Strongly coupled dense matter and hedgehog black holes

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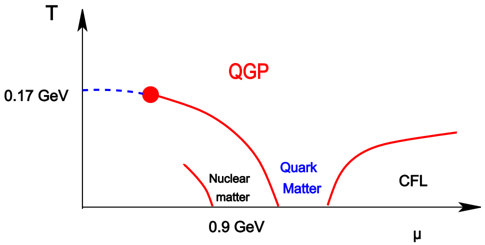
(work in progress with P. Benincasa)

Introduction

- The behaviour of cold dense baryonic matter is one of the outstanding issues in theoretical physics.
  Equation of state for bulk nuclear matter (e.g. Stephanov '07; Schäfer '05)



- Passing from nucleons to quarks requires non-perturbative treatment.
  Standard lattice techniques suffer from infamous sign problem. [Alternate approaches: Imaginary μ; stochastic quantization..]

- Another approach: theories without a sign problem,
  e.g. QCD with isospin chemical potential; theories with real matter respresentations.

- Models within the framework of the AdS/CFT correspondence
  Require large-N (colours) and large-N_f (flavours):

  Veneziano limit: $N_f \to \infty$ and $N \to \infty$ with $\frac{N_f}{N}$ fixed.

+ ....

$$\sim \sum c_{b,g} (N_f / N)^b N^{2-2g}$$

Not obvious if weakly coupled string dual exists when $\frac{N_f}{N} \sim 1$.

Outline

- Review $\mathcal{N} = 4$ theory with $\mathcal{N} = 2$ matter (D3-D7 system).
- Reducing the flavour group by "smearing".
- Phase structure at weak coupling.
- Strong coupling picture at finite baryon density.
- Outlook

Theory with fundamental matter: D3-D7 system

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	×	×	×	×						
D7	×	×	×	×	×	×	×	×		

- D3-D7 open strings: N_f hypermultiplets $i = 1, 2 \ldots N_f$, $(Q^i, \tilde{Q}_i) \rightarrow (N, \bar{N})$ of $SU(N)$. (Karch-Katz '02)
- $\mathcal{N} = 4$ theory coupled to $\mathcal{N} = 2$ matter

$$W = \sum_{i=1}^{N_f} \left(\sqrt{2}\, \tilde{Q}_i\, \Phi_3\, Q^i + m \tilde{Q}_i\, Q^i \right) + \sqrt{2}\, \mathrm{Tr}\, (\Phi_3\, [\Phi_1, \Phi_2]).$$

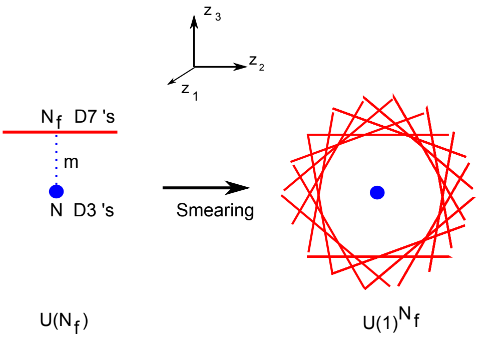
- Flavour symmetry: $U(N_f) \simeq U(1)_B \times SU(N_f)$
- R-symmetry: $SU(2)_R \times U(1)_R \times SU(2)_\Phi$

$\mathcal{N} = 1$ "smeared" theory: D-brane picture

(see Núñez-Paredes-Ramallo 2003 . . . , 2010 review)

- Six directions transverse to D3-brane:

$$z_1 = x_4 + ix_5\,, \qquad z_2 = x_6 + ix_7\,, \qquad z_3 = x_8 + ix_9$$



- Obtained by action of $\frac{SU(3)}{U(1)\times SU(2)}$ on the orientation vector $(0, 0, 1)$.

- As $N_f \to \infty$ the smearing "orbit" $\frac{SU(3)}{U(1)\times SU(2)} \simeq \mathbb{CP}^2$.

- New superpotential Yukawa couplings :

$$\sum_{a=1}^{3}\sum_{i=1}^{N_f} \lambda_i^a\, \tilde{Q}_i \Phi_a Q^i \to \int d\vec{X}\ \tilde{Q}_X\ \vec{X}^\dagger \cdot \vec{\Phi}\ Q^X$$

with $\vec{X} = \Omega\,(1,0,0)^T$ and $\Omega \in SU(3)$.

- Theory has $SU(3) \times U(1)$ global symmetry; and $U(1)^{N_f}$ whose diagonal combination is baryon number

- For $N_f \sim N_c$, perturbative β-function has Landau pole, $\beta_\lambda \sim \frac{N_f}{N}\lambda^2$. We must treat the theory with a UV cut-off.

Some weak-coupling intuition

- Perturbative study of gauge theories on $S^3 \times R$ can provide some intuition for what to expect.
- Studying large-N theories on finite volume is natural from the point-of-view of AdS/CFT correspondence.
- Most famously, Hawking-Page transition in AdS_5 has been connected to Hagedorn/deconfinement transition of free gauge theory on $S^3 \times S^1_\beta$. (Aharony-Marsano-Minwalla-Papadodimas-Van Raamsdonk '03)
- What is the thermodynamics of weakly coupled large-N theory with $N_f \sim N$ flavor fields?

- Lightest field on $S^3 \times S^1_\beta$ is the Wilson/Polyakov loop:

$$U = \exp i \oint_\beta A_0 \equiv (\alpha_1, \alpha_2 \ldots \alpha_N)$$

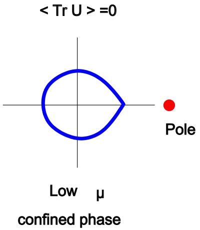
- Integrating out KK harmonics and matter fields results in a unitary matrix model

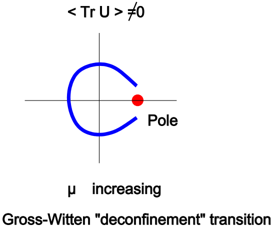
$$Z[U] = \int dU \exp \Big[-N_f \sum_\ell \Big(d_\ell \,\mathrm{Tr}\, \ln(1 - U\, e^{-\beta(\epsilon_\ell - \mu)}) + d_\ell \mathrm{Tr}\, \ln(1 - U^\dagger\, e^{-\beta(\epsilon_\ell + \mu)}) + \ldots \Big) \Big]$$

- When $N_f, N \to \infty$, complex saddle point configuration dominates the integral: $\{\alpha_i\}$ lie on a continuous contour in the complex plane. (Hands-Hollowood-Myers '10)

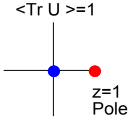
Fixed (low) T and varying μ

- $\{\alpha_i\}$ distributed on a contour $\mathcal{C}$ - density function $\rho(z)$ with pole(s):



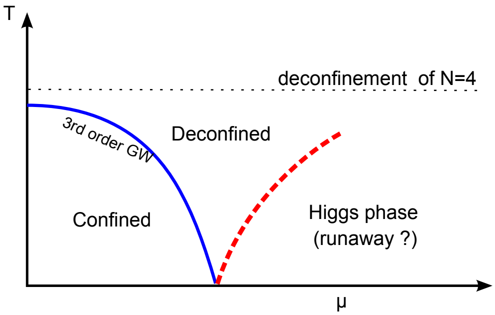


- When $\mu \to$ mass of lightest scalar mode, Bose-Einstein condensation occurs, and occupation number $\to \infty$.



- Theory makes transition to Higgs phase (akin to moving from "Coulomb to Higgs branch")

Phase diagram



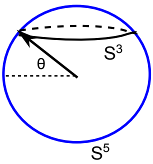
- The deconfined phase goes all the way to $T = 0$.
  Taken seriously, this suggests a dual black hole state at low temperatures.

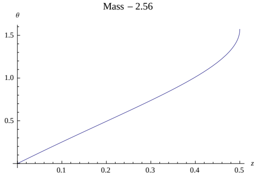
- The Higgs phase potential seems to be runaway at tree level.
  This could be corrected radiatively.

The "smeared D3/D7" system at strong coupling

- Start with the probe picture of D7-branes in $AdS_5 \times S^5$, wrapping $S^3 \subset S^5$.

- For massive flavours, the "slipping angle" $\theta \leftrightarrow$ fermion bilinear $\tilde{\psi}_i \psi^i$, smoothly caps off





- Writing S^5 as a $U(1)$-fibration over $\mathbb{CP}^2$

$$d\Omega_5^2 = ds^2_{\mathbb{CP}^2} + (d\psi + A_{\mathbb{C}P^2})^2$$

$SU(3) \times U(1)$ manifest; broken to $SU(2)_\Phi \times SU(2)_R \times U(1)_R$.

- $SU(3) \times U(1)$ is preserved by the smeared, backreacted solutions of N_f D7's obtained from

$$S = S_{IIB} - T_{\mathrm{D7}} \left(\int d^{10}x \sqrt{-g_{10}} |\tilde{\Omega}_2| + \int C_8 \wedge \tilde{\Omega}_2 \right),$$

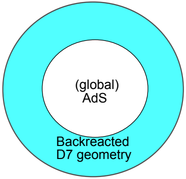
- $\tilde{\Omega}_2$ is the "smearing form" controlling the D7-distribution

$$dF_1 = -g_s \Omega_2 \qquad F_1 = N_f\, p(r)(d\psi + A_{\mathbb{C}P^2})$$

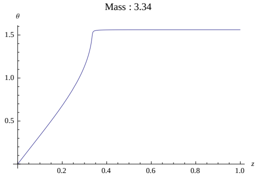
$$ds^2 = c_1 dr^2 + c_2 ds^2_{1,3} + c_3 ds^2_{\mathbb{C}P^2} + c_4 (d\psi + dA_{\mathbb{C}P^2})^2,$$

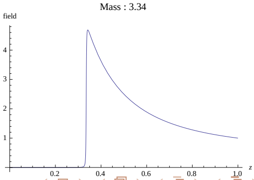
$F_5 = 16\pi N \alpha'^2 (1 + *) \Omega_5$ and dilaton exhibits UV Landau pole.

- With massive flavours for some $r < r_{crit}(m)$, the geometry is $AdS_5 \times S^5$, with constant dilaton and $F_1 = 0$ (no D7-branes).

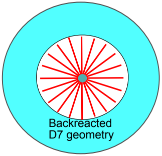


- What happens when chemical potential $\mu \neq 0$?
- For probes, $\mu \neq 0$ corresponds to a radial electric field F_{0r} in the DBI action. (Mateos-Myers-Thomson '06-'07; Karch-O'bannon '06-'07)
- For large enough quark density, the electric field induces F1-spike on the D7-brane $S_{\mathrm{D7}} \rightarrow nS_{\mathrm{F1}}[\mathrm{Nambu-Goto}]$,





• In global AdS this poses a potential problem due to Gauss's law; need baryon vertices to absorb the string flux:



We need to describe the combined D7-F1-D5(baryon vertex), a potentially complicated configuration.

• Two crucial simplifications:

- Smearing of D7-branes $\Longrightarrow$ smearing of strings.
- IIB equations automatically include flux sourced by D5-branes at the origin.

Smeared F1's and IR geometry

• Expect IR geometry to be sourced by a backreaction of strings only (no $D7's$ and $F_1 = 0$)

• First approximation: consistent $SO(6)$-symmetric smearing ansatz,

$$S = S_{IIB} - \frac{nNN_f}{2\pi\alpha'}\left(\int d^{10}x\sqrt{-g}|\tilde{\Omega}_8| + \int B_2 \wedge \tilde{\Omega}_8\right)$$

$$\tilde{\Omega}_8 = \Omega_3 \wedge \Omega_5$$

- The $SO(6)$ will be actually be broken by matching conditions with the UV flavour-brane background.

- The D7-brane physics is frozen/decoupled in this limit. Equivalent to looking for gravity dual of a state with $\mathcal{O}(N^2)$ static quarks in $\mathcal{N} = 4$ SYM.

The consistent IIB background

- With strings uniformly smeared on compact transverse space, how is Gauss's law for B_2 satisfied:
C-S term $\sim \int C_4 \wedge F_3 \wedge H_3$

- Equation of motion for B_2 allows $H_3 = B_2 = 0$, provided

$$\frac{nN_f N}{2\pi\alpha'}\Omega_8 = \frac{1}{32\pi G_{10}} F_5 \wedge F_3,$$

So, $\boxed{F_3 = \# \, n \, N_f \Omega_3} \leftrightarrow \boxed{nN_f \text{ D5-branes/baryons.}}$

- Therefore, we are looking at a high density state, energy density $\sim \mathcal{O}(N^2)$, containing $\mathcal{O}(N)$ baryons.

- $SO(6)$-symmetric ansatz for metric (Einstein frame)

$$ds^2 = -g_{tt}\, dt^2 + g_{rr} dr^2 + e^{2\sigma}\, d\Omega_3^2 + e^{2\eta}\, d\Omega_5^2$$

- Action for metric and dilaton,

$$S = \frac{N^2}{4} \int dr\, dt \sqrt{g_{rr} g_{tt}}\, e^{3\sigma+5\eta} \Big[R_{(2)} + g^{rr} \Big((3\sigma' + 5\eta')^2$$

$$-3\sigma'^2 - 5\eta'^2 - \frac{1}{2}\phi'^2 \Big) + 6e^{-2\sigma} + 20e^{-2\eta} - 8e^{-10\eta}$$

$$-Q^2\, e^{\phi}\, e^{-6\sigma} - 2\, Q\, e^{\phi/2} e^{-3\sigma-5\eta} \Big]$$

- $Q \equiv n \frac{2\sqrt{\lambda}}{\pi} \frac{N_f}{N}$

- Four equations and one constraint and we look for smooth solutions.

Hedgehogs

Similar systems have been studied in different contexts:

- Pure gravity (with negative cosmological constant) with a uniform distribution of strings stretching to the boundary.

  (Guendelman-Rabinowitz '91)

  This yields the so-called hedgehog black holes

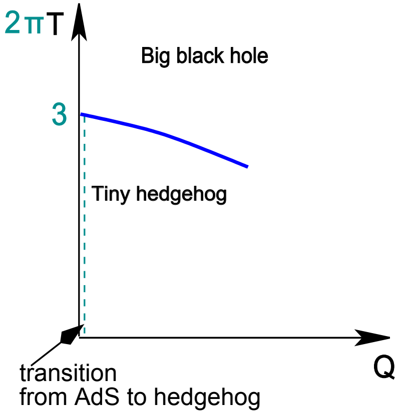
  $$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2.$$
  $$f(r) = (1 + r^2 - \frac{Q}{r} - \frac{c}{r^2}).$$

  The $1/r$ term is the Newtonian potential due to the string in $4+1$ dimensions.

- More recently, Headrick (2007), studied the same system in IIB, but without an F_3 flux.

Basic hedgehog

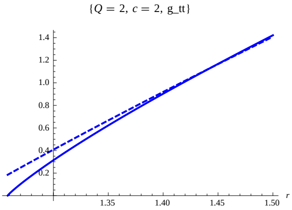
- Any small $Q \neq 0$ opens up a horizon, including at $T = 0$. Thus, there is a phase transition from thermal AdS to "tiny hedgehog black hole". (Headrick '07)

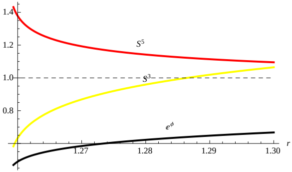


- Our solutions have hedgehog-like asymptotic behaviour, with two free integration constants

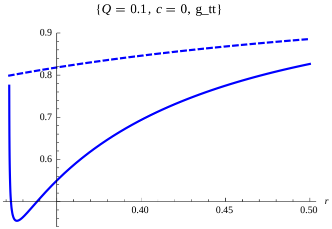
$$g_{tt} \to \left(1 + r^2 - \frac{5}{7}\frac{Q}{r} - \frac{c}{r^2} + \ldots\right)$$
$$\phi \to -\frac{Q}{3r^3} + \frac{a}{r^4} + \ldots$$

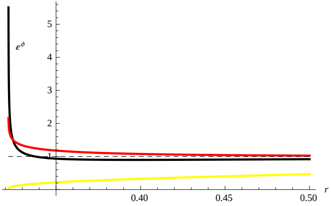
- c varies the temperature for a fixed Q, whilst a corresponds to the VEV of a $\Delta = 4$ boundary operator.





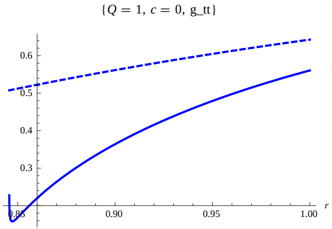
- Black hole solutions generically exist.
- Singular solutions:

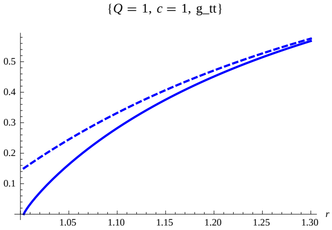




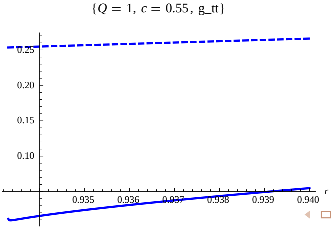
Do regular $T = 0$ solutions exist?

Increasing c cloaks the singular solutions





Extremal solutions?



Summary/outlook

- Determining (numerically) the $T - Q$ (and $T - \mu$) phase plot of the hedgehog configurations.
- Analytic approximations for the solutions, expanding outwards and inwards from the horizon and boundary respectively.
- Do extremal ($T = 0$) solutions exist?
- Obtaining the free energy for hedgehogs vs. $\langle \mathrm{Tr}\, U \rangle$, the Polyakov loop. This is what Headrick attempted in a different set-up, with mixed results.
- Stability, and possible phase transition to Higgs phase, when the horizon size of the hedgehog approaches the D7-brane distribution.
- Does the pure gravity + strings model provide a useful physical description of dense quark matter?