N^3 and Junctions in 6d (2,0) Theories

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M Theory

- strong coupling limit of Type IIA superstring theory
- D0 branes are KK modes => 11-dim
- 11d supergravity is low-energy dynamics
- purely quantum with 11d Planck constant as single parameter
- M2 and M5 branes as electric and magnetic objects of 3-form field: C_{MNP}

D3,M2 Branes

- On N D3 branes, 4d max susy gauge theory of gauge group SU(N):
  - 4d SCFT with dof N^2 =adjoint matter
- On N M2 branes, 3d SCFT of type A_{N-1}
  - dof : $N^{3/2}$ < N^2 (partially bounded..)
  - From D2 branes: 3d gauge theory and strongly interacting quantum theory
  - ABJM theory of $U(N)_{+k}$ x $U(N)_{-k}$

M5 Branes

- Single M5 brane: (2,0), $SO(5)_R$ Symmetry
  - Fields: $B_{\mu\nu}, \phi_I, \psi_A$: self-dual H=dB, *H=H
- (tensionless) self-dual strings

$$\hbar = 1$$

- Multiple N M5 branes: nonabelian, no covariant derivative: A_{N-1} type
- Two issues:
  - How to define the theory ???
  - How to account d.o.f. N^3 ???

ADE (2,0) Theories

- type IIB on R^{1+5} x K_3 near ADE type singularities: simple laced A_N, D_N, E_6, E_7, E_8
- N M5 branes: A_{N-1} type
- N M5+ OM5: D_N type
- SCFT, Nonabelian,

Gravity Dual

- gravitational solutions of N M2 and N M5 branes, respectively: Klebanov and Tseytlin

- Entropy:

$$S_{M_2} \sim N^{3/2} L^2 T^2$$

$$S_{M_5} \sim N^3 L^5 T^5$$

SO(5) Anomaly

- Chiral Fields: $B_{\mu\nu}$, ψ_A
- single M5 with SO(5) F and curvature R

$$I_8(1) = \frac{1}{48}\left[p_2(F) - p_2(R) + \frac{1}{4}(p_1(F) - p_1(R))^2\right],$$

- ADE type $$I_8[G] = r_G I_8(1) + c_G \times \frac{p_2(F)}{24},$$
- rank r_G and dimension d_G
- dual Coxeter number h_G
- anomaly coefficient $c_G = h_G * d_G$

N-Cubic

Group	r_G	d_G	h_G	$c_G/3$
$A_{N-1} = SU(N)$	$N-1$	N^2-1	N	$\frac{1}{3}N(N^2-1)$
$D_N = SO(2N)$	N	$N(2N-1)$	$2(N-1)$	$\frac{2}{3}N(2N-1)(N-1)$
E_6	6	78	12	312
E_7	7	133	18	798
E_8	8	248	30	2480

Table I: r_G, d_G, h_G and $c_G/3$ for simple-laced groups ADE

- For large N, $c_{A_{N-1}} \approx N^3$
- Much bigger than adjoint representation
- **Pants diagram**?

5D Gauge Theory

- (2,0) theory on a circle of radius R_6
- 5-dim N=2 susy gauge theory in 5-dim $\frac{8\pi^2}{g_5^2} = \frac{1}{R_6}$
- Instantons=Kaluza-Klein modes
- Instanton partons in the symmetric phase
- N^3 d.o.f. all hidden in KK modes???
- Strongly coupled above energy $\frac{1}{NR_6}$

Coulomb Phase

- Broken phase
- 1/2 BPS W-bosons= M2 brane wrapping the circle

  M2 branes between M5 branes, both of which wrap the circle

- 1/2 BPS monopole strings = anti-strings

  M2 branes between M5 branes

- 1/4 BPS waves on monopole strings

  waves on M2 branes between M5 branes

- 1/4 BPS dyonic instantons

  waves on M2 branes between M5 branes,

  where both branes wrap the circle

More BPS Objects?

BPS Equations

- SO(4) spatial rotational symmetry
- SO(5) R-symmetry
- Lock SO(4) and SO(4) of SO(5)

$$E_1 = \Gamma_{8127}P_+,\ E_2 = \Gamma_{8163}P_+,\ E_3 = \Gamma_{8246}P_+,\ E_4 = \Gamma_{8347}P_+,$$
$$E_5 = \Gamma_{8567}P_+,\ E_6 = \Gamma_{8253}P_+,\ E_7 = \Gamma_{8154}P_+,$$

$$F_{12} + F_{34} + F_{56} + F_{78} = 0$$
$$F_{13} + F_{42} + F_{57} + F_{86} = 0$$
$$F_{14} + F_{23} + F_{76} + F_{85} = 0$$
$$F_{15} + F_{62} + F_{73} + F_{48} = 0$$
$$F_{16} + F_{25} + F_{47} + F_{38} = 0$$
$$F_{17} + F_{35} + F_{64} + F_{82} = 0$$
$$F_{18} + F_{27} + F_{63} + F_{54} = 0$$

- 1/16 BPS Equation (Ho-Ung Yee, KL)

$$F_{ab} - \epsilon_{abcd}D_c\phi_d + i[\phi_a, \phi_b] = 0\,,\ D_a\phi_a = 0\,,$$
$$D_a^2\phi_5 - [\phi_a, [\phi_a, \phi_5]] = 0\,.$$

$a = 1, 2, 3, 4 \quad F_{a0} = D_a\phi_5$

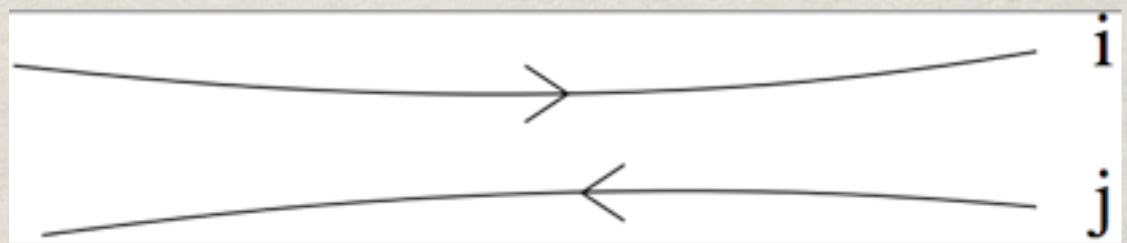
=> 4-dim Webs of Junctions

Monopole Strings

between i and j D4 branes

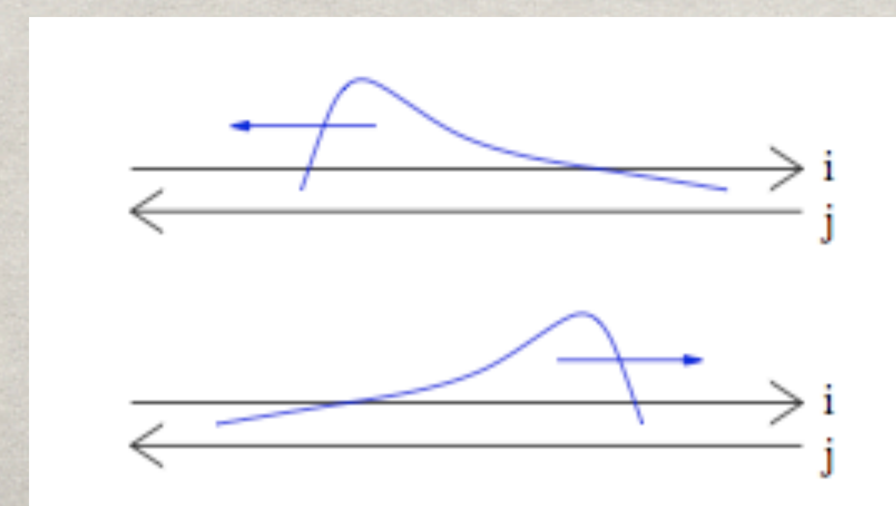
- 1/2 BPS monopole strings=anti-string

$$\Gamma^{1238}\epsilon = \epsilon, \quad F_{12} = D_3\phi_4, F_{23} = D_1\phi_4, F_{31} = D_2\phi_4, D_4\phi_4 = 0$$



- 1/4 BPS waves on monopole string

$$\Gamma^{40}\epsilon = \pm\epsilon, \quad F_{0i} = \pm F_{4i}, \; D_0\phi_4 = \pm D_0\phi_4$$



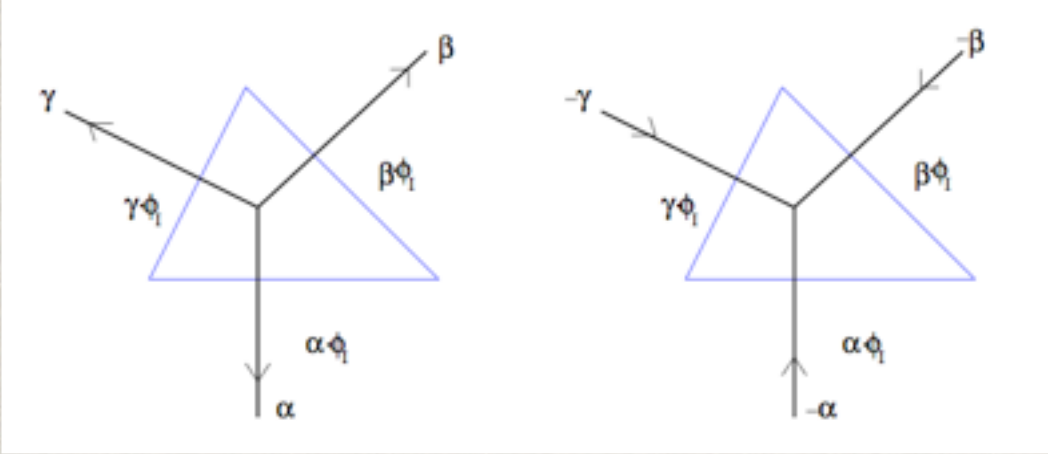
left moving $\neq$ right moving

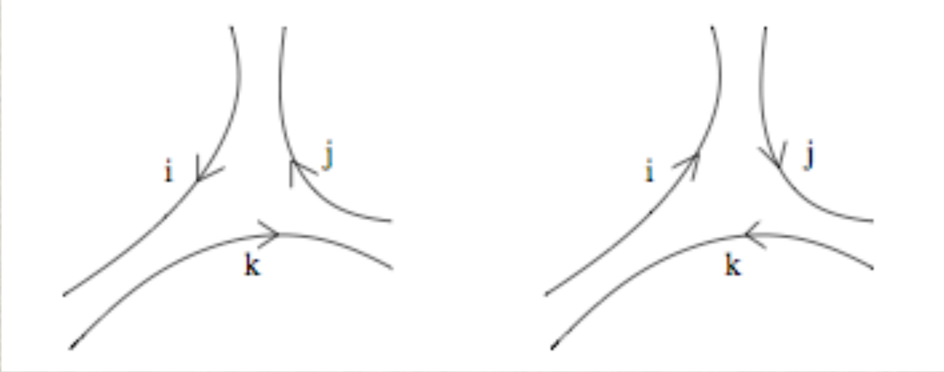
1/4 BPS Junctions

- Lock 34 to 78 direction

$$\Gamma^{1238}\epsilon = \epsilon, \Gamma^{1247}\epsilon = \pm\epsilon,\ \phi_1 = \phi_2 = 0,$$
$$F_{12} = D_3\phi_4 - D_4\phi_3, F_{23} = D_1\phi_4, F_{31} = D_2\phi_4$$
$$F_{41} = D_2\phi_3, F_{24} = D_1\phi_3, F_{43} = -i[\phi_4, \phi_3], D_3\phi_3 + D_4\phi_4 = 0$$

- junctions and anti-junctions between i,j,k D4 branes $\alpha + \beta + \gamma = 0,\ \alpha = e_i - e_j, \beta = e_j - e_k, \gamma = e_k - e_i$





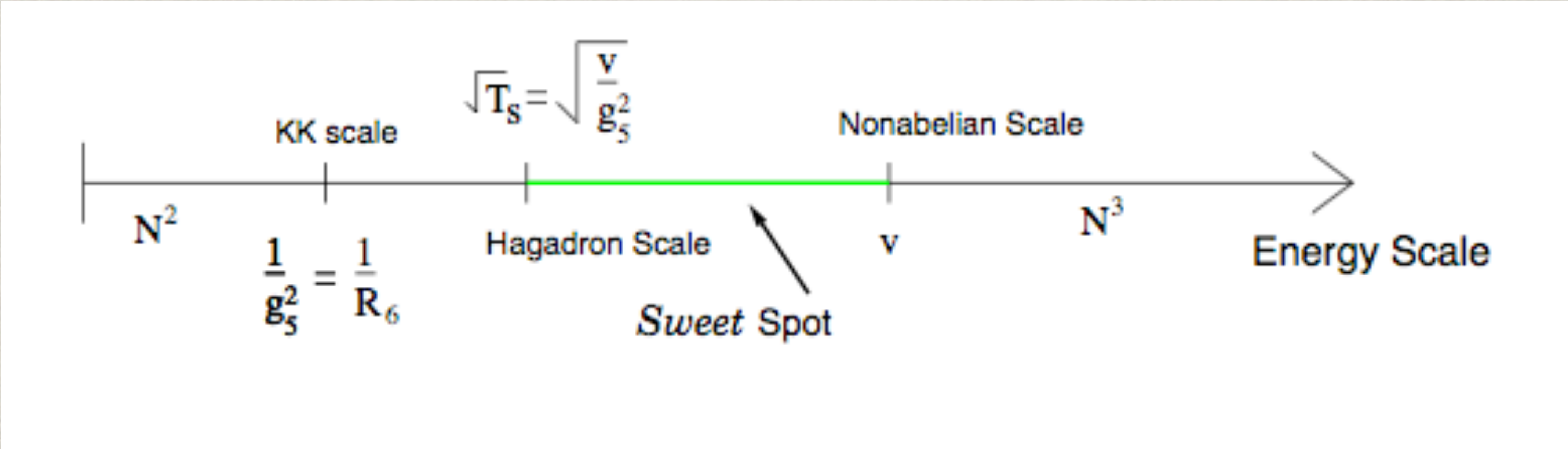
- tension balance: angle on 34 and 78 are locked

Lift to M5 branes
in Coulomb phase

- 1/2 BPS objects:
  - massless waves
  - self-dual strings
- 1/4 BPS objects:
  - left and right moving waves on self-dual strings
  - junctions and anti-junctions

Appearance of New D.O.F.

- Heating Up the 5-dim theory of the Coulomb phase in the limit $v >> 1/R_6$



Counting 1/2 BPS Objects

- massless particles r_G

- infinitely massive self-dual strings

$$(d_G - r_G)/2 = h_G r_G/2$$

- They cannot be in the adjoint representation.

Counting 1/4 BPS Objects in SU(N)

- N M5 branes, roots

$$\text{roots} = \{e_i - e_j\}$$

- l.m. and r.m. waves on strings connecting i and j M5

$$A = 2 * \frac{1}{2} N(N-1) = N(N-1)$$

- (anti)-junctions connecting i,j,k M5 branes

$$B = 2 * \frac{1}{6} N(N-1)(N-2)/6 = \frac{1}{3} N(N-1)(N-2)$$

- Total number

$$A + B = \frac{1}{3} N(N^2 - 1) = \frac{1}{3} c_{A_{N-1}}$$

$D_N = O(2N)$

- roots

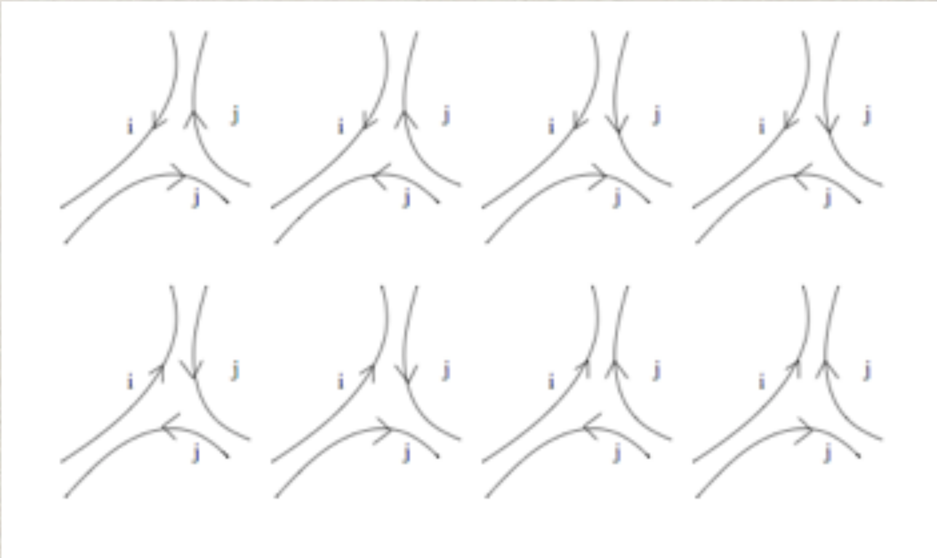
$$\{e_i \pm e_j\},\ i \neq j,\ i,j = 1,2 \cdots N$$

- waves on strings

$$A = 2 * 2 * \frac{1}{2} N(N-1) = 2N(N-1)$$

- junctions

$$B = 8 * \frac{1}{6} N(N-1)(N-2)$$
$$= \frac{4}{3} N(N-1)(N-2)$$



- total number

$$A + B = \frac{2}{3} N(N-1)(2N-1) = \frac{1}{3} c_{D_N}$$

E_6

- roots

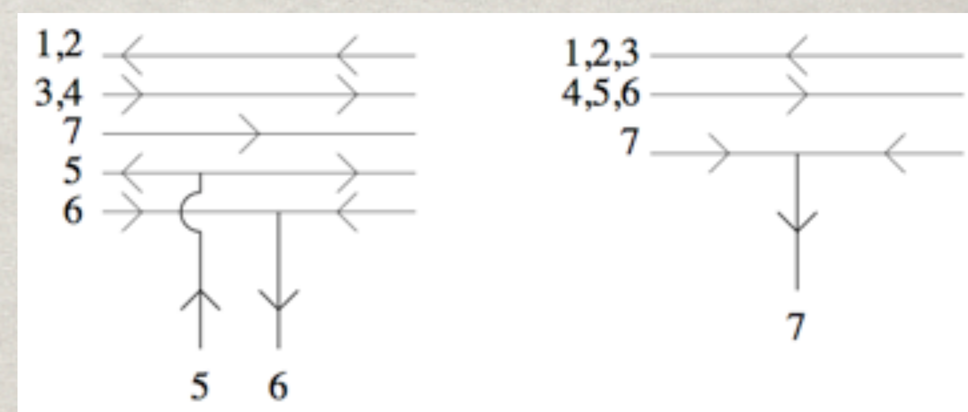
$$e_i - e_j,\ \ (i,j = 1,2,3,4,5,6),\ \ \pm\sqrt{2}e_7,$$
$$\frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4 \pm e_5 \pm e_6) \pm \frac{1}{\sqrt{2}}e_7$$
$$(3\ plus\ and\ 3\ minus\ for\ e_1,..e_6)$$

- waves on strings

$$A = 2*(\frac{1}{2}*6*5) + \left[2 + (\frac{1}{6}*6*5*4)*2\right] = 30 + 42 = 72$$

- junctions: su(6) type + new ones

$$B = 2*(\frac{1}{6}*6*5*4) + \left[2*(\frac{1}{2}*6*5)*(\frac{1}{2}*4*3)\right.$$
$$\left.+2*(\frac{1}{6}*6*5*4)*\frac{1}{2}\right] = 40 + [180+20] = 240$$



- total number

$$A + B = 72 + 240 = 312 = \frac{1}{3}c_{E_6}$$

E_7

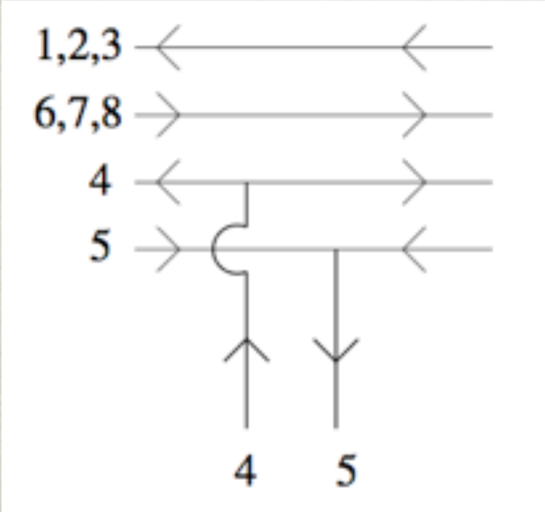
- roots= su(8) roots

$$e_i - e_j, (i, j = 1, 2, \cdots 8)$$
$$\frac{1}{2}(\pm e_1 \pm e_2 \cdots \pm e_8)$$
$$(4 \ plus \ and \ 4 \ minus)$$

- waves on strings

$$A = 2 * (\frac{1}{2} * 8 * 7) + \left[\frac{1}{4!} * 8 * 7 * 6 * 5\right]$$
$$= 56 + 70 = 126$$

- junctions = su(8) type+ new ones



$$B = 2 * (\frac{1}{6} * 8 * 7 * 6) + \Big[2 * (\frac{1}{2} * 8 * 7) * \frac{1}{2} * (\frac{1}{6} * 6 * 5 * 4))\Big] = 112 + 560 = 672$$

- total number

$$A + B = 168 + 630 = 798 = \frac{1}{3} c_{E_7}$$

E_8

- roots: D_8 roots + others

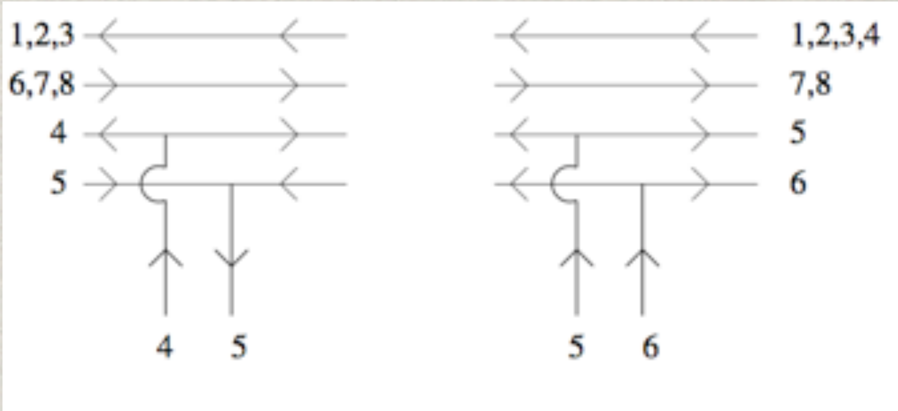
$$e_i \pm e_j, (i \neq j,\ i,j = 1,2,\cdots 8)$$
$$\frac{1}{2}(\pm e_1 \pm e_2 \cdots \pm e_8)$$
$$(even\ \#\ of\ minus)$$

- waves on strings

$$A = 4 * (\frac{1}{2} * 8 * 7) + 2^8/2 = 112 + 128 = 240$$

- junctions=D_8 type+ new

$$B = 8 * (\frac{1}{6} * 8 * 7 * 6) + \Big[(\frac{1}{2} * 8 * 7) * (\frac{1}{2} * 2^8 * \frac{1}{2})\Big] = 448 + 1792 = 2240$$



- total number

$$A + B = 560 + 1920 = 2480 = \frac{1}{3} c_{E_8}$$

Math

- Coxeter number= number of roots/rank

$$h_G = (d_G - r_G)/r_G, \quad d_G = r_G(h_G + 1)$$

- Coxeter=Dual Coxeter for simple laced groups
- Anomaly Coefficient $c_G = h_G d_G$
- Relation $$\frac{1}{3}c_G = \frac{1}{3}h_G(h_G+1)r_G = d_G - r_G + \frac{1}{3}h_G(h_G-2)r_G$$

- # of roots: wave on strings $A = d_G - r_G = h_G r_G$
- # of SU(3) imbedding= # of junctions

$$B = \frac{1}{3}h_G(h_G-2)r_G$$

Conclusion

- N degrees of freedom are revealed in the Coulomb phase
- Just Numerology?
- Find more evidence for these object is in the related theories (Toda, Sicilian, …)
- Ultimate understanding of (2,0) theories