36}^d \end{pmatrix}$$

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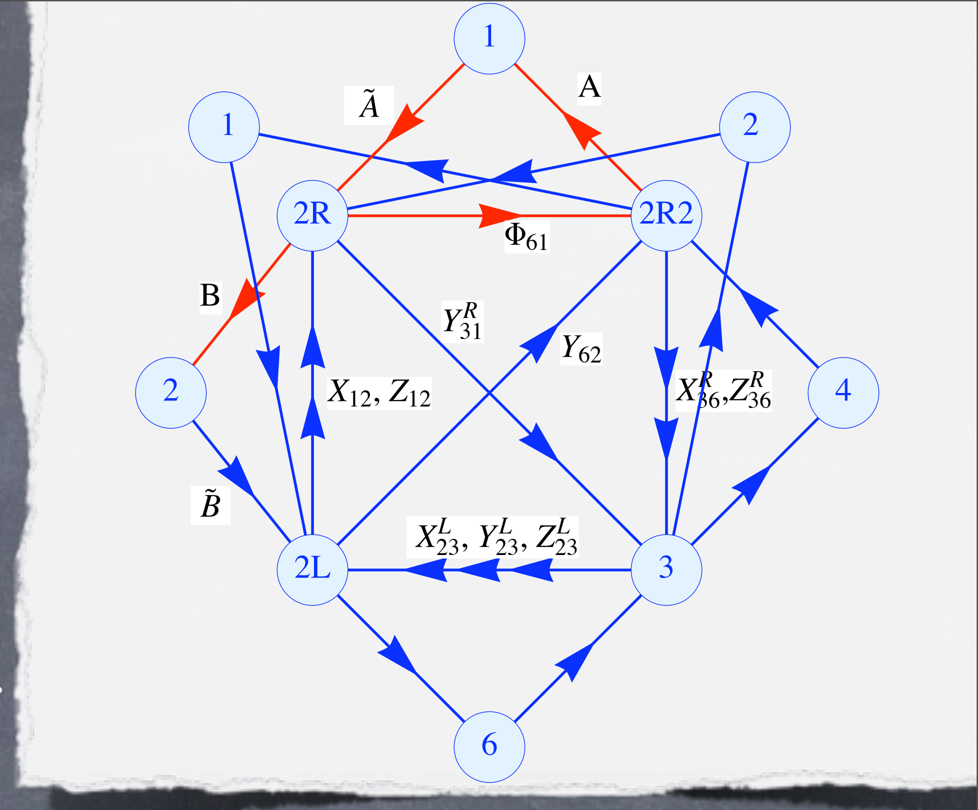

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The CKM is given in terms of ratios of Higgs vevs. dP1:

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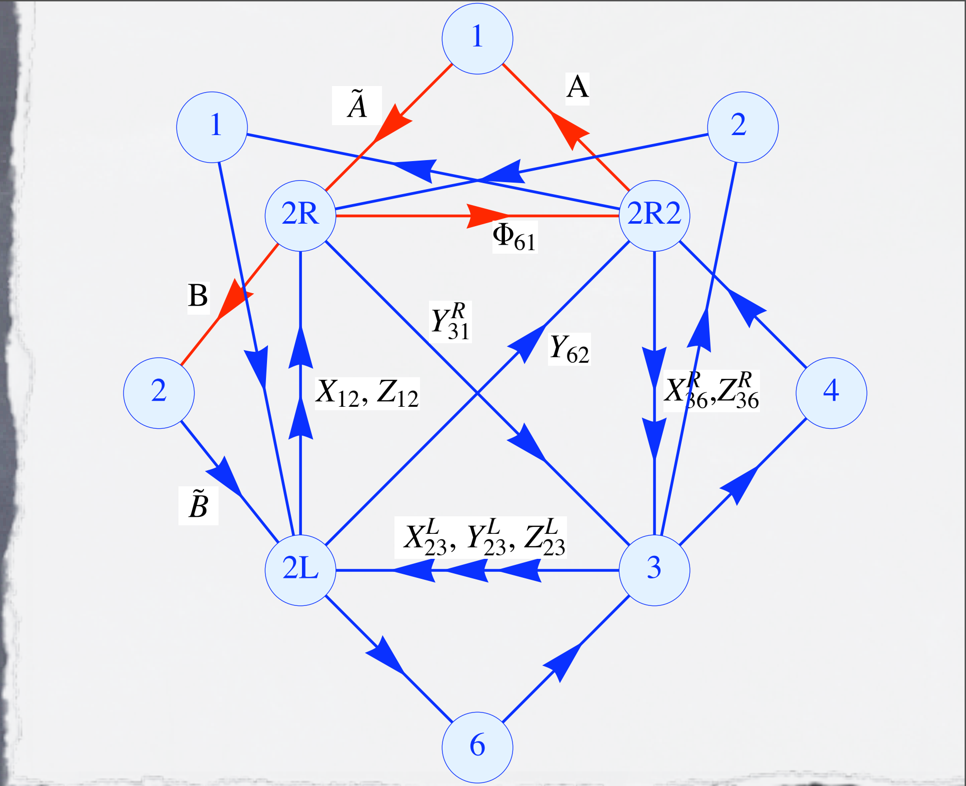

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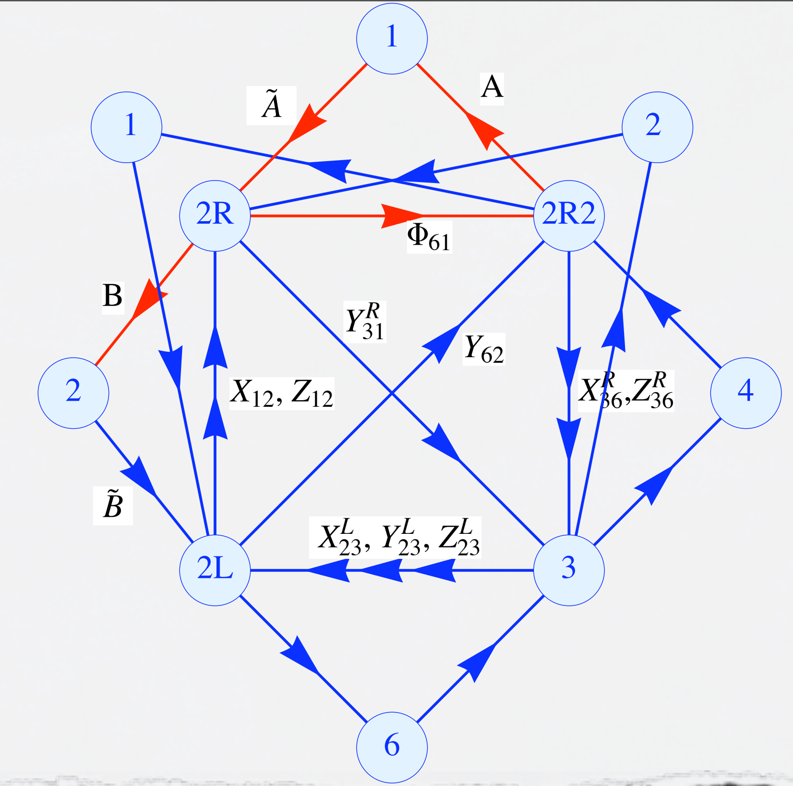
normalisation

A left-right model with the right CKM



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The CKM is given in terms of ratios of Higgs vevs. dP1:

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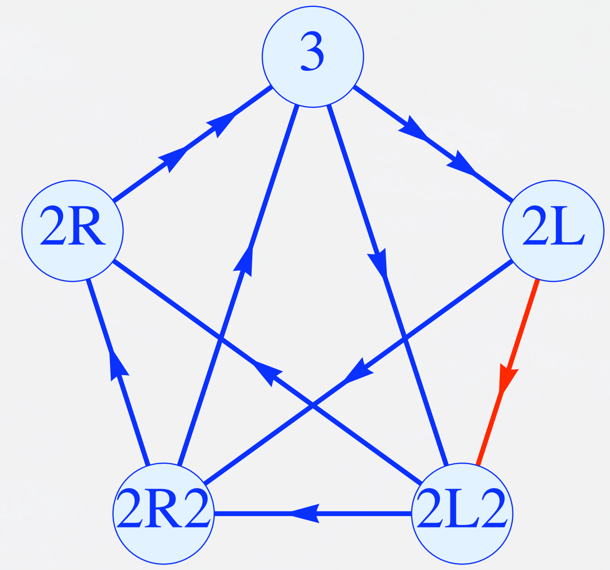
In dP1 we get almost the right CKM.

A left-right model with the right CKM



$$W = \begin{pmatrix} X^L_{43} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{14} & -Y_{64} \\ -Z_{14}\frac{\Phi_{61}\Psi_{42}}{\Lambda^2} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{64}\frac{\Psi_{42}}{\Lambda} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

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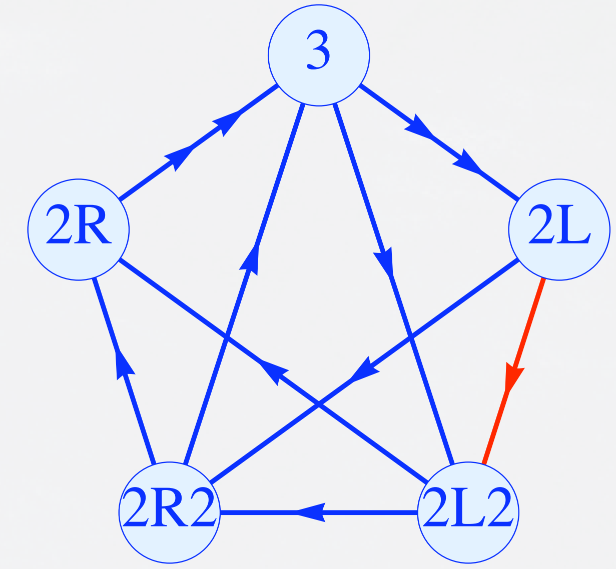

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After breaking of $U(2)_R$

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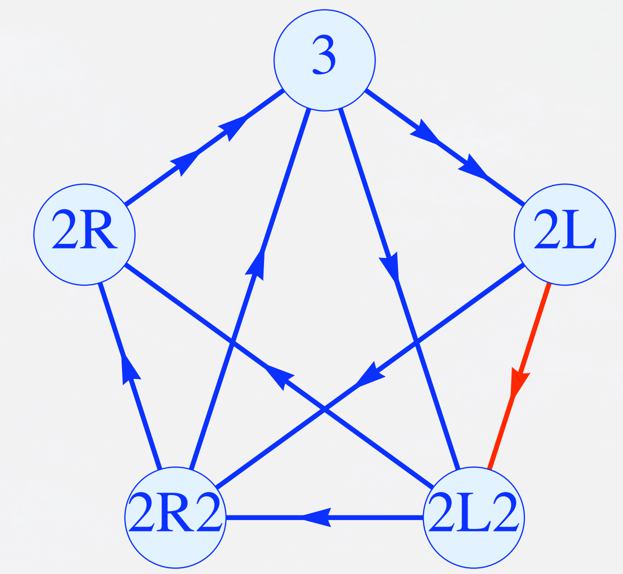
The CKM is given in terms of ratios of Higgs vevs. dP2:

$$V_{\rm CKM} = V_u^\dagger V_d = \begin{pmatrix} \frac{X^u_{12}\Phi_{61}}{Y^u_{64}\Lambda} & \frac{X^u_{12}Z^u_{14}}{(Y^u_{64})^2} & 1 \\ 1 & -\frac{Z^u_{14}\Phi_{61}}{\Lambda Y^u_{64}} & -\frac{X^u_{12}\Phi_{61}}{\Lambda Y^u_{64}} \\ \frac{Z^u_{14}\Phi_{61}}{Y^u_{64}\Lambda} & 1 & -\frac{X^u_{12}Z^u_{14}}{(Y^u_{64})^2} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ a\frac{\Lambda Y^d_{64}}{Z^d_{14}\Phi_{61}} & -b\frac{Z^d_{14}\Phi_{61}}{\Lambda Y^d_{64}} & 0 \\ a & b & 0 \end{pmatrix} = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}.$$

In dP1 we get almost the right CKM.

A left-right model with the right CKM



$$W = \begin{pmatrix} X^L_{43} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{14} & -Y_{64} \\ -Z_{14}\frac{\Phi_{61}\Psi_{42}}{\Lambda^2} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{64}\frac{\Psi_{42}}{\Lambda} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^u_{12} & -Y^u_{62} \\ -Z^u_{12}\frac{\varphi}{\Lambda} & 0 & X^u_{12}\frac{\varphi}{\Lambda} \\ Y^u_{62} & -X^u_{12} & 0 \end{pmatrix} \begin{pmatrix} X^u_{36} \\ Y^u_{31} \\ Z^u_{36} \end{pmatrix} + \begin{pmatrix} X^d_{23} \\ Y^d_{23} \\ Z^d_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^d_{12} & -Y^d_{62} \\ -Z^d_{12}\frac{v_d}{\Lambda} & 0 & 0 \\ Y^d_{62} & 0 & 0 \end{pmatrix} \begin{pmatrix} X^d_{36} \\ Y^d_{31} \\ Z^d_{36} \end{pmatrix}$$

The CKM is given in terms of ratios of Higgs vevs. dP2:

$$V_{\mathrm{CKM}} = V_u^\dagger V_d = \begin{pmatrix} \frac{X^u_{12}\Phi_{61}}{Y^u_{64}\Lambda} & \frac{X^u_{12}Z^u_{14}}{\left(Y^u_{64}\right)^2} & 1 \\ 1 & -\frac{Z^u_{14}\Phi_{61}}{\Lambda Y^u_{64}} & -\frac{X^u_{12}\Phi_{61}}{\Lambda Y^u_{64}} \\ \frac{Z^u_{14}\Phi_{61}}{Y^u_{64}\Lambda} & 1 & -\frac{X^u_{12}Z^u_{14}}{\left(Y^u_{64}\right)^2} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ a\frac{\Lambda Y^d_{64}}{Z^d_{14}\Phi_{61}} & -b\frac{Z^d_{14}\Phi_{61}}{\Lambda Y^d_{64}} & 0 \\ a & b & 0 \end{pmatrix} = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}.$$

In dP1 we get almost the right CKM. **In dP2 we get the right CKM.**

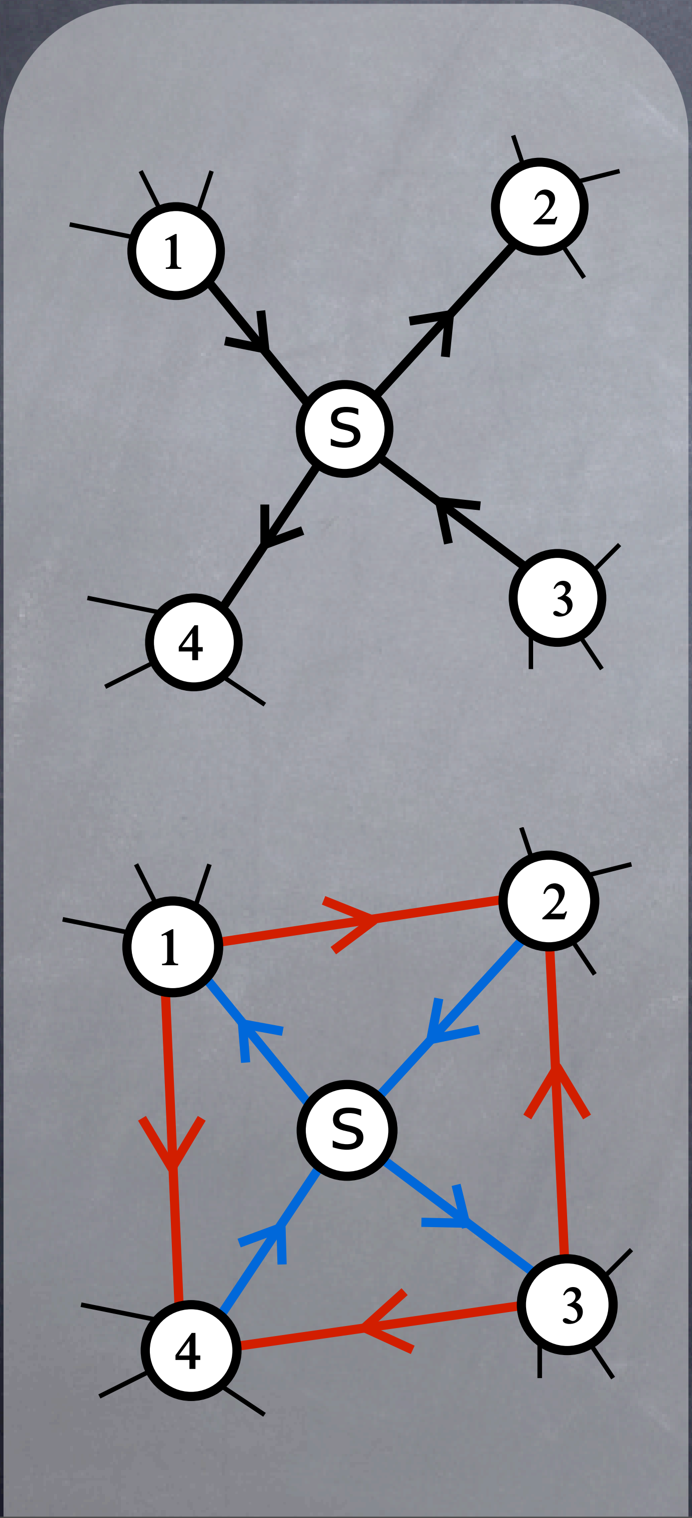
CP violation: with correct CKM, Jarlskog invariant $J \approx \epsilon^6$

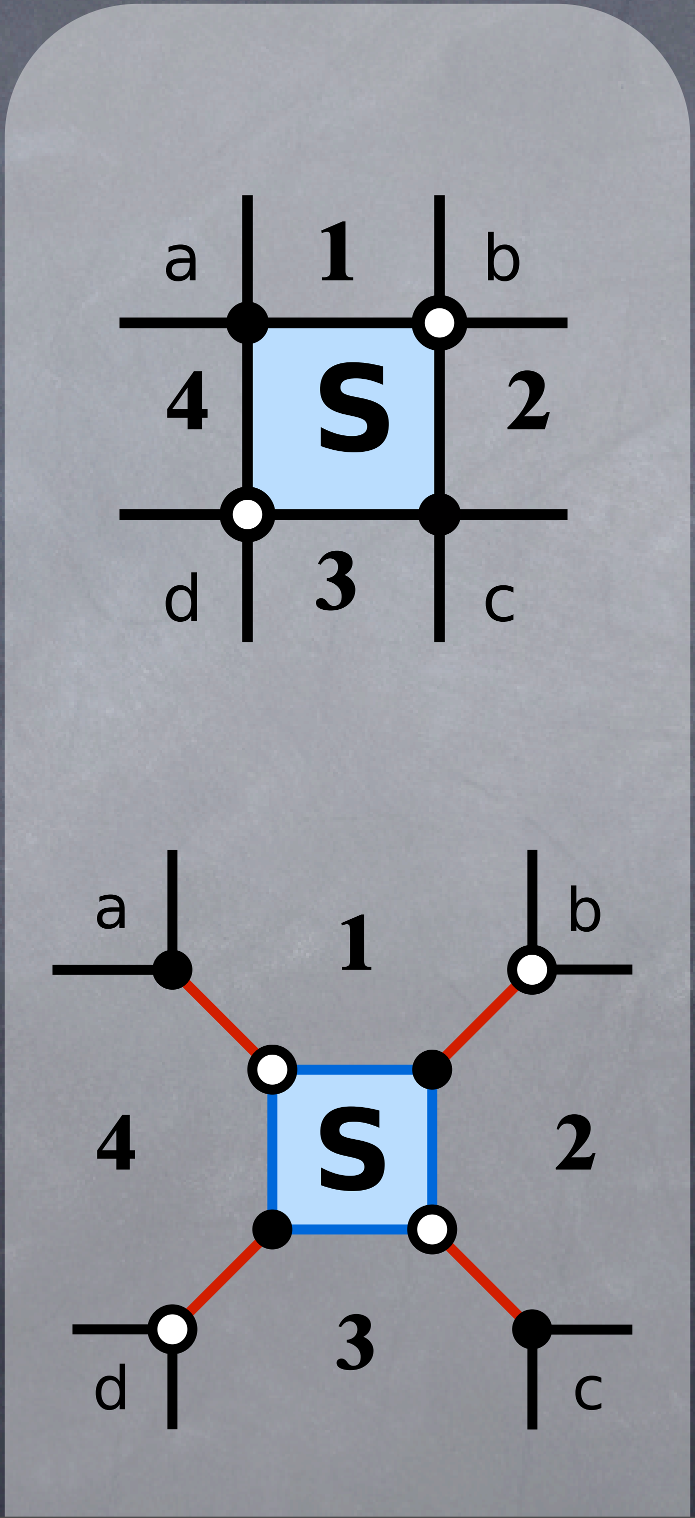
Summary

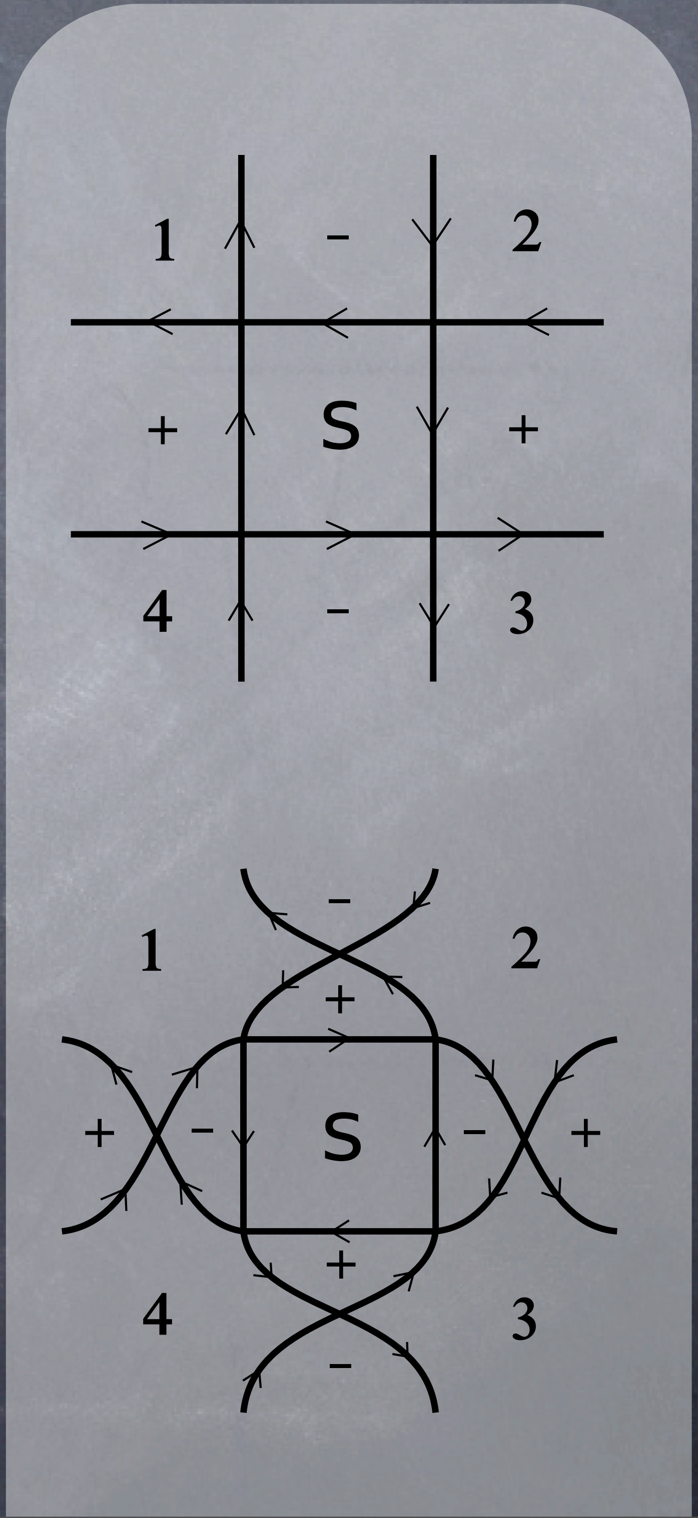
- D-branes at toric singularities interesting class of models:
- Upper bound of 3 families in toric singularities
- Mass Hierarchies are possible, generic structure (0, m, M).
- Sufficient structure for realistic CKM-matrix & CP-violation (concrete models with this structure)
- Open questions: compact models,
  a completely realistic local model...

kdmq: 1002.1790

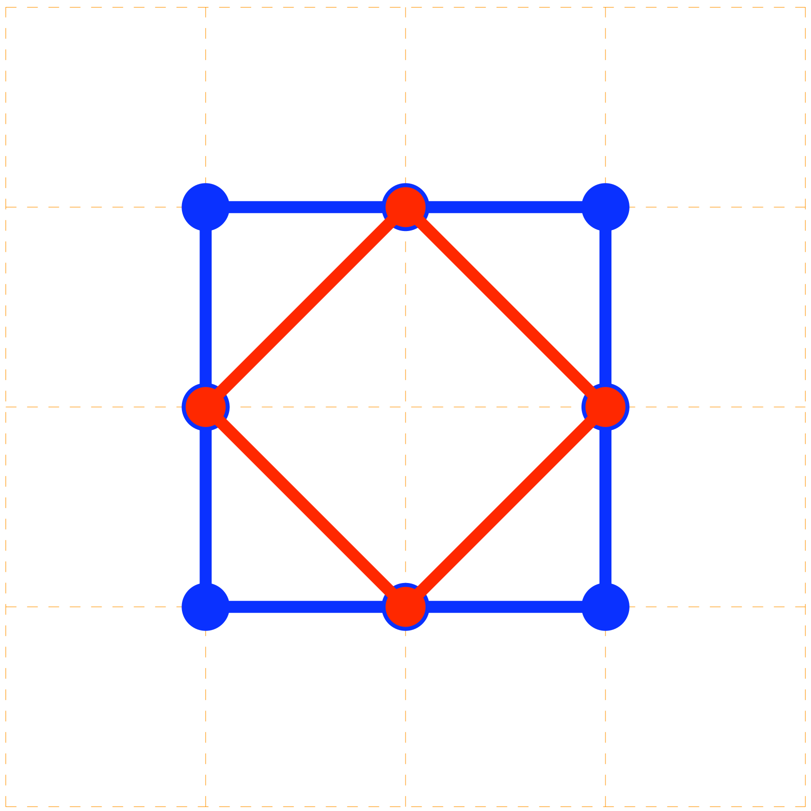
Seiberg duality in quivers and dimers

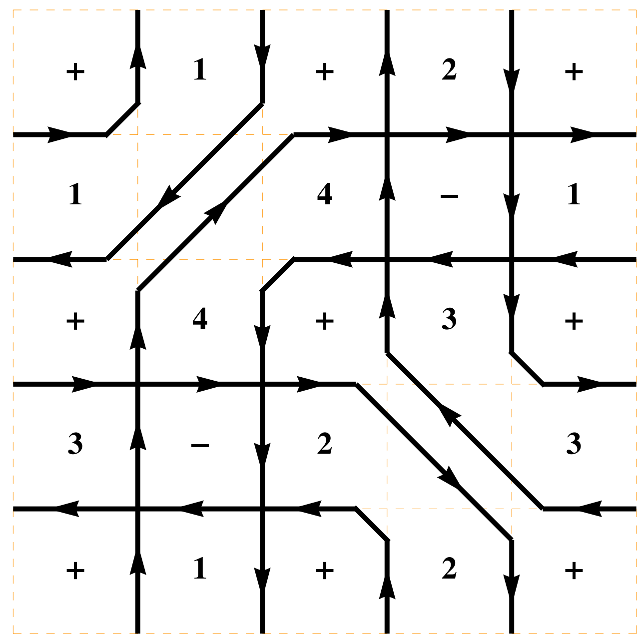


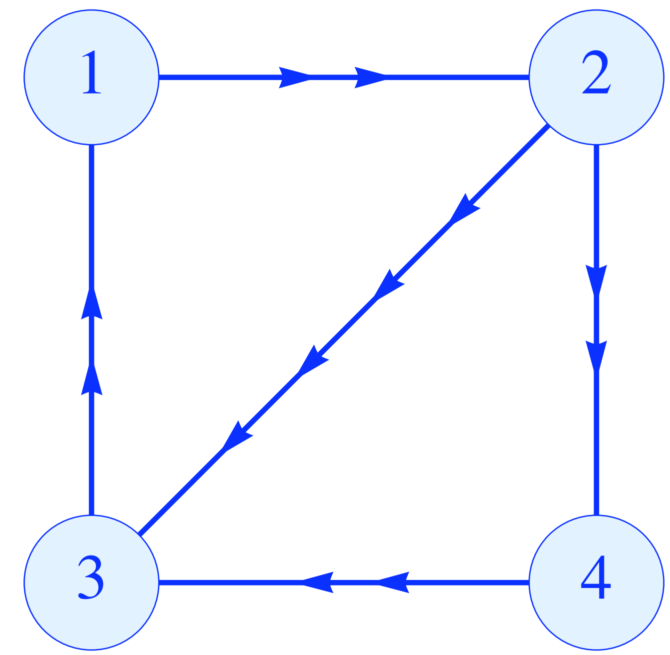


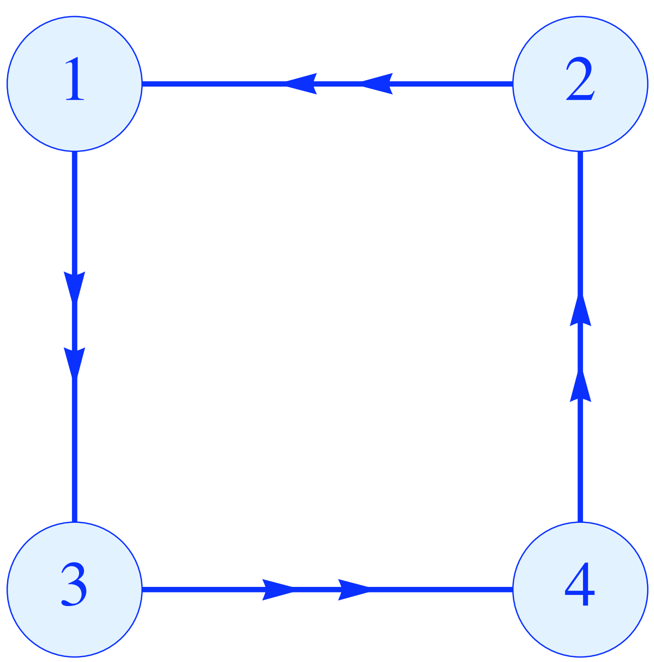


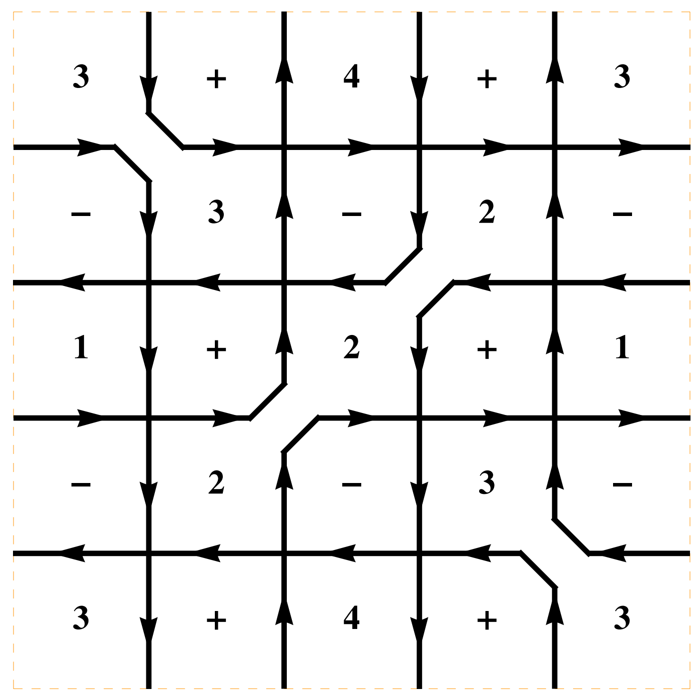
The zeroth Hirzebruch surface











The zeroth Hirzebruch surface