We claimed that the locally integrable function
\[ E(x, t) = \begin{cases} (4\pi t)^{-n/2} \exp(-|x|^2/4t), & t > 0, \\ 0, & t \leq 0, \end{cases} \]
is a fundamental solution to the Heat Operator \( P(D) = \partial_t - \Delta_x \) in the coordinates \((x, t) \in \mathbb{R}^n \times \mathbb{R}\). Since \( E \) is locally integrable it certainly defines an element of \( \mathcal{D}'(\mathbb{R}^{n+1}) \). In addition, a routine computation shows that for each \( t > 0 \)
\[ \frac{\partial E}{\partial t} - \Delta_x E = 0. \] (\( \star \))

According to the definition of the distributional derivative
\[ \langle P(D)E, \varphi \rangle = \langle E, P(-D)\varphi \rangle = -\int_0^\infty \left( \int E(x, t)(\varphi_t + \Delta_x \varphi) \, dx \right) \, dt \]
Since \( E \) is locally integrable, we can write the latter integral as
\[ -\lim_{\epsilon \to 0} \int_\epsilon^\infty \left( \int E(x, t)(\varphi_t + \Delta_x \varphi) \, dx \right) \, dt, \]
where the limit is taken from above. A note for intuition: we did this so we could integrate over a region \( t \geq \epsilon > 0 \) in which \( P(D)E = 0 \). On integrating by parts we see that
\[ \int_\epsilon^\infty \left( \int E(x, t)(\varphi_t + \Delta_x \varphi) \, dx \right) \, dt = \int_\epsilon^\infty \int \partial_t(E \varphi) \, dx \, dt - \int_\epsilon^\infty \left( \int \varphi(\partial_t - \Delta_x E) \, dx \right) \, dt. \]
The latter integral vanishes, from the observation in (\( \star \)), and the former integral is
\[ -\int E(x, \epsilon)\varphi(x, \epsilon) \, dx \]
by the fundamental theorem of calculus. In summary
\[ \langle P(D)E, \varphi \rangle = \lim_{\epsilon \to 0} \int E(x, \epsilon)\varphi(x, \epsilon) \, dx = \lim_{\epsilon \to 0} \frac{1}{(4\pi \epsilon)^{n/2}} \int e^{-|x|^2/4\epsilon} \varphi(x, \epsilon) \, dx. \]
On making the substitution \( x = 2\sqrt{\epsilon} y \) and applying the dominated convergence theorem, we find \( \langle P(D)E, \varphi \rangle = \varphi(0, 0) \) for each \( \varphi \in \mathcal{D}(\mathbb{R}^{n+1}) \). Hence \( P(D)E = \delta_0 \), i.e. \( E \) is a fundamental solution to the Heat operator, as claimed.