Mesons in the (Dymarsky-) Kuperstein-Sonnenschein holographic models

Matthias Ihl

Departamento de Física Teórica, Instituto de Física,
Universidade Federal do Rio de Janeiro

String Theory seminar, University of Cambridge, 28/10/2010

Introduction to gauge/gravity correspondence

Original AdS/CFT correspondence: $\mathcal{N} = 4$ SU(N_c) SYM in 3+1 dim. **dual to** type IIB superstring theory on $AdS_5 \times S^5$

- Look at D3-branes [Maldacena, hep-th/9711200] Two different points of view:
- Near horizon/brane limit of geometry created by stack of N_c coincident D3-branes:
  $ds^2 = h(r)^{-1/2} dx_{1,3}^2 + h(r)^{1/2}(dr^2 + r^2 d\Omega_5^2)$,
  $h(r) = 1 + \frac{L^4}{r^4}$, $L^4 = 4\pi g_s N_c L_s^4$, (N_c units of self-dual $F_{(5)}$ flux through S^5)
  $\alpha' \to 0$, $u = \frac{r}{\alpha'}$ fixed,
  $ds^2 = \left(\frac{u^2}{L^2} dx_{1,3}^2 + \frac{L^2}{u^2} du^2 + L^2 d\Omega_5^2 \right)$
- world volume theory of stack of N_c D3-branes at low energies: $\mathcal{N} = 4$ SU(N_c) SYM in 3+1 dim.
- Strong version of correspondence: Two theories are completely **equivalent** [Gubser, Klebanov, Polyakov hep-th/9802109; Witten hep-th/9802150]

Introduction to gauge/gravity correspondence

Towards (large N_c) QCD (top-down models)

- Break some (or all) of SUSY and conformal sym.
  - $\mathcal{N} = 1$: Klebanov-Strassler, Maldacena-Nunez
  - Finite temperature: AdS black holes
- Add flavor degrees of freedom
  - Probe limit: $N_f \ll N_c$ flavor branes, no backreaction on geometry, "quenched" approx. [Karch, Katz hep-th/0205236]
  - Beyond probe limit: include backreaction to leading $\mathcal{O}(N_f/N_c)$, e.g., [Burrington, Kaplunovsky, Sonnenschein 0708.1234], $N_f \sim N_c$, smearing technique, e.g., [Casero, Nuñez, Paredes hep-th/0602027; CFIW 0711.4878]
- Semi-realistic models of (flavor) chiral symmetry breaking, (deconfinement transition): Sakai-Sugimoto, [hep-th/0412141] , Backgrounds with constant dilaton (Klebanov-Witten, Klebanov-Strassler),

  [Kuperstein, Sonnenschein 0807.2897;Dymarsky, Kuperstein, Sonnenschein 0904.0988]

Sakai-Sugimoto model of holographic QCD

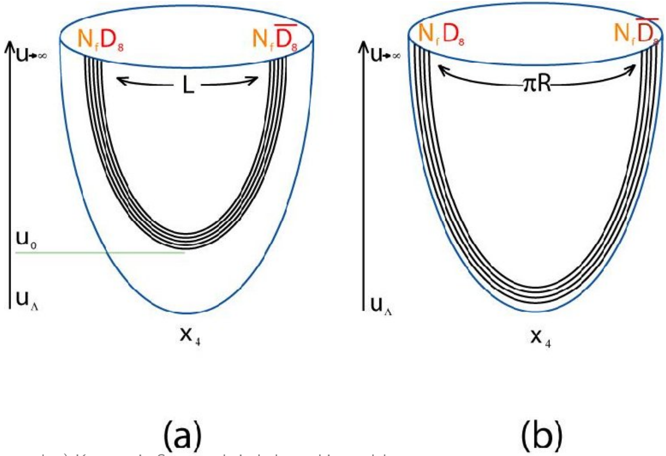
Introduction [Aharony, Sonnenschein, Yankielowicz hep-th/0604161]

- String theory dual to a field theory with chiral symmetry breaking and deconfinement.
- IIA string theory on circle x^4, N_c D4-branes filling 01234-directions, N_f D8-branes at $x_4 = 0$, N_f $\overline{D8}$-branes at $x_4 = L$.
- Stable configuration in decoupling (near-horizon) limit, (classical) massless spectrum of 3+1 $U(N_c)$ gauge theory coupled to N_f massless Dirac fermions.
- Global chiral symmetry visible as gauge theory on $D8 - \overline{D8}$-branes.

Sakai-Sugimoto model of holographic QCD

Zero temperature: Geometric picture of chiral symmetry breaking

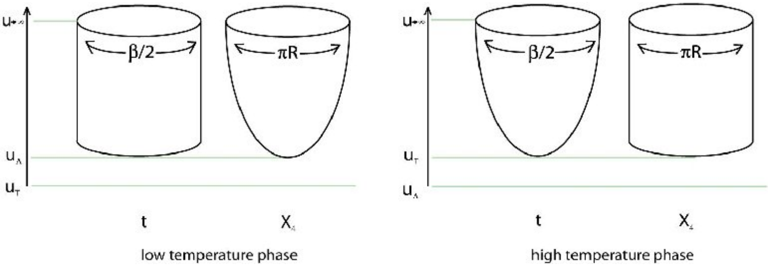
- Embedding $u(x_4)$; $u(0) = \infty$, $u(L) = \infty$, $\frac{du}{dx_4} = 0$ at $u = u_0$, χSB.



Sakai-Sugimoto model of holographic QCD

Finite temperature

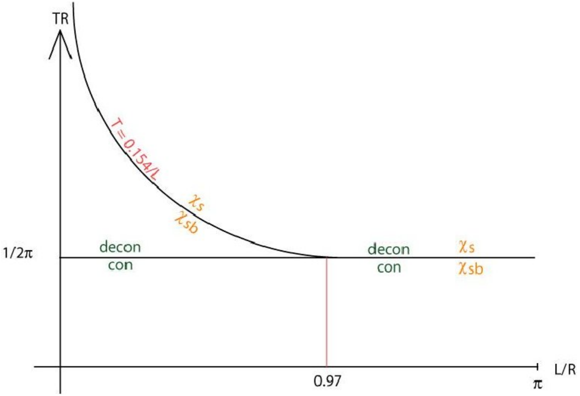
- Two smooth solutions: low temperature, confined phase $\leftrightarrow$ high temperature, deconfined phase (study $V_{q\bar{q}}$).



Sakai-Sugimoto model of holographic QCD

Phase diagram at finite temperature

- For small $\frac{L}{R} < 0.97$, deconfinement and χSB transition separate, for $\frac{L}{R} > 0.97$, single transition.



Motivation

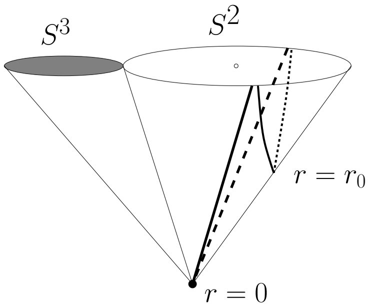
- Study holographic models with geometric realization of flavor chiral symmetry breaking other than Sakai-Sugimoto.
- Overcome some of the shortcomings of the D4/D8 model:
  - Compactification on S^1: $M_{gb} \sim M_{KK}$.
  - Dilaton blows up, UV completion is (2,0) superconformal theory in six dimensions.
- (D)KS: Field theory dual well-understood for arbitrary energies, no compactification!
- Compare phenomenology of hadronic physics to Sakai-Sugimoto and experiments.

Kuperstein-Sonnenschein model

[Kuperstein, Sonnenschein 0807.2897]

Klebanov-Witten background with non-SUSY $D7$-$\overline{D7}$ branes

- Near brane limit of $D3$-branes placed at tip of conifold Y^6 with base $T^{1,1}$.
- $D7$-$\overline{D7}$ brane embedding breaks SUSY of the background, yet is stable.
- Profile:



Kuperstein-Sonnenschein model

[Ballon Bayona, Boschi-Filho, Ihl, Torres 1006.2363]

Vector mesons in the KS model.

- Vector mesons (and pions) arise as fluctuations of the $U(N_F)$ gauge fields on the (probe) $D7$-branes.
  $$A_\mu(x,\tilde{z}) = \hat{\mathcal{V}}_\mu(x) + \hat{\mathcal{A}}_\mu(x)\psi_0(\tilde{z}) + \sum_{n=1}^{\infty} v_\mu^{(n)}(x)\psi_{2n-1}(\tilde{z}) + \sum_{n=1}^{\infty} a_\mu^{(n)}(x)\psi_{2n}(\tilde{z}),$$
- $A_{\tilde{z}} = 0$ gauge, gauge fields independent of S^3 directions, $A_\alpha \equiv 0$, $\alpha = 5, 6, 7$.
- Derive $3+1$ dim. effective action for these modes from 5d eff. action derived from $D7$-brane DBI action:
  $$S_{5d,\text{eff}} = -\kappa \int d^4x \int d\tilde{z}\ \text{tr}\left[\frac{1}{2} C(\tilde{z})\eta^{\mu\lambda}\eta^{\nu\rho}F_{\lambda\rho}F_{\mu\nu} + M_*^2 D(\tilde{z})\eta^{\mu\nu}F_{\mu\tilde{z}}F_{\nu\tilde{z}}\right].$$

Kuperstein-Sonnenschein model

Pions in the KS model.

- Different gauge: $A_\mu(x, \tilde{z}) = U^{-1}(x^\mu)\partial_\mu U(x^\mu)\psi_+(\tilde{z})$,
  $U(x^\mu) = \exp(i\Pi(x)/f_\pi)$, $\psi_+ = \frac{1}{2}(1+\psi_0)$.
- Skyrme action:

$$S_{\text{Skyrme}} = \int d^4x \left(\frac{f_\pi^2}{4}\text{tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32 e_S^2}\text{tr}\left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2 \right)$$

- The pion decay constant f_π ($\sim$ 93 MeV experimentally)

$$f_\pi^2 = 4\kappa M_*^2 \int d\tilde{z}\, D(\tilde{z})(\partial_{\tilde{z}}\psi_+)^2 = \frac{4\Gamma(3/4)}{\sqrt{\pi}\Gamma(1/4)}\kappa M_*^2 \approx 0.76\kappa M_*^2,$$

- Dimensionless parameter

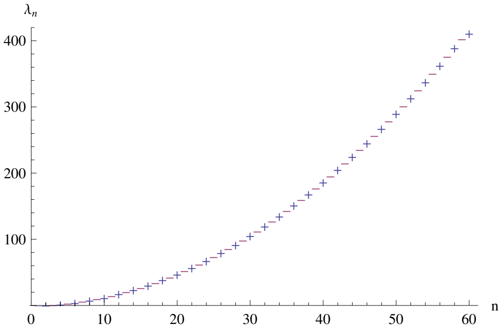
$$e_S^{-2} = 32\kappa \int d\tilde{z} \frac{1}{2} C(\tilde{z})\psi_+^2(\psi_+ - 1)^2 \approx 3.93\kappa.$$

- Choose $\kappa = 4.87 \times 10^{-3}$ to match f_π.

Kuperstein-Sonnenschein model

Spectrum of (axial-) vector mesons.

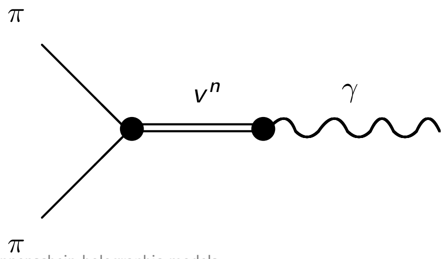
- Solve numerically EOM for wavefunctions: $-(C(\tilde{z}))^{-1}\partial_{\tilde{z}}(D(\tilde{z})\partial_{\tilde{z}}\psi_n(\tilde{z})) = \lambda_n \psi_n(\tilde{z})$.
- Masses given by $\lambda_n = M_n^2/M_*^2$, where $M_*(r_0) = 1527$ MeV in order to match $M_1 = M_\rho = 775$ MeV.



Kuperstein-Sonnenschein model

Pion (and vector meson) form factors.

- Can numerically compute decay and coupling constants, e.g.,
  $g_{v^n} = \kappa M_{v^n}^2 \int_{-\infty}^{+\infty} d\tilde{z} C(\tilde{z}) \psi_{2n-1}(\tilde{z})$,
  $g_{v^n\pi\pi} = \kappa \frac{M_{v^n}^2}{2f_\pi^2} \int_{-\infty}^{+\infty} d\tilde{z}\, C(\tilde{z}) \psi_{2n-1}(\tilde{z})(1-\psi_0^2)$,
- $\mathcal{V}\pi\pi$ coupling vanishes, leading to vector meson dominance in the pion form factor.
- Corresponding Feynman diagram:

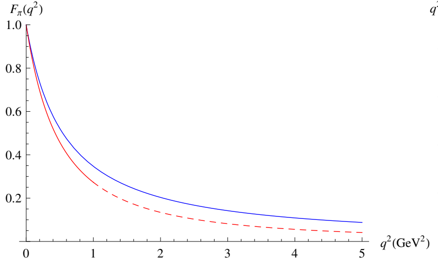


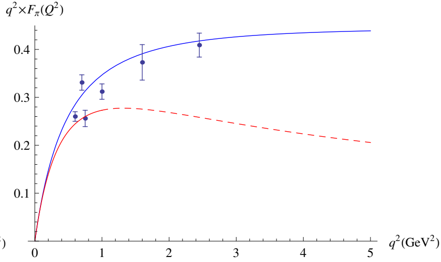
Kuperstein-Sonnenschein model

Pion form factor.

- Pion form factor: $F_\pi(q^2) = \sum_{n=1}^{\infty} \frac{g_{v^n} g_{v^n \pi\pi}}{q^2 + M_{v^n}^2}$.
- $F_\pi(0) = \sum_{n=1}^{\infty} g_{v^n} g_{v^n \pi\pi} = 1$, same sum rule that implies vanishing of $\mathcal{V}\pi\pi$ coupling.
- large q^2: $F_\pi(q^2) = q^{-2} \sum_{n=1}^{\infty} g_{v^n} g_{v^n \pi\pi} \left(1 - \frac{M_{v^n}^2}{q^2} + \mathcal{O}(q^{-4})\right)$.
- Compare to QCD results and Sakai-Sugimoto model.

[Jefferson F(pi) Collaboration, nucl-ex/0607005,0607007]

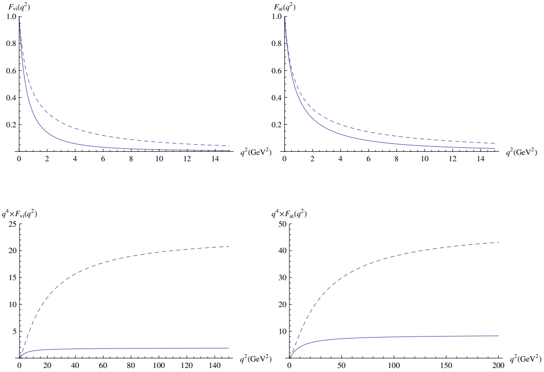




Kuperstein-Sonnenschein model

(Axial-) vector meson form factors.

- Similar results hold for the (axial-) vector meson form factors.
- Replace external pions with (axial-) vector meson lines in Feynman diagram; large q^2 behavior $\sim q^{-4}$.



Dymarsky-Kuperstein-Sonnenschein model

[Dymarsky, Kuperstein, Sonnenschein 0904.0988]

Non-conformal generalization based on Klebanov-Strassler model (deformed conifold), ISD bg.

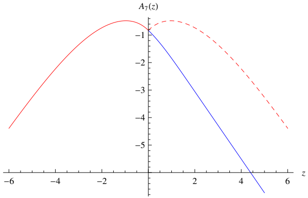
- D3-brane geometry, warped product $ds^2_{10} = h^{-1/2} dx_\mu dx^\mu + h^{1/2} ds^2_6$, CY3 manifold Y^6.
- const. dilaton $e^\Phi = g_s$, selfdual RR 5-form $\widetilde{F}_5 = (1 + \star_{10}) dC_4$.
- ISD condition $G_3 = i \star_6 G_3$, $G_3 = F_3 + i e^{-\Phi} H_3$.
- KS solution: N D3-branes and M fractional D3-branes. Fluxes $\int_{T^{1,1}} F_5 = N$, $\int_{S^3} F_3 = M$,
  dual gauge theory: $\mathcal{N} = 1$ $SU(N+M) \times SU(N)$, running couplings, Seiberg duality cascade.
- Difficult to find classical D7-brane embedding.
  Exception: SUSY
  - Internal 4-cycle Σ_4 holomorphic.
  - Gauge invariant field strength $\mathcal{F}_0 = P[B_2] + 2\pi\alpha' F$ anti-selfdual ASD.

Dymarsky-Kuperstein-Sonnenschein model

[Sakai, Sonnenschein hep-th/0305049, Dymarsky, Kuperstein, Sonnenschein 0904.0988]

Construction of non-BPS solution.

- ISD background: non-BPS (anti-) $D7$-brane solutions via 1) minimizing the volume of the embedding (subject to some boundary conditions incompatible with holomorphicity) and 2) finding an(y) ASD (SD) background gauge field $\mathcal{F}_0 = \pm \star_4 \mathcal{F}_0$.
- emb.: $\theta_1 = \frac{\pi}{2}$, $\phi_1 = \phi_1(\tau)$, ASD/SD gauge fields: $A^{(0)}_{5,6,7}(\tau)$.
- study extremal case $\tau_0 = 0$: ϕ_1= const., $A^{(0)}_5 = A^{(0)}_6 = 0$



Mesons in the DKS model

[Ihl, Torres, Boschi-Filho, Ballon-Bayona 1010.0993]

As before, can study vector and scalar mesons.

- For $\tau_0 = 0$, can introduce $z = \tau \sin\phi_1 = \pm\tau$, $y = \tau\cos\phi_1 = 0$.
- Normalization: $\kappa \int dz C(z)\psi_n(z)\psi_m(z) = \delta_{mn}$.
- EOM: $\psi_n''(z) + \frac{D'(z)}{D(z)}\psi_n'(z) + \frac{C(z)}{D(z)}\lambda_n\psi_n(z) = 0,$.
- But now coefficient functions more complicated:

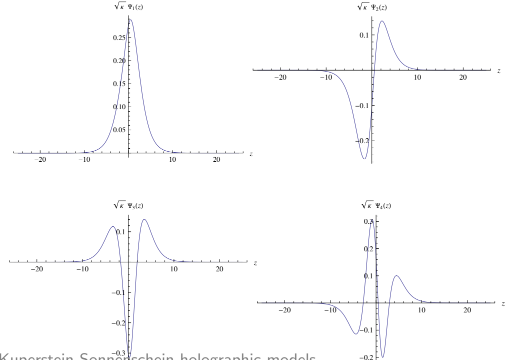
$$\frac{D'(z)}{D(z)} = \frac{3(|z|\coth z - 1) - \sinh^2|z|}{3(\sinh|z|\cosh z - |z|)} + \tanh|z|,$$

$$\frac{C(z)}{D(z)} = \frac{h(|z|)}{K^2(|z|)} + \frac{2}{3}\frac{(A_7(z) - b(|z|))^2}{K(|z|)^4\cosh^2 z}.$$

Results for vector mesons.

- Use shooting technique: Frobenius expansion for large z, exponential fall-off $\psi_n(z) \sim e^{-\frac{2}{3}|z|}$.

n	1	2	3	4	5	6
λ_{2n-1}	0.1310	0.4785	1.1081	2.0267	3.2238	4.6948
λ_{2n}	0.2582	0.7785	1.5552	2.6004	3.9217	5.5214



Mesons in the DKS model

Results for vector mesons.

- $M_n^2 = \lambda_n M_*^2$, four-dimensional vector mesons masses.
- Can match $v^{(1)}(x^\mu)$ with $\rho(770)$ meson $\Longrightarrow M_* = 2141$MeV.
- $k = 1$ corresponds to $a^{(1)} \sim a_1(1260)$, $k = 2$ to $v^{(2)} \sim \rho(1450)$, etc.

k	$\left(\frac{\lambda_{k+1}}{\lambda_1}\right)_{DKS}$	$\left(\frac{\lambda_{k+1}}{\lambda_1}\right)_{KS}$	$\left(\frac{\lambda_{k+1}}{\lambda_1}\right)_{SS}$	$\left(\frac{\lambda_{k+1}}{\lambda_1}\right)_{exp.}$
1	1.97	2.68	2.4	~ 2.51
2	3.65	5.36	4.3	~ 3.56

Mesons in the DKS model

Results for scalar mesons.

- Two indep. 5d scalar fluctuation modes: $\delta\theta_1/\delta\phi_1$ and a_7, that give rise to scalar mesons in the dual FT.

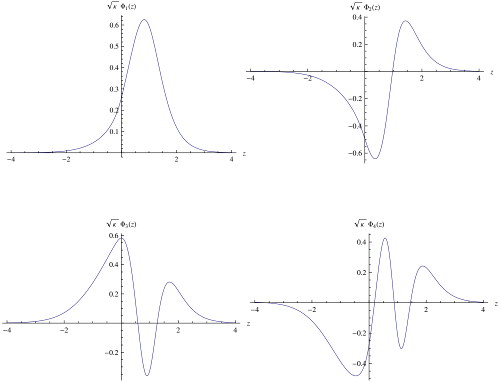
n	1	2	3	4	5	6
$\widetilde{\lambda}_n = \frac{\widetilde{M}_n^2}{M_*^2}$	0.1690	0.2889	0.4167	0.6186	0.8177	1.0837

n	1	2	3	4	5	6
$\overline{\lambda}_n = \frac{\overline{M}_n^2}{M_*^2}$	0.093	0.300	0.544	0.812	1.136	1.553

Mesons in the DKS model

Results for scalar mesons.

- Similar results: Parity broken due to presence of non-trivial A_7.
- Wavefunctions fall off as $e^{-\frac{4}{3}|z|}$ (or even faster).



Mesons in the DKS model

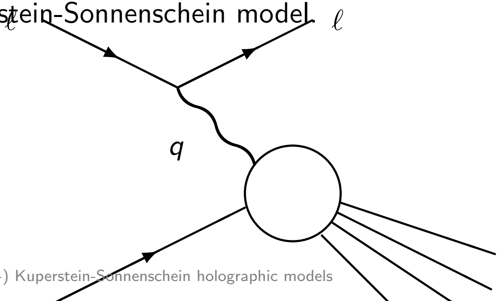
Discussion of results.

- Parity breaking in z-direction: Implications for 4D physics? Violation of charge conjugation?
- Parity restored for $|z| \to \infty$, due to $A_7 \to -\sqrt{\frac{3}{2}}|z|$.
  Can identify axial-vector/ vector and pseudo-scalar/scalar mesons.
- First example where the lightest scalar meson is lighter than the lightest vector meson.
  $\Longrightarrow$ Potentially interesting implications for (large N_c) holographic nuclear physics: [Kaplunovsky, Sonnenschein 1003.2621]
  In the far zone: $V_{\text{att.}} \propto -\frac{\exp(-rM_{sc.})}{r}$, $V_{\text{rep.}} \propto +\frac{\exp(-rM_{vec.})}{r}$,
  Sakai-Sugimoto: $M_{\text{light.sc.}} \sim 2M_{\text{light.vec.}}$.

(Dymarsky-) Kuperstein- Sonnenschein model

Ongoing research.

- Holographic baryons/nucleons in DKS model:
  - Sakai-Sugimoto: λ-dep. flavor physics, baryon size $\rho \sim \lambda^{-1/2} \ll 1/M_{meson}$, baryons as small ADHM instantons.
  - DKS: 5D κ is λ-indep., baryon size $\rho \sim 1/M_{meson}$, but much more complicated.
- Drell-Yan process (dilepton production in p-p scattering) in KS model.
- Deep inelastic scattering of pions and mesons in the Kuperstein-Sonnenschein model.



(Dymarsky-) Kuperstein- Sonnenschein model

Conclusions and Outlook.

- Some meson (and pion) physics of (D)KS models seems to reproduce QCD results well.
- Need to study more phenomenology of (Dymarsky-) Kuperstein-Sonnenschein models, Ongoing research.
  - $\tau_0 > 0$, $\tau_0 \to \infty$.
  - S-parameter, holographic (walking) technicolor? [Mintakevich, Sonnenschein 0905.3284]
  - finite temperature? [Mia, Dasgupta, Gale, Jeon 0902.1540]
- Identify/classify other (type IIB) models of flavor chiral symmetry breaking and their phenomenology, hopefully without asymmetric background gauge fields.